dicates the proven value of the quark model in resonance classification. The generally dichotomous situation here is of course essentially that envisaged for quarks by Gell-Mann,[1] that one may approach the problem either in a purely dynamical bootstrap model or in a model based on fundamental entities.

Finally, since the quark model is equally as successful in baryon classification as in meson classification, one may hope that eventually dual-resonance models for baryons will reach the stage where one can as strongly demonstrate the coincidence of dual- and quark-model predictions for baryonic spectra, as one has in the present paper for the mesonic case.

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[1]M. Gell-Mann, Phys. Lett. 8, 214 (1964).

[2]G. Zweig, CERN Reports No. TH 401 and No. TH 402, 1964 (unpublished).

[3]For a recent review of the status of the quark model, see J. L. Rosner, Phys. Rep. 11C, 189 (1974).

[4]For detailed agreement one may add phenomenological spin and spin-orbit terms to the quark-model mass operator; see R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966* (Univ. of California Press, Berkeley, Calif., 1967), p. 215.

[5]A. Neveu and J. H. Schwarz, Nucl. Phys. B31, 86 (1971); A. Neveu, J. H. Schwarz, and C. B. Thorn, Phys. Lett. 35B, 529 (1971).

[6]See Neveu, Schwarz, and Thorn, Ref. 5.

[7]P. H. Frampton, Phys. Rev. D 9, 487 (1974).

[8]Note that in the A_6 amplitude, Eq. (62) of Ref. 7, only six of fifteen different terms give nonspurious contributions at $\alpha_\pi = 1$ when $a = 0$. This presumably ensures that in the present Eq. (6) the new 1^{+-} state which couples with normal strength for $a = 1 - \alpha_\rho(0) = \frac{1}{2}$ is nonspurious.

[9]It is worth remarking that M. Gell-Mann (private communication) suggests that one should consider the internal and external states of the dual-resonance model as mass-degenerate SU(3) nonets. We differ from Gell-Mann's philosophy in expecting that SU(3) mass breaking and physical intercepts $\alpha_\rho(0) = \frac{1}{2}$, $\alpha_\pi(0) = 0$ and $\alpha_{K^*(890)}(0) = \frac{1}{4}$, $\alpha_K(0) = -\frac{1}{4}$ should be already in the Born amplitude. For this reason, in the present dual-model analysis we consider directly only the nonstrange isospin multiplets contained within the SU(3) classification.

Quantum Dynamics of the Relativistic String

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The equations characterizing the orthogonal gauge do not restrict the spatial degrees of freedom. This observation leads to Poincaré invariance in $s+1$ dimensional space for any s. The absence of a tachyonic ground state and of ghosts is derived from algebraic consistency.

The literature on the quantum dynamics of the relativistic string has until now been beset by the difficulty that it permitted Poincaré invariant solutions only in a critical number of space-time dimensions larger than four.[1] The source of this difficulty has been the dependence of some of the generators of the Lorentz group on the operators L_n of the Virasoro algebra. This dependence, in turn, resulted from the elimination of one of the spacelike degrees of freedom by means of the orthogonal gauge conditions. The null plane coordinates were used for this purpose. It is therefore suspect.[2]

The quantum dynamics of the string will be formulated here in a Poincaré invariant but not manifestly covariant way. This permits a separation of the internal dynamics from the trivial uniform motion of the free string as a whole.

Algebraic restrictions are imposed on the Poincaré generators by the desire to have an algebra of operators which includes the Poincaré generators P^μ and $M^{\mu\nu}$ as well as a canonical (center-of-mass) position operator of the string at $\tau = 0$, $\bar{X}^\mu$. This algebra is supposed to contain the Poincaré algebra as a subalgebra, as well as the canonical algebra of $\bar{X}^\mu$ and P^μ. Because of the

dependence of $M^{\mu\nu}$ on $\bar{X}^\mu$, it is well known[3] that $\bar{X}^\mu$ cannot be a four-vector because $\bar{X}^0$ is a Casimir operator of this algebra. Thus, *the canonical algebra is necessarily noncovariant,* viz.[4]

$$[\bar{X}^\mu, \bar{X}^\nu]=0, \quad [P^\mu, P^\nu]=0, \quad [\bar{X}^\mu, P^\nu]=ig^{\mu\nu}-i(P^\mu/P^0)g^{\nu 0}. \tag{1}$$

It takes account of P^0 as the generator of time translations, $t=\bar{X}^0$.

