Mechanism for the Hard–X-Ray Emission in Vacuum Spark Discharges

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A possible mechanism is discussed for the hard–x-ray emission observed in vacuum spark plasmas. The mechanism is based on the hypothesis that in the process of a sausage instability in a plasma pinch the plasma can have very high resistivity due to the constriction and strong turbulence, so that the conduction current is virtually cut off. In such a case strong electric fields should appear which could accelerate the electrons and ions to produce hard x rays as energetic as 20 times the discharge potential. The kinetic energies of ions and electrons are calculated.

The observation of hard–x-ray emissions in a linear discharge is an old one. In the early experiments on thermonuclear fusion a number of groups[1] reported observing hard–x-ray emission in deuterium linear discharges, usually accompanied with neutron production.[2] Recent experiments[3] show that a vacuum spark discharge produces a series of x-ray emissions with spectra whose energy increases as the current grows. One interesting fact is that the quanta of the hard x ray which comes out at near the peak value of the current have energies of several hundred keV at a discharge voltage of 10 to 20 kV. There must be some acceleration mechanism for the charged particles to gain the high kinetic energy necessary for producing the hard–x-ray radiations.

However, the mechanism of this physical process has not been well established. The dynamical behavior of the charged particles involved in the pinched plasma is complex.[4] A possible mechanism may be due to a quasistationary turbulent electric field arising during changes of the magnetic field near a sausage instability in the pinch. This was discussed by several authors[5]; however, the particle kinetic energies estimated by their theory do not agree with the observed experimental values.

In this report we shall propose a different mechanism to account for the hard–x-ray emission by taking into consideration the experimental facts. We shall pay particular attention to the evidence that hard–x-ray emission coincides in time with abrupt changes in the discharge current. A summary of the experimental results is given in the following.

A typical experimental apparatus[3] used for linear discharges consists of a pair of electrodes, a cathode and an anode of dimensions 3 cm and 0.7 cm, respectively, with a separation of 1 cm. The cathode is made of a tungsten alloy and the anode is made of iron. The pressure in the discharge tube is of the order of 10^{-6} Torr. The maximum current is approximately 200 kA and the gap potential is 10 to 20 kV.

Among the results which those experiments established, the following are relevant to the present discussion:

(1) In the early stage, a plasma is formed on the surface of the anode. The plasma constituents are highly ionized, high-Z atoms of the anode material, electrons, and some neutrals. The plasma moves toward the cathode.

(2) As the current grows, the plasma is constricted because of a sausage instability and plasmas of minute size (15–50 μm in radius and elongated somewhat axially) are produced.[6]

(3) The estimated values, obtained by pressure-balance calculations, for the strongly pinched minute plasmas are as follows: The electron density may be as high as 10^{21} cm^{-3}, the temperature T_e is ~ 10 keV, and the azimuthal magnetic field B is of the order of 2×10^7 G.[3]

(4) Such minute plasmas emit short bursts of intense x radiation which consist of a continuum spectrum, as well as line spectra arising from the ions. The hard–x-ray component often has energy greater than 500 keV.[3]

(5) The rising current reaches a peak value of approximately 200 kA in a time interval of 2 to 4 μsec. The current curve shows several abrupt changes: a sharp dip followed by a sharp rise within a time interval on the order of nanoseconds. The detected x-ray emission roughly coincides in time with the abrupt changes in the current and have, typically, a pulse half-width of about 20 nsec.[3,6] The hard–x-ray emissions are found in the later stages.

(6) The sharp rise of current after a dip in (5) is believed to be due to short bursts of accelerat-

ed iron ions moving with velocities about 5×10^7 cm/sec (kinetic energy of 80 keV) and of electrons.[3]

The experimental observation that the discharge current drops sharply in a time duration on the order of nanoseconds, roughly coinciding with the hard-x-ray emission, indicates that the conductivity of the plasma is momentarily decreased by a large amount. A plausible reason for this may be attributed to the anomalous resistivity in a turbulent plasma which arises in the presence of electrostatic instabilities, such as ion-acoustic or two-stream instabilities.[7,8] In order to determine which microinstability is responsible, or whether some other mechanism is more likely, further investigations both experimentally and theoretically are needed. However, in this report we conjecture that the plasma resistivity is greatly increased by some of the effects discussed above and we will focus our discussion on what will happen when the conduction current is momentarily drastically reduced or virtually cut off, as observed in the experiments.

