[12]E. Nelson, in *Partial Differential Equations, Proceedings of the Twenty-Third Symposium in Pure Mathematics*, edited by D. Spencer (American Mathematical Society, Providence, R. I., 1973).

[13]S. Albeverio and R. Høegh-Krohn, Commun. Math. Phys. 30, 171 (1973).

[14]C. Deutsch and M. Lavand, Phys. Rev. A 9, 2598 (1974).

[15]F. Guerra, L. Rosen, and B. Simon, "Euclidean $P(\Phi)_2$ Field Theory. . ." (to be published).

[16]J. Fröhlich, "Schwinger Functions and Their Generating Functionals, I, II" (to be published).

Charged-Particle Multiplicity in π^--Nucleus Interactions at 100 and 175 GeV/c*

W. Busza, J. E. Elias,† D. F. Jacobs, P. A. Swartz, and C. C. Young
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

M. R. Sogard
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 6 January 1975)

The space-time evolution of particle production at high energies was investigated by measuring the charged multiplicity in π^--nucleus collisions at 100 and 175 GeV/c. We find that (a) the forward multiplicity (in the πp center-of-mass system) is independent of the target nucleus; (b) the backward multiplicity is approximately proportional to the nuclear thickness; and (c) the data are consistent with the assumption that the absorption cross section of the incident particle characterizes the multiplication process.

Experiments on multiparticle production using nucleons as targets give data only on the asymptotic states produced and thus cannot yield direct information on the space-time development of the production process. To probe the early evolution of the production, it is necessary to interact with the process in its initial stages of development. Reasonable estimates[1] indicate that in the 100-GeV energy range the formation of multiparticle states may take place over distances of many femtometers in the laboratory frame of reference, i.e., in distances greater than the mean free path of hadrons in nuclear matter. Thus targets of complex nuclei seem to offer a viable method for studying the creation process.

We have studied the development of hadronic showers inside nuclear matter by measuring the multiplicity of charged relativistic particles as a function of angle and nuclear size in π^--nucleus collisions. The experiment was carried out in the M6 beam line at the Fermi National Accelerator Laboratory. Details of the experimental method and of the analysis of the data will be published elsewhere.[2]

The target was surrounded by twelve Lucite hodoscopes forming a truncated cone with its axis along the beam line. Directly downstream of the target, at the narrow end of the cone, a high-resolution Lucite Cherenkov counter was placed. Relativistic charged particles ($\beta \gtrsim 0.85$) produced at large angles were counted in the hodoscopes, while those at small angles were counted through the pulse height in the forward Cherenkov counter. The distance from the target to the forward counter determined the angular ranges covered by the two sets of counters.

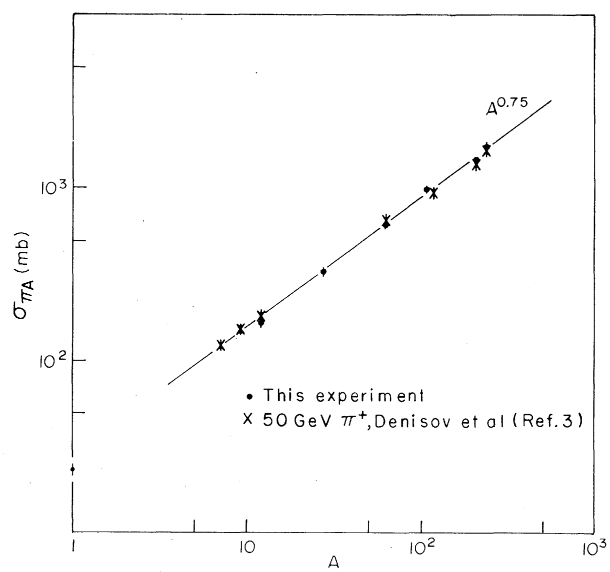

FIG. 1. πA absorption cross sections.

