lieve the results reasonably establish the existence of axially asymmetric shapes at the first barrier, we are not yet able to make a definitive statement regarding the question of reflection symmetry.

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[11]In fitting to the data we found that taking other sets of single-particle levels resulted in a variation of the appropriate decay widths by less than a factor of 2. This is because in the energy region of interest (below ~ 6 MeV) the shell and pairing effects tend to offset each other. In contrast, the enhancement of ~20 in the level densities due to axial asymmetry was crucial for obtaining absolute fits to the data. This enhancement at barrier A changed not only the overall normalization for Γ_f/Γ_n but also the energy dependence, because now for most cases barrier A determines the low-energy behavior while for energies of more than 2–3 MeV above threshold Γ_f/Γ_n is dominated by barrier B where $\rho(E)$ is rising more slowly.

[12]In cases where $E_A < E_B$ it is also possible to fit the experimental results with axially symmetric level densities but the values obtained for E_A are much lower then the values for neighboring nuclei. This possible ambiguity exists for ^{231}Pa, ^{233}Pa, ^{231}U, ^{232}U.

Evidence for the D State of ^{4}He

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From a study of experimental phase shifts for $p+{}^4$He elastic scattering at proton energies below 50 MeV we conclude that small D-state admixtures to the dominant S-state configuration exist in the ^{4}He ground state. This result is obtained by evaluating a forward dispersion relation for the $p+{}^4$He spin-flip scattering amplitude.

It has been recognized long ago that in the presence of noncentral forces the ground-state wave function of ^{4}He can be a mixture of $^1S_0$, $^3P_0$, and $^5D_0$ contributions.[1-4] Attempts to include tensor forces in calculations of the ^{4}He binding energy[1,2,4] have shown that a D-state admixture of 2–10% to the dominant S-state configuration can be expected to exist. P-state contributions should be much smaller, since they enter only in second order.[2-4]

To our knowledge, no experimental evidence for D-state contributions has ever been presented. Since ^{4}He has no spin, a D-state admixture does not give rise to a quadrupole moment. The only way to investigate such a configuration is to remove a nucleon from the ^{4}He ground state and to determine its orbital angular momentum. However, since the D-state contribution is small, its effects on any such process will be masked by those of the dominant S-state configuration, unless a process is studied to which the latter cannot contribute.

Such a selective process is the trinucleon exchange[5] in $N+{}^4$He spin-flip scattering as shown in Fig. 1. A nucleon spin-flip is only possible in this exchange scattering if the orbital angular momentum l at the vertices is not zero.[6] Since l is just the (asymptotic) orbital angular momentum of a nucleon in ^{4}He, the amplitude for this trinucleon exchange is proportional to ground-state admixtures with $l \neq 0$, or, neglecting P states, to

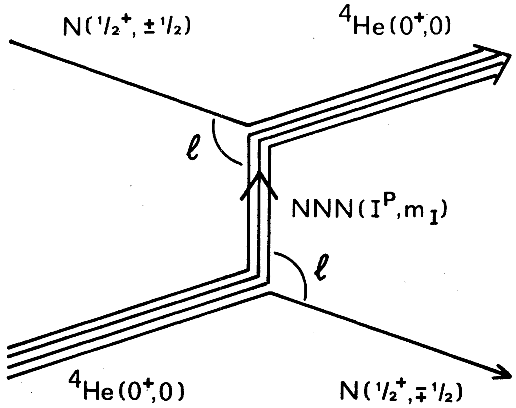

FIG. 1. Trinucleon exchange in $N+{}^4$He spin-flip scattering. The particles are labeled with their spins, parities, and spin projections. The exchanged trinucleon can have $I^{\pi}=\frac{1}{2}^-,\frac{3}{2}^-,\frac{3}{2}^+,\frac{5}{2}^+,\ldots$. The orbital angular momentum l at the vertices must be different from zero to make the process possible.

the D-state probability. Thus we can investigate the D state if we find a way to isolate the trinucleon exchange from the other contributions to $N+{}^4$He spin-flip scattering, e.g., that caused by the spin-orbit interaction in the compound nucleus.

We can achieve this separation in a model-free manner by evaluating a forward dispersion relation (FDR) for the $N+{}^4$He spin-flip scattering amplitude $\tilde{g}$. The use of this technique in nuclear physics has been developed in the last five years[7,8] and has been shown to yield reliable results.[9-11] It is based solely on conjectures about fundamental symmetries and the analyticity of scattering amplitudes. We use the fact that the FDR separates the scattering contributions from the upper and lower sheets of the complex energy plane.[7,8] This amounts to eliminating exactly the contributions of all the positive-energy states from the scattering amplitude, so that one is left with the contributions from negative energies. These are caused by exchange processes[7,10] and by bound states, if such exist.