A rough estimation will give us a quick judgment whether the above idea is reasonable or not. Consider the Maxwell equation

$$\nabla\times\vec{\mathrm{B}}=\mu_0\vec{\mathrm{J}}+\mu_0\epsilon_0\,\partial\vec{\mathrm{E}}/\partial t. \tag{1}$$

The second term on the right-hand side is the displacement current which is negligible before the conduction current is cut off. At the current cutoff we have $\vec{\mathrm{J}}=0$, and then the magnetic field in space will induce the electric field

$$\vec{\mathrm{E}}(t)=(\mu_0\epsilon_0)^{-1}\int\nabla\times\vec{\mathrm{B}}\,dt, \tag{2}$$

or

$$\vec{\mathrm{E}}_z(t)\sim(\mu_0\epsilon_0)^{-1}\int(B/a)\,dt\sim(\mu_0\epsilon_0)^{-1}(ct/a)B, \tag{3}$$

where a is the plasma radius and t is the characteristic time for the plasma constituents. The magnetic field is evaluated to be $B=\mu_0I_0/2\pi a\simeq2\times10^7$ G ($a=10$ μm) which gives the right order of magnitude found in the experiments. The characteristic time will be on the order of less than a nanosecond, as the duration of the current dip indicates. Let us assume, very roughly, $ct/a\sim10^4$; then E_z is approximately 10^{14} V/cm which is clearly strong enough to accelerate the charged particles and produce the observed hard x radiation.[9] We note here that the time dependence of B in Eq. (3) was ignored for this estimation, and that the actual value of $E_z(t)$ will be much less since B decreases rapidly after the current cutoff.

In order to make the *actual* problem mathematically tractable, we shall approximate the problem as follows. We assume that the minute plasma is formed by the sausage instability in the pinch. This case may be idealized as shown in Fig. 1. The minute plasma is sandwiched between two conducting plasmas which are assumed to be infinite in extent. The minute plasma in the preturbulent stage is a good conductor and a current I_0 is flowing through it, with a current density $J_0(r)$ over the cross section of the constricted plasma.

The magnetic field created by this current is given by

$$B(r)=(\mu_0/2\pi r)\int_0^r J_0(r)2\pi r\,dr. \tag{4}$$

The relatively weak fields can be neglected in the preturbulent stage. Let us assume now that at the instant $t=0$, the conduction current is cut off. Then, the magnetic field distributed in space according to Eq. (4) will excite a displacement current exactly equal to the conduction current which existed before cutoff. This means that at $t=0$

$$\epsilon_0\left.\frac{\partial E(r,t)}{\partial t}\right|_{t=0}=J_0(r), \tag{5}$$

and that at a later time t the electric field is given by the two-dimensional wave equation

$$\frac{\partial^2E}{\partial x^2}+\frac{\partial^2E}{\partial y^2}-\frac{1}{c^2}\frac{\partial^2E}{\partial t^2}=0. \tag{6}$$

Thus, the electric field, arising in the conduction-current break, is obtained as the solution

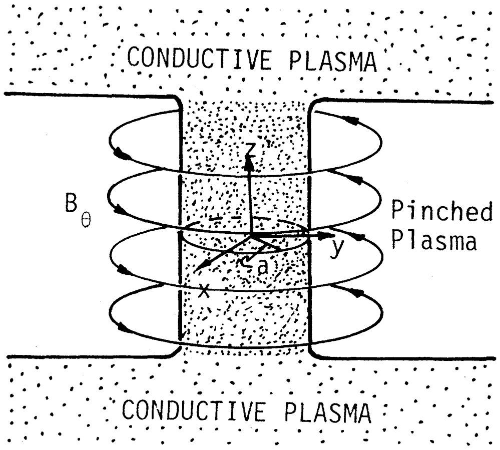

FIG. 1. An idealized model for pinched plasma ($m=0$, sausage instability). a is the radius of the plasma and B_θ is the azimuthal magnetic field strength due to the current I_0.

of Eq. (6) with the initial conditions given by Eq. (5) and $E(x, y, t)|_{t=0}=0$. By applying Fourier transforms and taking proper contours for integration, we obtain the solution

$$E(x, y, t) = \frac{1}{2\pi\epsilon_0 c}\int_0^{ct} \frac{r\,dr}{[(ct)^2 - r^2]^{1/2}} \int_0^{2\pi} J_0(x', y')\,d\theta, \qquad (7)$$

where $x' = x + r\cos\theta$ and $y' = y + r\sin\theta$.

We assume here that the initial distribution of current density has the form

$$J_0 = \begin{cases} I_0/\pi a^2, & r < a, \\ 0, & r > a. \end{cases} \qquad (8)$$

Then, with introduction of a dimensionless quantity $\tau = ct/a$, the electric field on the axis is found to be

$$E(0, t) = \begin{cases} (I_0/\pi\epsilon_0 ca)\tau \text{ when } \tau < 1, \\ (I_0/\pi\epsilon_0 ca)[\tau - (\tau^2 - 1)^{1/2}] \text{ when } \tau > 1. \end{cases} \qquad (9)$$

In this field the charged particles can be accelerated.