Data were collected at both 100 and 175 GeV/c for C, CH_2, Al, Cu, Ag, Pb, and U targets. Hydrogen results were obtained from the CH_2-C difference. We have data for various angular regions: 0–3.5°, 0–26°, and 26–110°. The laboratory angles of 3.5 and 26° correspond to laboratory pseudorapidities $\eta = -\ln\tan\theta_{lab}/2$ of 3.5 and 1.5. The corresponding angles in the πp center-of-mass system for a $\beta = 1$ particle are 48 and 149° at 100 GeV/c and 61 and 156° at 175 GeV/c.

Figure 1 shows our values of the absorption cross sections and those of Denisov *et al.*[3] The two are in good agreement and follow approximately an $A^{0.75}$ power law predicted by a simple optical-model calculation in which a Woods-Saxon distribution of nucleons is assumed.

Our values for $\langle n\rangle_H$, the average charged multiplicity of relativistic ($\beta \gtrsim 0.85$) particles in inelastic $\pi^- p$ collisions, are 6.5 ± 0.4 at 100 GeV/c and 7.7 ± 0.5 at 175 GeV/c. Hydrogen-bubble-chamber measurements[4] are 6.80 ± 0.14 at 100 GeV/c, 7.34 ± 0.10 at 147 GeV/c, and 8.02 ± 0.12 at 205 GeV/c for *all* charged secondaries.

Figure 2 shows the variation with nuclear size of the observed relative multiplicity $\langle n\rangle_A/\langle n\rangle_H$ for four angular regions. The nuclear "size" is measured in terms of $\bar{\nu}$, the average number of absorption mean free paths encountered by the incident particle in going through a nucleus. It is given by $\bar{\nu} = A\sigma_{hN}/\sigma_{hA}$, where A is the atomic weight, and σ_{hN} and σ_{hA} are the absorption cross sections of the incident hadron on a nucleon and nucleus, respectively.[5] For σ_{hN} and σ_{hA} we used the average of our results and those of Refs. 3 and 4.

The most striking result is that there is no increase in the number of particles produced for $\theta_{lab} \leq 3.5°$, while at large angles the multiplicity is approximately proportional to nuclear thickness. A pion encounters on the average 3 mean free paths in a uranium nucleus and yet in the forward cone ($\theta_{lab} \leq 3.5°$), which contains almost half the produced particles for πp interactions, the multiplicity in a π-uranium collision is the same as that in a πp collision. These results support the old observation from cosmic-ray physics[6] and the more recent emulsion measure-

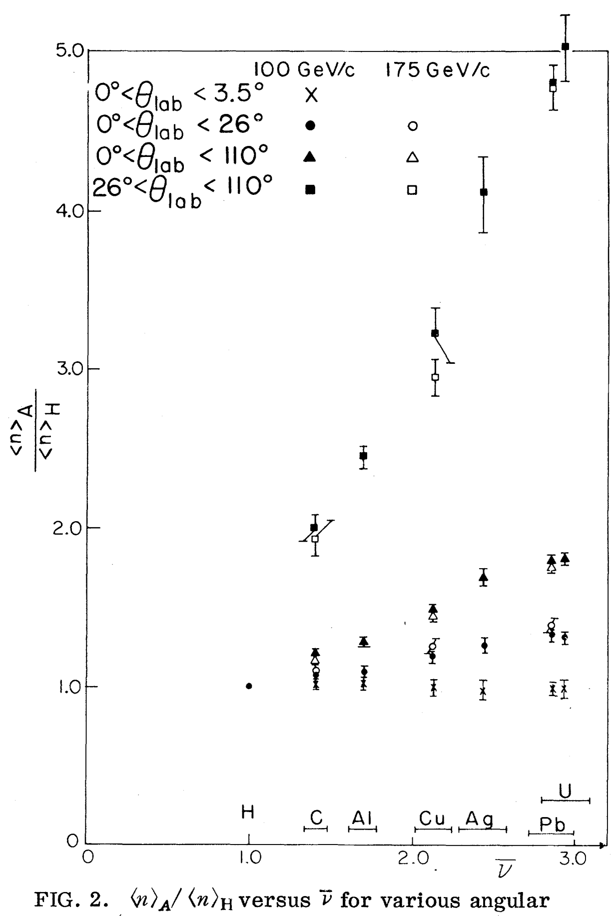

FIG. 2. $\langle n\rangle_A/\langle n\rangle_H$ versus $\bar{\nu}$ for various angular ranges. $\langle n\rangle_A$ is the average charged multiplicity of relativistic particles produced in an inelastic collision with nucleus A, and $\bar{\nu}$ is the average number of absorption mean free paths encountered in the nucleus. Errors on $\bar{\nu}$ are shown at the bottom of the graph. The errors on $\langle n\rangle_A/\langle n\rangle_H$ are statistical only.