The equations of motion and boundary conditions of the string are satisfied by the solution

$$x^\mu(\sigma,\tau)=\bar{X}^\mu+\frac{P^\mu\tau}{l}+\frac{i}{\sqrt{\pi}}\sum_{n\neq 0}\frac{\alpha_n{}^\mu}{n}\cos\sigma_n\exp(-i\tau_n). \tag{2}$$

The string is assumed to have length l, $\sigma_n=n\pi\sigma/l$, $\tau_n=n\pi\tau/l$, and $\alpha_{-n}{}^\mu=(\alpha_n{}^\mu)^\dagger$. The quantities $\alpha_0{}^\mu$ are not defined and occur nowhere. From this one obtains $x'\equiv\partial x/\partial\sigma$ and $\dot{x}\equiv\partial x/\partial\tau=p$. Capital letters will indicate the integrated quantities:

$$X^\mu\equiv\int_0^l x^\mu d\sigma, \quad P^\mu\equiv\int_0^l p^\mu d\sigma; \tag{3}$$

the bar indicates the average: $\bar{X}\equiv X/l$, $\bar{P}\equiv P/l$.

The canonical equal-time commutation relations for the space components of the fields ($k, l=1, 2, \ldots, s$ in $s+1$ dimensional space-time)

$$[x^k(\sigma,\tau), x^l(\sigma',\tau)]=0, \quad [p^k(\sigma,\tau), p^l(\sigma',\tau)]=0, \quad [x^k(\sigma,\tau), p^l(\sigma',\tau)]=i\delta^{kl}\delta(\sigma-\sigma') \tag{4}$$

can be satisfied provided the internal modes are independent of the center-of-mass kinematics,

$$[\alpha_n{}^k, X^\mu]=0, \quad [\alpha_n{}^k, P^l]=0, \tag{5}$$

and satisfy the standard commutation relations with one another,

$$[\alpha_m{}^k, \alpha_n{}^l]=m\delta^{kl}\delta_{m+n,0}. \tag{6}$$

The Lorentz generators now become

$$M^{\mu\nu}=:\int_0^l d\sigma(\tfrac{1}{2}\{x^\mu,p^\nu\}-\tfrac{1}{2}\{x^\nu,p^\mu\}):=\tfrac{1}{2}\{\bar{X}^\mu,p^\nu\}-\tfrac{1}{2}\{\bar{X}^\nu,p^\mu\}+\tfrac{1}{2}i\sum_{n\neq 0}n^{-1}:\{\alpha_n{}^\mu,\alpha_{-n}{}^\nu\}:. \tag{7}$$

They do *not* depend on the Virasoro operators

$$\Lambda_n\equiv\tfrac{1}{2}\sum_{0\neq m\neq n}\alpha_m\cdot\alpha_{n-m}, \tag{8}$$

but they do require the knowledge of the commutation relations of the $\alpha_n{}^0$ with the other operators.

Now physically the $\alpha_n{}^k$ in (7) describe a spin angular momentum of the string as a whole,

$$S^{kl}\equiv i\sum_{n\neq 0}n^{-1}\alpha_n{}^k\alpha_{-n}{}^l, \quad k\neq l. \tag{9}$$

If the $\alpha_n{}^0$ were independent of $\vec{P}$ this would mean that the string at rest is endowed with internal structure which involves not only the axial vector $\vec{S}$ ($S^k\equiv\frac{1}{2}\epsilon^{klm}S^{lm}$), but also a polar vector $\vec{S}'$ with components

$$S'^k\equiv\tfrac{1}{2}\sum_{n\neq 0}n^{-1}\{\alpha_n{}^0,\alpha_{-n}{}^k\}.$$

If the string were charged this would mean a static electric dipole moment in addition to a magnetic moment associated with $\vec{S}$. Thus the $\alpha_n{}^0$ must depend on the $\vec{P}$ in such a way that $\vec{S}'$ vanishes in the rest system. One finds

$$\alpha_n{}^0=\vec{\alpha}_n\cdot\vec{P}(P^0+M)^{-1}, \quad n>0. \tag{10}$$

The derivation of this result involves a comparison of the canonical representation of the Lorentz generators

$$\vec{J}=\vec{X}\times\vec{P}+\vec{S}, \tag{11}$$

$$\vec{N}=\bar{X}^0\vec{P}-\tfrac{1}{2}\{\vec{\bar{X}},P^0\}+(\vec{S}\times\vec{P})/(P^0+M) \tag{12}$$

with the expression (7) where $J^k=\frac{1}{2}\epsilon^{klm}M^{lm}$ and $M^{0k}=N^k$.

The Poincaré generators consist of the Lorentz generators, (11) and (12), and the four-vector $P^\mu\equiv((\vec{P}^2+M^2)^{1/2},\vec{P})$. The commutators of all ten of these are known from the above equations once the commutators involving the operator M are known. To determine the operator M we must use the orthogonal gauge conditions. The orthogonal gauge is characterized by the two equations

$$p\cdot x'+x'\cdot p=0, \tag{13}$$

$$p^2+x'^2=0. \tag{14}$$

Equation (13) is satisfied provided (classically)

$$G_n \equiv \Lambda_n + (1/2\sqrt{\pi})(\alpha_n \cdot p + p \cdot \alpha_n) = 0, \quad n \neq 0, \quad (15)$$

while Eq. (14) requires also

$$M^2 \equiv -P_\mu P^\mu = 2\pi\Lambda_0. \quad (16)$$

At this point the following important observations must be made.

(a) The string model is crucially dependent on the mathematical implementation of a relativistic position operator, as is evident from the algebra, as well as from (2). If $\bar{X}^k$ is the Newton-Wigner position operator for the localizability of the center of mass, it singles out a preferred coordinate system: A relativistic system can be localized at any one time only relative to *one* Lorentz frame. It is most reasonable to assume this frame to be the frame in which the system as a whole is at rest, the center-of-mass frame.

(b) As is evident from (1) the canonical representation of the Poincaré algebra yields an X^μ which is *not* a four-vector. Neither are α^μ, $x^\mu(\sigma, \tau)$, or $p^\mu(\sigma, \tau)$. But $P^{\mu\nu}$ and $M^{\mu\nu}$ *are* covariant by construction [using (10)]. The gauge conditions (13) and (14) are therefore *not* Lorentz form invariant. Moreover, one can verify explicitly that the G_n of (15) do not commute with the boost operator $\vec{N}$. If the gauge operators G_n vanish in one frame, then $U(\Lambda)G_nU^{-1}(\Lambda)$ will also vanish, i.e., they will vanish in all frames. This is the classical case. The quantum case, however, permits only $G_n|\rangle = 0$, i.e., a gauge constraint on the states. In that case $G_n|\rangle = 0$ does not imply $UG_nU^{-1}|\rangle = 0$, unless G_n commutes with U. Therefore, in quantum dynamics these constraints hold in the form $G_n|\rangle_{\text{c.m.}} = 0$, where $|\rangle_{\text{c.m.}}$ is a center-of-mass state. With respect to any other Lorentz frame $|\rangle_\Lambda \equiv U(\Lambda)|\rangle_{\text{c.m.}}$ the corresponding constraint operator will be $G_n{}^\Lambda \equiv U(\Lambda) \times G_nU^{-1}(\Lambda)$ so that $G_n{}^\Lambda|\rangle_\Lambda = U(\Lambda)G_n|\rangle_{\text{c.m.}} = 0$ also. It then follows that the $\alpha_n \cdot p$ terms of (15) never contribute to the constraints.

(c) The condition (13) leads to a normal ordered operator, but (14) does not. In fact, the vacuum expectation value of Λ_0, (8), is infinite. The quantum mechanical interpretation of (14) can therefore be made only within an undetermined additive multiple of the identity.

With this is mind (15) and (16) become

$$L_n = 0, \quad n \neq 0, \quad (15')$$

and

$$M^2 = 2\pi L_0 + m_0{}^2, \quad (16')$$

where

$$L_n = \tfrac{1}{2} \sum_{0 \neq m \neq n} : \vec{\alpha}_m \cdot \vec{\alpha}_{n-m} : .$$

While (16′) thus provides us with the mass operator, (15′) cannot be satisfied as an operator relation as mentioned above. It can hold only as a condition on the state vectors,

$$L_n|\varphi\rangle = 0, \quad n > 0. \quad (17)$$

These vectors are necessarily the c.m. system description. The corresponding matrix elements are

$$x_{\text{c.m.}}{}^\mu = \left[t + \langle M \rangle \frac{\tau}{l}, \frac{i}{\sqrt{\pi}} \sum_{n \neq 0} \frac{1}{n} \langle \vec{\alpha}_n \rangle \cos\sigma_n \exp(-i\tau_n) \right]$$

$$p_{\text{c.m.}}{}^\mu = \left[\frac{\langle M \rangle}{l}, \frac{\sqrt{\pi}}{l} \sum_{n \neq 0} \langle \vec{\alpha}_n \rangle \cos\sigma_n \exp(-i\tau_n) \right]. \quad (18)$$