Using the formula

$$d(mv)/dt = eE, \qquad (10)$$

where m is the mass of the particle (it must be the relativistic mass when the velocity is relativistic), and assuming that the particle was at rest when $t=0$; we obtain, for $\tau > 1$,

$$v(t) = (\mu_0 I_0/2\pi m)\{\tau^2 - \tau(\tau^2 - 1)^{1/2} + \ln[\tau + (\tau^2 - 1)^{1/2}]\}, \qquad (11)$$

and in the nonrelativistic case

$$z(t) = (\mu_0 I_0/2\pi m)\{\tau \ln[\tau + (\tau^2 - 1)]^{1/2} + \tfrac{1}{3}[\tau^3 - (\tau^2 - 1)^{3/2}] - (\tau^2 - 1)^{1/2}\}. \qquad (12)$$

For $\tau \gg 1$ we have, approximately,

$$v(t) = (\mu_0 I_0/2\pi m)[\ln(2\tau) + \tfrac{1}{2}] \qquad (13$$

and

$$z(t) = (\mu_0 I_0/2\pi m)(a/c)\tau[\ln(2\tau) - \tfrac{1}{2}]. \qquad (14)$$

We are now ready to estimate the kinetic energies of the charged particles accelerated in the electric field. First we consider the acceleration of ions. We shall let the conduction current just before the cutoff be 100 kA. Taking the mass of an ion to be that of the iron atom, M, we obtain for the coefficient in the above equations

$$\mu_0 I_0/2\pi M \cong 3.4\times 10^4 \text{ m/sec.}$$

From this value and Eq. (14) we can determine the factor τ by assuming $z = 100$ μm (the approximate length of acceleration) and $a = 10$ μm (the plasma radius). We obtain a value of $\tau \simeq 9\times 10^3$, and the velocity v_{ion} and kinetic energy $\mathcal{E}_{\text{ion}}$ of iron ions become

$$v_{\text{ion}} \simeq 3.5\times 10^5 \text{ m/sec}$$

and

$$\mathcal{E}_{\text{ion}} \simeq 40 \text{ keV},$$

which are in good agreement with the observed values.[3]

For electrons, the coefficient becomes

$$\mu_0 I_0/2\pi m_0 \simeq 3.5\times 10^9 \text{ m/sec.}$$

This suggests that the calculation for the electrons must be done by using the relativistic treatment. Equations (11) and (13) still hold for the electrons with the relativistic mass $m = m_0/(1 - v^2/c^2)^{1/2}$. The equation corresponding to Eq. (14) will be found as

$$z(t) = \int_0^t dt \frac{v^*[\ln(2\tau) + \frac{1}{2}]}{\{1 + (v^{*2}/c^2)[\ln(2\tau) + \frac{1}{2}]^2\}^{1/2}}, \qquad (15)$$

where $\tau = ct/a$ and $v^* = \mu_0 I_0/2\pi m_0$. Since $v^* \gg c$, we obtain approximately

$$z(t) \cong ct.$$

With the same dimensions of the plasma as above we find the factor $\tau = 10$ and the kinetic energy of accelerated electrons

$$\mathcal{E}_e = \frac{m_0 c^2}{(1 - v^2/c^2)^{1/2}} - m_0 c^2 \simeq 40 m_0 c^2 \simeq 20 \text{ MeV.}$$

This is the maximum kinetic energy attainable for electrons. Actually the kinetic energy of the electrons in the plasma will vary depending on their location. Furthermore, the x rays are produced by these fast electrons when they interact with the plasma itself or with the anode material. This calculation shows that the x-ray quanta can have energies up to 20 MeV. Reports of evidence for 0.3–1-MeV x-ray quanta are consistent with the above calculations.[1,3]

The results obtained above for the kinetic energy of the plasma particles indicate a rapid reestablishment of the conduction current, as observed in the rapid rise time (~1 nsec) of the current curve after a dip.

The exact position of the source of the hard-x-ray emission has not been exactly determined by experiment. The electrons accelerated in the strong electric field either will interact with the minute plasma itself in a collective manner[10] or will hit the anode. Either way makes it possible to produce hard-x-ray radiation.

The treatment discussed here will also be correct in the case when the conduction current is not cut off completely. The essential features obtained from the model discussed here will not be altered significantly even if we consider more difficult, realistic configurations.

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[10] Collisional interactions are negligible since the mean free paths for binary collisions are too long compared with the size of the minute plasma. However, the Debye length is 2.5×10^{-2} μm which is much smaller than the plasma size, and also the electron plasma frequency is of the order of 10^{15}/sec. This indicates that the collective interactions of the particles and plasma are most probable.

Penetration of Slow Waves into a Dense Plasma Using a Phased Wave-Guide Array*

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We report experimental evidence of wave coupling and penetration to the interior of a high-density magnetoplasma using a double-wave-guide antenna as a slow-wave structure. Reflection coefficients as low as 4% were observed. The experiment used 9-cm microwaves with a plasma of 11-cm diam. The relevance to lower-hybrid heating of tokamak fusion plasmas is discussed.

It is becoming clear that Ohmic heating alone will probably not be sufficient to increase tokamak plasmas to ignition temperatures in a thermonuclear reactor.[1,2] Supplementary heating of toroidal plasmas through the application of rf power is an especially interesting approach since, below frequencies of about 3 GHz, there are available today sources of cw power that are sufficient to heat even the largest proposed toroidal devices. In this regime of available power one of the most promising characteristic plasma frequencies for heating is the lower-hybrid frequency. Today's toroidal devides are of such dimension that the free-space wavelength corresponding to the low-