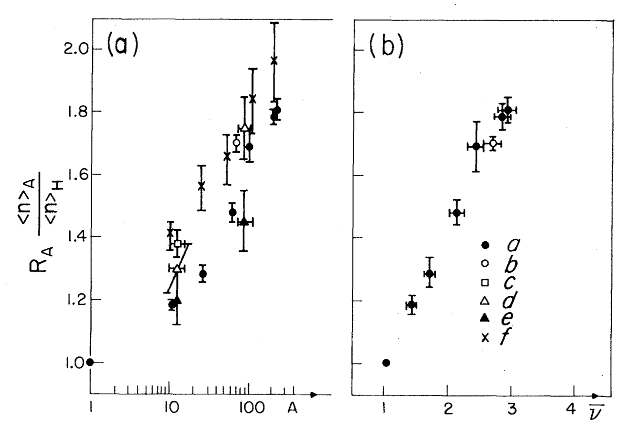

FIG. 3. (a) R_A versus A. Errors on all measurements are statistical only. (b) R_A versus $\bar{\nu}$. Errors include all systematic and statistical uncertainties. The data for both (a) and (b) are as follows: a, this experiment, average of 100 and 175 GeV π^-; b, world average of p-emulsion at 200 GeV (Ref. 7); c, p-emulsion (light elements) at 200 GeV (Ref. 8); d, p-emulsion at 69 GeV (Refs. 9 and 10); e, π^--emulsion at 60 GeV (Ref. 10); f, cosmic-ray data of Vishwanath *et al.* (Ref. 6), consisting of 30% pions and 70% protons at 80 to 500 GeV.

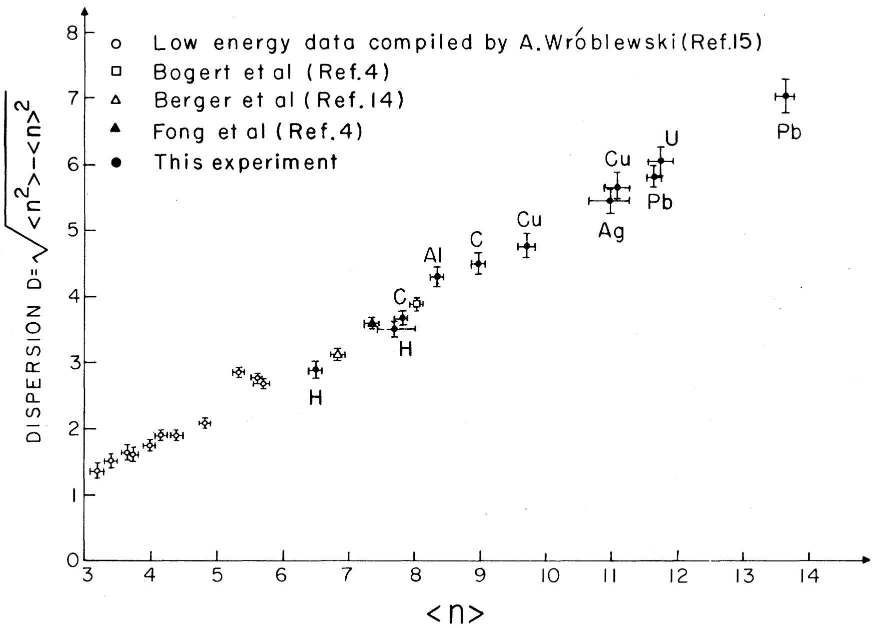

FIG. 4. Dispersion $D=[\langle n^2\rangle - \langle n\rangle^2]^{1/2}$ versus $\langle n\rangle$.

ments[7] at Fermilab that the average multiplicity grows slowly with the atomic weight of the target, that the rise is primarily in the target fragmentation region, and that $R_A = \langle n\rangle_A/\langle n\rangle_H$ (all angles) is independent of energy.

The dependence of the multiplication process on the nature of the incident particle is shown in Fig. 3, where various measurements of R_A are plotted (a) versus the atomic weight A and (b) versus $\bar{\nu}$. In Fig. 3(b) the only data shown are those for which the identity of both the incident particle and the target nucleus are well defined, i.e., for which $\bar{\nu}$ is known. In Fig. 3(a), for a given value of A the value of R_A for incident pions is smaller than that for incident protons. On the other hand, Fig. 3(b) shows that for pions and protons the results are similar when plotted as a function of $\bar{\nu}$. This comparison suggests the striking conclusion that the relevant parameter which describes the multiplication process is the absorption cross section of the incident particle, and not that of the secondaries.

Finally, in Fig. 4 we show the dispersion $D = [\langle n^2\rangle - \langle n\rangle^2]^{1/2}$ versus the average charged multiplicity $\langle n\rangle$. It is interesting to note that in π-nucleus interactions D depends linearly on $\langle n\rangle$ in the same way as in πp interactions.[11,12] In neither is it Poisson-like, where $D \propto \langle n\rangle^{1/2}$.

The observed slow increase in multiplicity with nuclear size and the energy independence of R_A rule out all cascade models[13] which assume that in hadronic collisions the asymptotic multiparticle final states are formed in a distance short compared to the mean free path of hadrons in nuclear matter.

Our data are in excellent agreement with the energy-flux-cascade model of Gottfried.[14] In our energy range, Gottfried predicts no difference in multiplicity between nuclei and hydrogen for

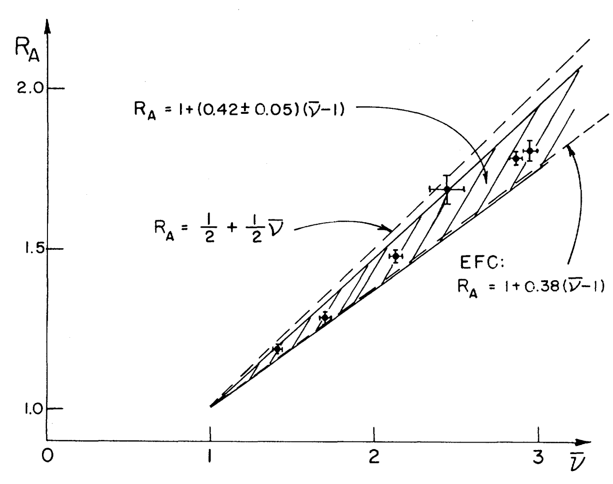

FIG. 5. R_A versus $\bar{\nu}$: comparison with theory. Best fit to our data gives $R_A = 1 + (0.42 \pm 0.05)(\bar{\nu} - 1)$, where the error includes all systematic and statistical uncertainties in R_A and $\bar{\nu}$. The broken lines are two theoretical predictions (see text).

rapidities greater than 2.5; we observe $\langle n\rangle_A/\langle n\rangle_{\rm H} = 1+(-0.005\pm 0.050)(\bar{\nu}-1)$ for $\eta > 3.5$, and $\langle n\rangle_A/\langle n\rangle_{\rm H} = 1+(0.17\pm 0.07)(\bar{\nu}-1)$ for $\eta > 1.5$. Gottfried predicts $\langle n\rangle_A/\langle n\rangle_{\rm H}(\text{all } \eta) = R_A = 1+0.38(\bar{\nu}-1)$, independent of energy; we find $R_A = 1+(0.43\pm 0.05)(\bar{\nu}-1)$ at 100 GeV/c and $R_A = 1+(0.41\pm 0.06)(\bar{\nu}-1)$ at 175 GeV/c. The above quoted errors are not only statistical; they include all systematic effects as well as the error on $\bar{\nu}$.