For $N+{}^4$He spin-flip scattering the trinucleon exchange shown in Fig. 1 is the only source of negative-energy contributions[12] since there exists no bound state in the five-nucleon system. Thus, the FDR evaluated for this amplitude will isolate exactly those terms which are proportional to the D-state probability. Any nonzero contributions will indicate a D-state admixture to the ground state of ^{4}He.

We have applied these ideas to the experimental $p+{}^4$He spin-flip amplitude $\tilde{g}$ as determined from the scattering data by phase-shift analyses.[13-17] Many accurate $p+{}^4$He scattering experiments have been performed in the last fifteen years, particularly with polarized beams. Hence the spin-flip amplitude $\tilde{g}$ is well known below approximately 65-MeV proton energy.

The evaluation of the FDR is straightforward, following the methods (and notation) outlined previously.[8,10,11] The trinucleon-exchange contribution is represented[10,11] by the quantity $\tilde{\Delta}(E)/E$ and has been determined from the FDR as

$$\frac{\tilde{\Delta}(E)}{E}=\frac{1}{E}\operatorname{Re}\tilde{g}(E)-\frac{1}{\pi}\,\mathrm{P}\int_0^{\infty}\frac{\operatorname{Im}\tilde{g}(E')}{E'(E'-E)}\,dE'.$$

In Fig. 2 the result is shown as a function of the proton lab energy for the available sets of experimental phase shifts.[13-17] For comparison we have also plotted the exchange contribution to scattering without spin-flip, taken from earlier work.[10] The uncertainties introduced by the evaluation of the FDR, in particular by the lack of information about $\operatorname{Im}\tilde{g}(E)$ above 65 MeV, are of the order of a few percent. It is evident from the figure that nonvanishing exchange contributions to $p+{}^4$He spin-flip scattering and hence a D-state admixture to the ^{4}He ground state do indeed exist. As expected, the contribution to spin-flip scattering is only about 5–10% of that to spin-nonflip scattering, since the latter process involves both the S and D states of ^{4}He rather than the D state only.

Three of the four low-energy phase-shift sets[14-16]

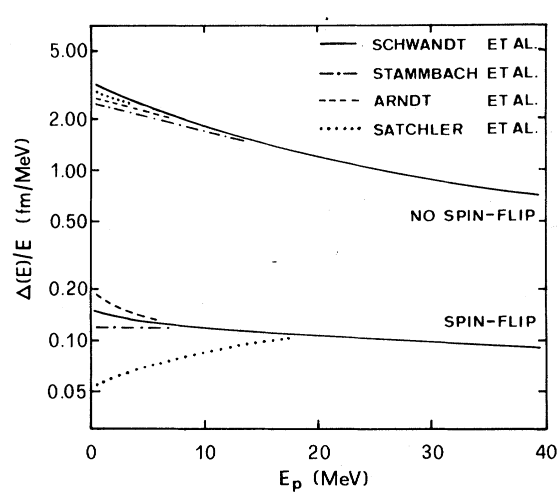

FIG. 2. Trinucleon-exchange contributions to $p+{}^4$He scattering with and without spin-flip. At low energies, the curves have been calculated by using the experimental phase shifts of Refs. 13–16 as indicated in the figure.

yield similar results. The set given by Satchler *et al.*[13] deviates strongly from the others. This is caused by the particular parametrization used in Ref. 13. The authors attempted to reproduce the scattering data by a potential model. In such a model, exchange contributions to the scattering as in Fig. 1 cannot be included. They must be "faked" by the introduction of a bound state at the appropriate energy to yield the proper singularity structure of the amplitude.[7,8] The potential used indeed has an (unphysical) bound state at -15 MeV. This is a pure S state since tensor forces were not considered. Taken all by itself, the phase-shift set of Ref. 13 will therefore yield no exchange contribution in our FDR analysis. In practice, however, the parametrization of Ref. 13 extends only up to 20 MeV, so that the phenomenological phase shifts of Ref. 17 were used in the FDR above that energy. The resulting mixture still yields a contribution which, however, tends towards zero at low energies.