The string can now be completely solved in the c.m. system. The constant can always be chosen so as to avoid a tachyonic ground state or any zero-mass excited state.[5] The condition (17) restricts the total Fock space to a physical subspace Φ on which the Poincaré algebra in $3+1$ dimensions (or $s+1$ dimensions) is consistently realized. This follows from the fact that on Φ, L_n commutes with $\vec{S}$, $\vec{P}$, $\vec{X}$, and L_n ($n \geq 0$). The multiplets in Φ can be constructed explicitly from the Fock representations of the spherical harmonics $Y_s{}^{M_s}$ confirming the SO(3) invariance. For $\vec{\alpha}$ with s components (9) describes the (integer) spin operator in that dimensionality and analogous statements hold.

Finally, one can define a Hamiltonian operator which yields the desired equations of motion for the $\vec{x}(\sigma, \tau)$ and $\vec{p}(\sigma, \tau)$,

$$H \equiv \int_0^l d\sigma \tfrac{1}{2}(\vec{p}^2 + \vec{x}'^2) = (1/2l)(\vec{P}^2 + 2\pi L_0). \quad (19)$$

Thus, $\pi L_0/l$ is the Hamiltonian for internal motion. Because of (16′)

$$H = (1/2l)P_0{}^2. \quad (20)$$

One verifies the consistency which requires

$$[\alpha_m{}^k, L_n] = m\alpha_{m+n}{}^k. \quad (21)$$

A full report will be published elsewhere.

I am indebted to Professor Paul Frampton for valuable discussions.

[1]P. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. B56, 109 (1973), and older literature quoted therein.

[2]A recent paper by A. Patrascioiu, Lett. Nuovo Cimento 10, 676 (1974), seems to point in the same direction.

[3]See, e.g., J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970); L. Foldy, Phys. Rev. 102, 568 (1954).

[4]Our metric $g^{\mu\nu}$ has diagonal elements $(-1,+1,+1,+1)$.

[5]No state (including the ground state) can have zero eigenvalue of the mass operator M^2 because this would contradict the existence of a position operator which we have assumed all along. In contradistinction to the usual theory our first excited state has therefore three triplet states rather than only two. All the other allowed states are identical with the usual string theory.

COMMENTS

Activation Energy Spectrum in Myoglobin—a Comment*

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Carbon monoxide rebinding to myoglobin after photodissociation is nonexponential below 150 K; we explain it as being due to an activation-energy spectrum. Fang has criticized our analysis as being ambiguous; we show here that his arguments do not invalidate our conclusion.

We have recently presented evidence for an activation-energy spectrum in the rebinding of carbon monoxide to myoglobin after a photoflash.[1] Fang[2] has criticized our treatment, claiming that (1) two discrete activation energies can explain our data and (2) the spectrum can have resulted from an implicit assumption in our analysis. In the present comment we answer Fang's criticism.

We fit our rebinding data by a function $H(t) = (1+t/t_0)^{-n}$, where t_0 and n are temperature-dependent parameters. We interpret this nonexponential behavior by assuming the existence of an energy spectrum $g(E)$ so that

$$H(t) = \int dE\, g(E) \exp(-\lambda t), \qquad (1)$$

where the rate λ depends on temperature T and activation energy E through the Arrhenius relation, $\lambda(T) = A\exp(-E/kT)$. Now to the criticism.

(1) Fang interprets our fit, Eq. (1), by introducing Arrhenius relations for t_0 and n, thus using only two activation energies instead of a spectrum. The Arrhenius relation, however, has a physical meaning only for rates; applying it to the dimensionless parameter n without justification is meaningless.

(2) Fang remarks that Eq. (1) implies a first-order reaction, but that $H(t)$ can also be explained by assuming a higher order and a different spectral function $g'(E)$. This remark is correct, but the assumption of a first-order reaction is justified by the fact that one myoglobin molecule binds one carbon monoxide molecule and by our observation that the reaction rate is independent of ligand-molecule concentration.

It is certainly not excluded that an alternate explanation for the nonexponential rebinding can be found, but we feel that Fang does not propose a viable substitute for our activation-energy spectrum.

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[1]R. H. Austin, K. Beeson, L. Eisenstein, H. Frauenfelder, I. C. Gunsalus, and V. P. Marshall, Phys. Rev. Lett. 32, 403 (1974).

[2]P. H. Fang, Phys. Rev. Lett. 33, 1515 (1974).