In Fig. 5 we compare our results with the energy-flux-cascade model and also with $R_A = \frac{1}{2} + \frac{1}{2}\bar{\nu}$ as predicted by other two-step models.[15]

We thank the Brookhaven National Laboratory staff and S. Redner for help in detector tests; and the Fermilab staff, especially S. Ecklund and P. Koehler, and the collaborators of Fermilab Experiments No. 69 and No. 96 for their cooperation. The help of the staff of the Laboratory for Nuclear Science at Massachusetts Institute of Technology is greatly appreciated, in particular T. Lyons and B. Nelson. Finally, we thank M. Deutsch, H. Feshbach, J. Friedman, H. Kendall, L. Rosenson, and V. Weisskopf for their constant support.

*Work supported through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(11-1) 3069, and by the National Science Foundation.

†Present address: Fermi National Accelerator Laboratory, Batavia, Ill. 60510.

[1]For example, suppose T^* is a characteristic particle production time in the center-of-mass frame. In the laboratory frame it will be $T = \gamma_{\rm c.m.}T^* \approx (E/2M_p)^{1/2}T^*$, where E is the energy of the incident particle. The production will thus take place over a distance of $l \approx c \times (E/2M_p)^{1/2}T^*$. It is reasonable to assume that for multiparticle production $T^* \gtrsim \hbar/m_\pi$ which at 100 GeV leads to $l \gtrsim 10$ fm. See also discussions by L. W. Jones, University of Michigan Report No. UM-HE 70-1 (unpublished), and A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972).

[2]W. Busza *et al.*, in Proceedings of the Topical Meeting on High Energy Collisions Involving Nuclei, Trieste, Italy, September, 1974 (to be published).

[3]S. P. Denisov *et al.*, Nucl. Phys. B61, 62 (1973).

[4]J. Erwin *et al.*, Phys. Rev. Lett. 32, 254 (1974); D. Fong *et al.*, to be published; D. Bogert *et al.*, Phys. Rev. Lett. 31, 1271 (1973).

[5]We thank A. Białas for pointing out the formula for $\bar{\nu}$.

[6]M. F. Kaplon, D. M. Ritson, and W. D. Walker, Phys. Rev. 90, 716 (1953); E. Lohrmann and M. W. Teucher, Nuovo Cimento 25, 957 (1962), and earlier references there; E. L. Feinberg, Phys. Rep. 5C, 237 (1972); A. Gurtu *et al.*, Tata Institute Report No. TIFR-BC-74-6 (unpublished), and earlier references there; P. R. Vishwanath *et al.*, to be published.

[7]Gurtu *et al.*, Ref. 6; J. Hébert *et al.*, Phys. Lett. 48B, 467 (1974); J. Babecki *et al.*, Phys. Lett. 47B, 268 (1973).

[8]P. L. Jain *et al.*, Phys. Rev. Lett. 33, 660 (1974); M. G. Antonova *et al.*, Contribution to the Seventeenth International Conference on High Energy Physics, London, England, 1–10 July 1974 (to be published).

[9]Babecki *et al.*, Ref. 7.

[10]K. M. Abdo *et al.*, Joint Institute for Nuclear Research, Dubna, Report No. E1-8021, 1974 (unpublished).

[11]A. Wróblewski, Acta Phys. Pol. B 4, 857 (1973).

[12]E. L. Berger *et al.*, Nucl. Phys. B77, 365 (1974).

[13]P. M. Fishbane and J. S. Trefil, Phys. Rev. D 3, 238 (1971); Dar and Vary, Ref. 1; I. Z. Artykov, V. S. Barashenkov, and S. M. Eliseev, Nucl. Phys. B6, 11 (1968).

[14]K. Gottfried, private communication, and Phys. Rev. Lett. 32, 957 (1974).

[15]P. M. Fishbane and J. S. Trefil, Phys. Lett. 51B, 139 (1974); G. Calucci, R. Jengo, and A. Pignotti, Phys. Rev. D 10, 1468 (1974). See also A. S. Goldhaber, Phys. Rev. Lett. 33, 47 (1974); A. Białas and W. Czyż, Phys. Lett. 51B, 179 (1974); Dar and Vary, Ref. 1; L. Van Hove, in Proceedings of the Topical Meeting on High Energy Collisions Involving Nuclei, Trieste, Italy, September, 1974 (to be published); E. S. Lehman and G. A. Winbow, Phys. Rev. D 10, 2962 (1974).