It is easy to demonstrate that the phase shifts of Satchler *et al.*[13] are in fact incompatible with accurate experimental data. To this end we have performed several measurements using the polarized-proton beam available at the Nuclear Physics Laboratory of the University of Wisconsin in Madison. In Fig. 3 we show some of our experimental results. The measured quantity is the c.m. scattering angle θ at which the polarization of protons scattered from ^{4}He changes sign. We have plotted $\Phi-\theta$ as a function of proton energy,

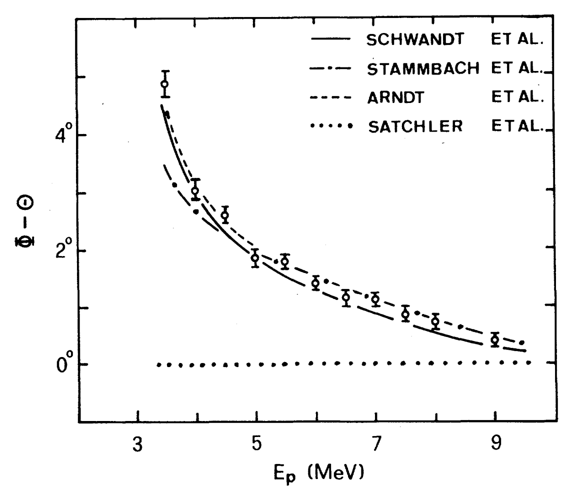

FIG. 3. The scattering angle θ where the proton polarization in $p+{}^4$He changes sign. The quantity $\Phi-\theta$ is shown as measured and as predicted by using the phase shifts of Refs. 13–16. The angle Φ is the prediction for θ of Satchler *et al.* (Ref. 13).

where Φ is the prediction for θ calculated from the parameters given by Satchler *et al.*[13] The data are in strong disagreement with this particular set of phase shifts, while the other parametrizations fit the measurement quite well.

Having demonstrated the existence of a D-state admixture to the ^{4}He ground state, we want to comment on the possibility of determining the corresponding spectroscopic factor from our analysis. In principle, information about the asymptotic normalization of the D-state wave function can be obtained by an extrapolation of the FDR to negative energies.[7-11] In the present case we have the fundamental problem of determining where to extrapolate to, i.e., to specify the removal energy of a proton from the D state. Because of angular momentum conservation, when a proton is removed from the D state, the remaining trinucleon cannot be a triton, but must be excited into the continuum with an *a priori* unspecified amount of additional energy. This makes it impossible at present to obtain quantitative spectroscopic information. Since calculations can be performed which reproduce the S-state properties of ^{4}He quite well,[18,19] we suggest that they also be used to predict a model wave function for the D-state with which our results could then be compared. Secondly, the existing experimental information is not precise enough to encourage an extrapolation, as can be seen from Fig. 2. Accurate polarization data at very low energies (preferably below 1 MeV) are needed to determine the $p+{}^4$He p-wave scattering lengths and effective ranges.

In this work the investigation of nuclear bound-state properties by applying a forward dispersion relation to scattering data has proven to be a sensitive tool. We doubt that other proposed techniques based on the analyticity of scattering amplitudes[20-22] could yield similar results, because there the influence of singularities on the lower E sheet, e.g., of resonances, cannot be eliminated. In addition, Coulomb effects are easily handled in the FDR approach,[8,10,11] while they give rise to considerable difficulties with other techniques.[20,21]

In conclusion we have investigated $p+{}^4$He scattering in a model-independent manner, based on no particular assumptions about the interaction of protons with ^{4}He. From the existing experimental information we have been able to deduce the existence of a non-S-state admixture to the ground state of ^{4}He. Further experimental and theoretical progress is needed to obtain quantitative spectroscopic information.

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Quantized "Sine-Gordon" Equation with a Nonvanishing Mass Term in Two Space-Time Dimensions

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For $\epsilon^2 < 4\pi$ the $\lambda \sin(\epsilon\Phi + \theta)$ quantum field model corresponding to the "sine-Gordon" equation with a nonvanishing mass term in two space-time dimensions exists and yields a field theory satisfying the Wightman axioms. For $|\lambda|$ small enough, the Euclidean Green's functions of this model have a convergent Feynman perturbation series. For λ large enough and positive, ϵ positive, and $\theta = 0$, the $\Phi \to -\Phi$ symmetry is presumably dynamically broken.

In this note I present rigorous results on the quantized "sine-Gordon" equation

$$(\Box + m^2)\varphi(x, t) = \lambda{:}\sin[\epsilon\varphi(x, t) + \theta]{:} \qquad (1)$$

in two space-time dimensions. Here m is the bare mass which is assumed to be strictly positive, unless otherwise stated; θ is an angle; ϵ is a positive, real number; and the bare coupling constant λ is an arbitrary real number. The colons designate the usual free-field Wick, or normal, ordering.

The field equation (1) is an interesting laboratory for constructive field theory, in several respects: