The results obtained above for the kinetic energy of the plasma particles indicate a rapid reestablishment of the conduction current, as observed in the rapid rise time (~1 nsec) of the current curve after a dip.

The exact position of the source of the hard-x-ray emission has not been exactly determined by experiment. The electrons accelerated in the strong electric field either will interact with the minute plasma itself in a collective manner[10] or will hit the anode. Either way makes it possible to produce hard-x-ray radiation.

The treatment discussed here will also be correct in the case when the conduction current is not cut off completely. The essential features obtained from the model discussed here will not be altered significantly even if we consider more difficult, realistic configurations.

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[10] Collisional interactions are negligible since the mean free paths for binary collisions are too long compared with the size of the minute plasma. However, the Debye length is 2.5×10^{-2} μm which is much smaller than the plasma size, and also the electron plasma frequency is of the order of 10^{15}/sec. This indicates that the collective interactions of the particles and plasma are most probable.

Penetration of Slow Waves into a Dense Plasma Using a Phased Wave-Guide Array*

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We report experimental evidence of wave coupling and penetration to the interior of a high-density magnetoplasma using a double-wave-guide antenna as a slow-wave structure. Reflection coefficients as low as 4% were observed. The experiment used 9-cm microwaves with a plasma of 11-cm diam. The relevance to lower-hybrid heating of tokamak fusion plasmas is discussed.

It is becoming clear that Ohmic heating alone will probably not be sufficient to increase tokamak plasmas to ignition temperatures in a thermonuclear reactor.[1,2] Supplementary heating of toroidal plasmas through the application of rf power is an especially interesting approach since, below frequencies of about 3 GHz, there are available today sources of cw power that are sufficient to heat even the largest proposed toroidal devices. In this regime of available power one of the most promising characteristic plasma frequencies for heating is the lower-hybrid frequency. Today's toroidal devides are of such dimension that the free-space wavelength corresponding to the lower-hybrid frequency is comparable with the plasma cross section. Thus the intriguing possibility arises that one may couple the rf energy by means of open-ended wave-guide apertures in the vacuum vessel wall[3,4] rather than by an internal coil structure, which is characteristic of lower-frequency regimes.

The basic questions concerning any rf-heating technique are wave penetration and dissipation mechanisms. This paper deals exclusively with the problem of penetration, that is, the coupling interaction that takes place from the electromagnetic wave in the wave guide to the essentially electrostatic wave in the plasma that must be generated for heating.

Considerable literature exists concerning the theory of wave penetration into the dense interior of a magnetized plasma. Several authors[5-7] have shown that the lower-hybrid-resonance layer is accessible only for waves that have a sufficiently large component of refractive index parallel to the magnetic field, i.e.,

$$n_\parallel = k_\parallel c/\omega \geqslant (1+\omega_{peo}^2/\omega_{ce}^2)^{1/2}. \qquad (1)$$

A possible way to obtain waves with the proper values of $n_\parallel$ is with an array of phased wave guides. Such a system is being suggested for use on the Joint European Tokamak[8] and a similar system is planned for the ATC Tokamak at Princeton. Recently[9] there has been much controversy over the nature of wave-guide systems for achieving penetration. This controversy arises in part because a dramatic change in the mode structure must occur in the narrow low-density region around $\omega_{pe} \approx \omega$ and simple theoretical methods of analysis cannot be used. Once launched inside the plasma, the wave is refracted by the density gradient. The wave vector $\vec{k}$, which is parallel to the magnetic field just inside the $\omega_{pe}=\omega$ layer, becomes outward radial when the wave reaches the resonant layer (neglecting warm-plasma and nonlinear effects). However, because of the strong anisotropy, the ray (group-velocity) trajectory is initially inward radial but refracts to become parallel to the magnetic field. If the accessibility condidtion (1) is not met, or if the wave frequency is above the mean gyrofrequency $(\omega_{ci}\omega_{ce})^{1/2}$, the ray penetrates to a certain maximum density and then returns toward the plasma edge. It is well known that a single wave guide, oriented with the rf electric field parallel to the confining magnetic field, generates the ordinary electromagnetic mode, which is reflected at the $\omega_{pe}=\omega$ layer. However, by phasing the wave guides and penetrating the outer ($\omega_{pe}<\omega$) layer of the plasma by tunneling, one should generate the plasma wave that undergoes the lower-hybrid resonance in sufficiently dense plasmas. In this paper we experimentally test these concepts concerning the penetration of the $\omega_{pe}<\omega$ layer, and we are not concerned directly with the lower-hybrid resonance as such. We work with $\omega \gg \omega_{LH}$, the frequency being chosen such that the free-space wavelength is comparable with the plasma diameter (as in the tokamak plasmas) and large compared with the thickness of the $\omega_{pe}<\omega$ layer.

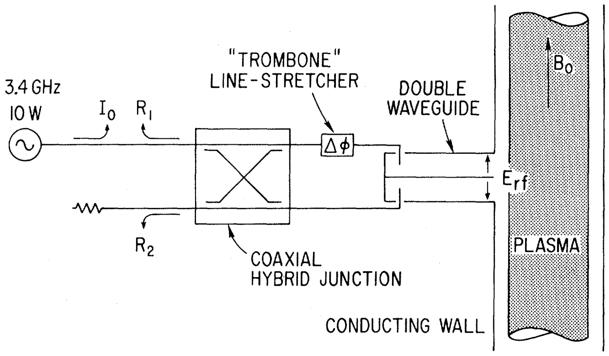

FIG. 1. Circuit of microwave excitation of double-wave-guide antenna. Transmitted and reflected signals are monitored by directional couplers I_0, R_1, and R_2.

Our antenna consists of a standard open-ended WR-229 (2.29 in.×1.14 in.) wave guide that has been divided by a central partition parallel to the broad wall. The two guides are driven in the usual TE_{10} mode at a frequency of 3.42 GHz. The relative aperture phase $\Delta\varphi$ is controlled by a line stretcher, as shown in Fig. 1. Signals reflected from the open ends of the wave guides are monitored by two directional couplers R_1 and R_2. The Fourier spectrum of the nominal aperture illumination is easily calculated. When the wave guides are driven in phase ($\Delta\varphi=0$), the spectrum peaks at $n_\parallel=0$ and, as in a single wave guide, the radiation is emitted mainly in the forward direction. When the wave guides are driven out of phase ($\Delta\varphi=180°$), the spectrum peaks at $|n_\parallel|=2.2$ and has a null at $n_\parallel=0$. The far-field free-space radiation pattern is derived from only that portion of the Fourier spectrum for which $|n_\parallel|<1$. Consequently when $\Delta\varphi=0$ the apertures are efficient radiators in the forward direction, but when $\Delta\varphi=180°$ they are inefficient radiators into free space, the residual radiation being nearly perpendicular to the antenna axis. This effect can also be understood in terms of interference between waves that are reflected internally from each open-ended wave guide and those that are cross-coupled between wave guides. The phases of these return waves are such that they destructively interfere for $\Delta\varphi = 0$ and constructively interfere for $\Delta\varphi = 180°$.

The antenna was mounted flush with the cylindrical conducting liner (11.4 cm diam) of the H-1 linear device[10] and was oriented such that the electric field of the TE_{10} wave in the wave guide was parallel to the static magnetic field (~ 7 kG). The plasma was prepared in argon gas by a 600-μsec, 7-kW, rf pulse; the electron density ($\sim 10^{12}$ cm^{-3}) was measured by an 8-mm microwave interferometer. A probe, which was movable in both axial and radial dimensions, was used both for Langmuir density-profile measurements and to sample the wave signal in the plasma. The wave amplitude was observed by a square-law video detector, and the phase was obtained by mixing the probe signal with a fixed reference signal and observing the resulting interference.

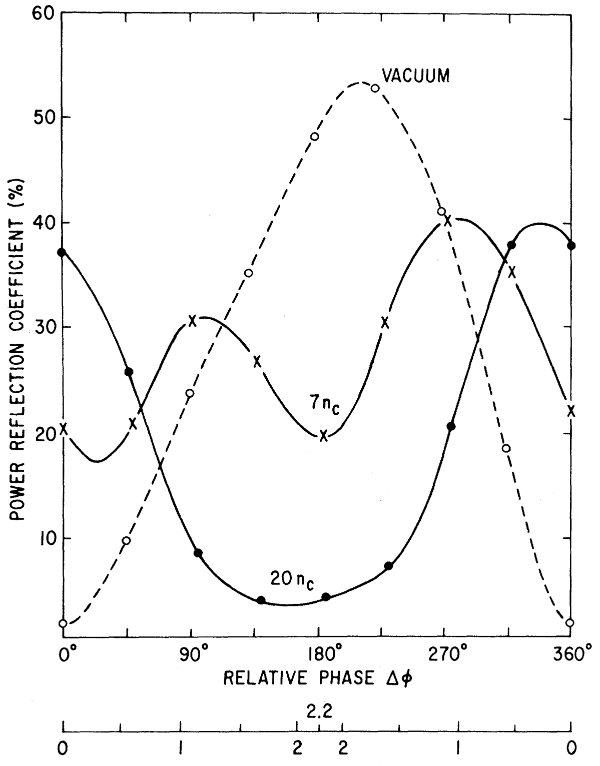

FIG. 2. Power-reflection coefficient as a function of relative aperture phase. The parameter is the peak electron density normalized to the critical density for the operating frequency, $n_c = \epsilon_0 m_e \omega^2/e^2$. The lower abscissa scale shows the peak values of the $n_{\parallel}$ Fourier spectrum calculated from the nominal aperture fields.

Figure 2 shows the time-averaged reflection coefficients of the antenna as a function of relative aperture phase for different plasma densities. In the absence of a plasma the reflection coefficient is about 3% when the two apertures are driven in phase ($\Delta\varphi = 0$) and is a maximum of 54% when $\Delta\varphi \approx 180°$.

For a succession of plasmas of increasing densities, the reflection coefficient increases when $\Delta\varphi = 0$, but decreases when $\Delta\varphi = 180°$, the minimum reflection coefficient observed being about 4%. The results can be understood as follows. Ordinary waves with $|n_{\parallel}| < 1$ can propagate only in densities lower than the critical density, $n_c = \epsilon_0 m_e \omega^2/e^2$, and undergo reflection at the depth where $\omega_{pe} = \omega$. However, waves with $|n_{\parallel}| > 1$ are evanescent in densities less than critical but can propagate in higher densities provided that mode conversion takes place. Thus, as the peak density of the plasma increases, more reflection is observed for waves with $|n_{\parallel}| < 1$ ($\Delta\varphi = 0$) since the critical layer moves closer to the antenna, while less reflection is observed for waves with $|n_{\parallel}| > 1$ ($\Delta\varphi = 180°$) as the thickness of the evanescent layer decreases and tunneling to the propagating region occurs. Consistent results have been obtained when the antenna was withdrawn small distances from the chamber wall, thereby artificially increasing the distance between the wave guides and the $\omega_{pe} = \omega$ layer.

Figure 3(b) presents radial scans of the wave amplitude at a succession of axial positions for $\Delta\varphi = 180°$, showing the progress of a wave packet from the mouth of the wave guides into the plasma, for the density profile of Fig. 3(a). At these densities the wave packet characteristically had a well-defined leading edge and there was no evidence of surface wave at the plasma boundary once the wave packet had progressed into the plasma. We infer that, for $\Delta\varphi = 180°$, a large percentage of the energy radiated from the antenna was coupled into plasma waves.

Axial scans with the signals fed into an interferometer permitted the axial refractive index $n_{\parallel}$ to be measured and the wave fronts to be mapped, as shown in Fig. 3(c). The measured $n_{\parallel}$ in the interior of the plasma is approximately 2.1 when $\Delta\varphi = 180°$. The experimental results for the trajectory of the wave packet are consistent with computer calculations [also shown in Fig. 3(c)] of the ray trajectory of the slow wave derived from WKB methods using the experimentally mea-

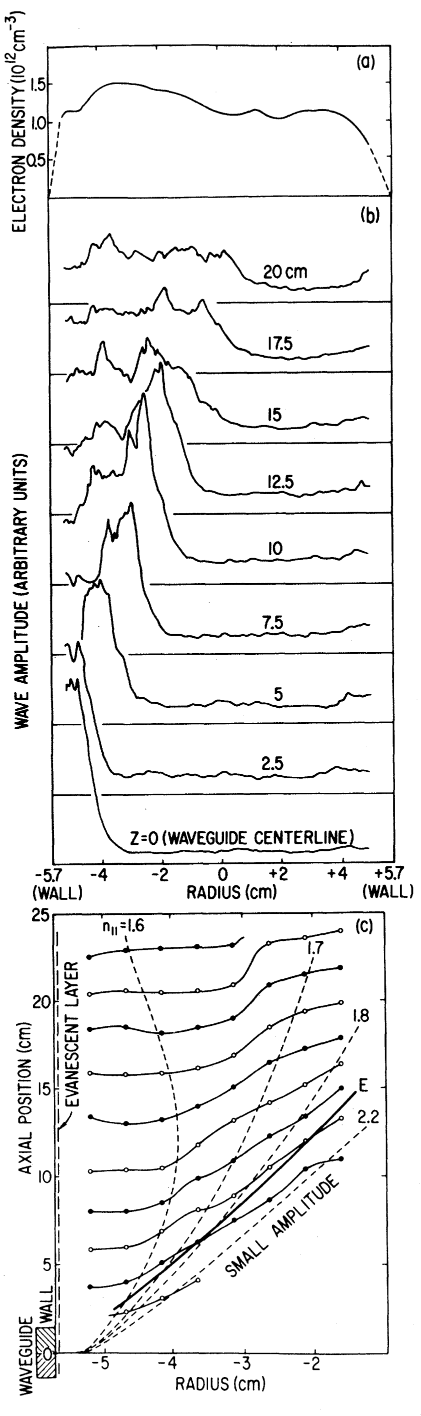

FIG. 3. (a) Electron density as a function of radius, $n_{\text{max}} = 10n_c$. (b) Observed wave magnitude as function of radius in successive planes downstream from the antenna ($z = 0$); $\Delta\varphi = 180°$, $\omega_{ce}/\omega = 5.7$. (c) Experimental points defining contours of constant phase (phase difference between contours is 180°). Heavy solid curve E, locus of observed maximum wave amplitude. Dashed curves, ray trajectories computed from WKB theory for various $n_\parallel$ values. Note the change in radial scale from (b).

sured electron-density profile. The wave normals, constructed from the experimental phase data, are found to be inclined at an angle of 85° with respect to the group velocity, again consistent with the WKB results which predict an angle of 80° for $n_\parallel = 2.1$ and our conditions. The computations also show that the total refractive index is $n \approx 5$ and that the ratio of longitudinal to transverse electric fields is ~ 7, thus implying that the wave is essentially electrostatic. Quantitative differences between the observed and calculated angles and trajectories are probably due to the mathematical model which assumes a one-dimensional plasma slab, as well as the limitations of WKB theory when the dimensions are comparable with the wavelength.

Variation of the aperture phase $\Delta\varphi$ affected the amplitude of the probe signal, in rough agreement with the measured antenna reflection coefficients, but had little systematic influence on the shape of amplitude scans similar to those in Fig. 3(b). The leading edge of the wave packet follows the so-called resonance cone,[3,11] corresponding to the ray trajectory in the limit $n_\parallel \to \infty$. As expected, both the trajectory of the leading edge and the value of $n_\parallel$ in the peak-density region down-stream from the leading edge depended upon the experimental electron-density profile. For instance, at a lower density ($\sim 3n_c$) the measured index was $n_\parallel \approx 1.5$. The thickness of the $\omega_{pe} < \omega$ layer (~ 1 cm) in this case was sufficient to reduce the coupling by tunneling, and there was evidence of a surface wave in addition to the penetrating wave packet.

In tokamak experiments the thickness of the evanescent layer ($\omega_{pe} < \omega$) will be smaller than in the present experiment. The total refractive index, however, will rapidly reach values much larger than those encountered in this experiment. We may thus expect more stringent coupling conditions between the wave guides and the plasma interior, and some tailoring of the electron-density profile near the plasma edge may be required. Nevertheless, it is quite clear from the present work that the wave-plasma-surface interaction is such that a phased array of wave guides can be highly effective in achieving penetration into a dense plasma.

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Effective Mass and *g* Factor of Interacting Electrons in the Surface Inversion Layer of Silicon*

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The effective mass and the *g* factor of quasiparticles near the Fermi surface of a two-dimensional electron gas are calculated in the random-phase approximation and in the Hubbard approximation, and are compared with the experimental results for an inversion layer on a (100) surface of silicon.

When a sufficiently strong electric field is applied normal to the surface, the electrons in the inversion layer of a metal-insulator-semiconductor (MIS) structure form an essentially two-dimensional interacting electron gas.[1] Because the electron concentration can be experimentally varied over a wide range, this system is a useful testing ground for approximate methods of calculating the effect of many-body correlations on measurable properties of the system. The g factor and the effective mass m^* of quasiparticles near the Fermi surface are two such properties. They have been measured over a wide range of concentrations for a (100) surface inversion layer of silicon by Fang and Stiles,[1] and by Smith and Stiles.[2] The measured values of both quantities are considerably larger than their counterparts in intrinsic bulk silicon, and they increase as the electron density is lowered. Janak,[3] and later Suzuki and Kawamoto,[4] evaluated the enhancement of the g value and of the effective mass caused by electron correlations within the framework of a static approximation which neglects the frequency dependence of the dielectric function. Chaplik[5] has calculated the effective mass by using an approximate dielectric function. Though some of the results obtained by these authors are in qualitative agreement with experiment, the many-body approximations employed are equivalent to the simplest approximations in three-dimensional systems.[6] Since such approximations are incapable of yielding quantitative results, these calculations cannot be used as a test of the most refined many-body approximations, nor of the assumption built into the two-dimensional model of a semiconducting inversion layer. In the present paper we calculate m^* and g^* by using the dynamic random-phase approximation (RPA) and the Hubbard approximation (HA). The HA and slight modifications of it are considered the most accurate many-body approximations for a three-dimensional electron gas. Both the RPA and the HA are discussed clearly by Rice,[7] and the reader is referred to his paper for general background.

The total energy of the interacting electron