Observation of a Profile on a Superfluid Helium Film on a Rotating Substrate*

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Simultaneous measurements of the thickness at two locations of a He II film adsorbed on a rotating substrate have been made. From the thickness difference we deduce that for $T > 2.1$ K the film has a solid-body velocity field, whereas for $T = 1.23$ K the velocity is zero until the linear velocity of the substrate exceeds the translational critical velocity of the film.

We have measured the angular velocity dependence of the thickness of superfluid helium films at three locations on a rotating substrate using capacitance probes. Osborne[1] first performed the analog of this experiment for bulk superfluid helium by studying the profile of the fluid in a rotating container. The observed temperature-independent parabolic profile implies a solid-body velocity field in contradiction to the potential flow demanded by Landau's hydrodynamics.[2] The problem was resolved by Feynman[3] in terms of quantized vortices which enable the fluid to sustain a solid-body velocity field macroscopically while microscopically satisfying $\nabla \times \vec{v}_s = 0$ except at the singular points at the cores of the vortices.

Two previous attempts to observe the behavior of superfluid films on rotating substrates have led to conflicting conclusions. Little and Atkins[4] rotated a cylinder, partly immersed in stationary bulk liquid, and used an optical technique to measure the thickness of the film near the center of the exposed flat end. No thickness change was observed at 1.2 K up to $\omega \simeq 20$ sec^{-1}, and they tentatively proposed that no vorticity had been created in the superfluid film. Van Spronsen *et al.*[5] measured the change in height of bulk liquid in a capillary at the center of the rotating substrate and inferred changes in the profile of the film. They concluded that the film as a whole participates in the rotation.

Our results clearly demonstrate the following:

(1) There is a pronounced overall thinning of the film as the angular velocity of the substrate is increased which means that it is impossible to infer the profile from measurements of thickness at only one point.

(2) At 2.15 K, where $\rho_n/\rho \sim 0.89$, the profile in the film depends on the angular velocity of the substrate in a way which is completely consistent with a classical calculation using the highly Thickness-dependent Van der Waals attractive force.

(3) At 1.2 K, where $\rho_s/\rho \sim 0.97$, a difference in thickness between the center and the radius r does not appear until the *linear* velocity of the substrate ωr exceeds the translational critical velocity of the film. This contrasts with the bulk results, and is in conflict with the thermodynamic critical angular velocity for the formation of the first vortex, and the conclusions of Van Spronsen *et al.*[5] We infer that up to a certain radius r_c given by v_c/ω, where v_c is the critical velocity of the film, the superfluid film is at rest, while for $r > r_c$ vortices perpendicular to the film exist allowing a thickness difference to be maintained.

The capacitance cell is formed by two aluminum plates of 6.0 cm diam, polished with 1-μm powder, which are spaced 25 μm apart by a Mylar spacer, the plates being clamped together by screws. The lower plate contains electrically isolated Al plugs 5.20 mm in diameter forming the lows of the capacitance cell, placed at the center of the axis of rotation and at radii of 1.4 and 2.0 cm (C_0, C_1, and C_2, respectively). The upper plate serves as a common high, and contains a 0.6-mm-diam hole at a radius of 2.0 cm which permits the formation of a film inside the cell when helium gas is admitted in the copper can that surrounds it. With the common high, any two capacitances can be measured simultaneously by driving two GR-1615A capacitance bridges with one generator, there being no detectable crosstalk between the bridges. A 30-V, 5-kHz driving voltage was used. The values of C_0, C_1, and C_2 were 7.7, 6.8, and 6.7 pF, respectively. The experimental cell could be rotated continuously up to ~35 sec^{-1}, the electrical signals being brought to the stationary electronics via copper-graphite slip rings. The resolution in thickness during rotation was ~2 Å. The temperature was held constant to better than 100 μK during the runs.

The helium films were formed by slowly admitting helium gas into the stationary cell while monitoring the capacitances. With the appropriate correction for the contribution of the helium vapor to the capacitances, the film thickness could be deduced. After formation of the film, the angular rotation of the cell was increased in steps of ~2 $\sec^{-1}$. Typically at least 2 min were spent at each ω, though no significant relaxation effects were observed.

For a film of 330 Å static thickness, Fig. 1 shows the variation of the thickness with ω at the center and at a radius of 2 cm at $T=2.12$ and 1.23 K. The development of a thickness difference between the probing locations is clearly demonstrated by the separation of the data for $r=0$ and $r=2$ cm. The variation of the thickness at the center does not appear to depend on the temperature. To calculate it requires an equation expressing conservation of mass in the whole can (not simply within the capacitance cell) plus knowledge of the surface profile. Using a simplified topological model of the can and the profile discussed below [Eq. (1)] we have obtained good agreement with the observed experimental values of the thinning.[6]

The difference in changes in thickness at the two locations as ω is increased is shown in Fig. 2 for the two temperatures. At $T=2.12$ K, the data can be described very well by calculating the difference $d(2,\omega)-d(0,\omega)=\Delta d_{20}(\omega)$ from

$$\alpha\left(\frac{1}{d^4(0,\omega)}-\frac{1}{d^4(2,\omega)}\right)=\tfrac{1}{2}m\omega^2r^2 \qquad (1)$$

which is derived by assuming a solid-body velocity field in the film. The Van der Waals constant α used for the calculated curve shown in Fig. 2 is 5.7×10^{-43} erg cm^4 which is in reasonable agreement with the value of 7.8×10^{-43} obtained by Hemming from static film-thickness measurements.[7] Note that the thinning of $d(0,\omega)$ with rotation must be taken into account and that this produces a rounding off in the curve. In some cases it can even cause the difference in thickness to decrease again at very high ω. There is no critical behavior and the results are reproducible.

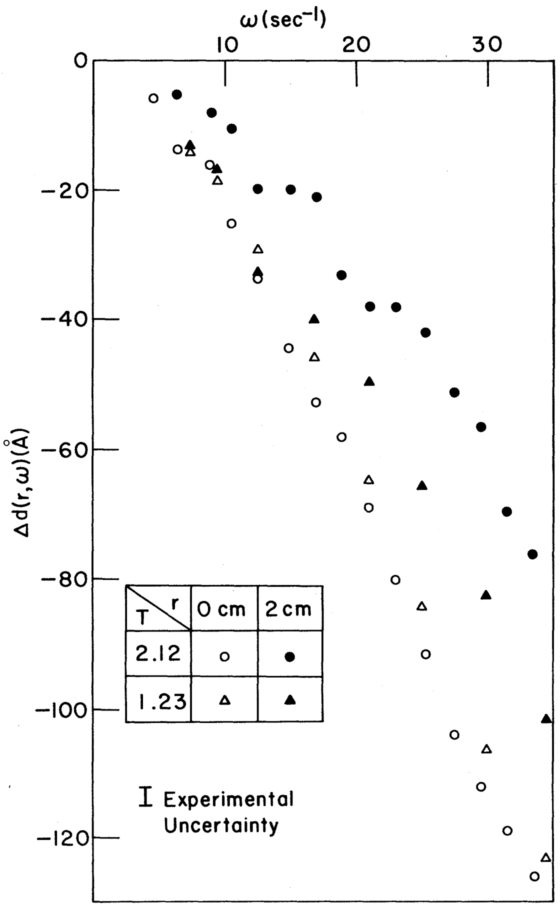

FIG. 1. Changes in thickness of a He II film of 330 Å static thickness with increasing angular rotation of the substrate.

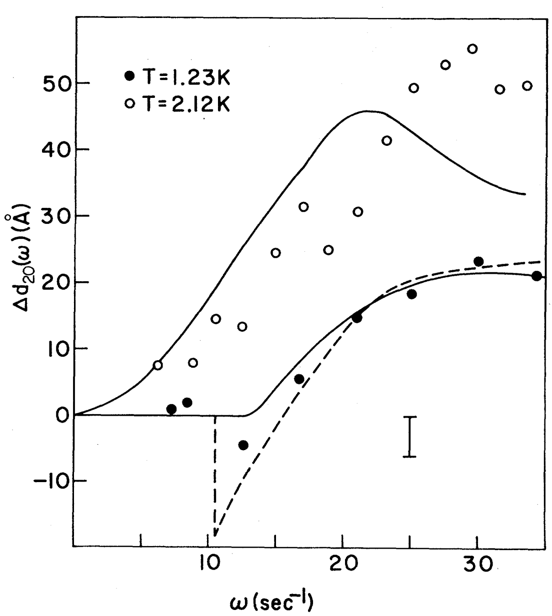

FIG. 2. Difference in thickness decrease between the center and $r=2$ cm as ω is increased. Solid line for $T=2.12$ K is a fit to the data using Eq. (1) with $\alpha=5.7\times10^{-43}$ erg cm^4. For the low-temperature data, the solid line is drawn by eye, and the dashed line is based on the model where $v=\omega r$ for $r>r_c$.

At $T=1.23$ K the data for the virgin film (no previous rotational history) show that the thickness decreases uniformly at the two probing locations until the angular velocity of the cell exceeds a certain critical angular velocity ω_c. We associate the appearance of a positive $\Delta d_{20}(\omega)$ for $\omega > \omega_c$ with the creation of quantized vortices which bring the superfluid into motion. For the film shown in Fig. 2, $\omega_c \sim 14$ sec^{-1}, corresponding to a substrate critical velocity of 28 cm/sec at the outside probe, in agreement with experiments on the onset of dissipation in translational flow for films of similar thickness.[8] This implies that the relevant parameter for the creation of vorticity in the film is the linear velocity ωr of the substrate rather than the thermodynamic critical angular rotation of the container for the formation of the first vortex $\omega_{c1} = (\hbar/mR^2) \times \ln(R/a)$ which we have shown to be approximately the same for both bulk helium and film.[9] Measurements of ω_c at $r=2$ cm for films of different thickness indicate that ω_c increases as the film gets thinner; for $d \lesssim 200$ Å the thickness at $r=2$ cm never exceeded that at the center even up to $\omega = 32$ sec^{-1}. Hysteresis effects have been observed, but as usual were not reproducible. However, there was a consistent reduction in the value of ω_c in subsequent rotations if in the initial rotation $\omega > \omega_c$.

We suggest the following model for the superfluid helium film on a rotating substrate. As ω is increased from 0 the film remains at rest (even though in our experiment it gets thinner[10]) until the *linear velocity* of the substrate exceeds the critical velocity $v_c(d)$ at the thickness d. Vortices are then formed and a thickness difference is supported outside the critical radius $r_c = v_c/\omega$. The profiles we observed were as stable as the electronics for periods up to 30 min.

The lack of a thickness difference for $r < r_c$ indicates that $v=0$ in this region. For $r > r_c$, our analysis is based on the equation

$$-\frac{\kappa}{2\pi}\left\langle \frac{\partial}{\partial t}(\varphi_1 - \varphi_2)\right\rangle = \mu_1 - \mu_2, \qquad (2)$$

where φ_i is the phase of the order parameter (quantum mechanical viewpoint) or the velocity potential (classical approach) at r_i, and $\mu_i = \mu_0 + v_i^2/2 - \alpha/md^4(r_i, \omega)$ is the chemical potential.[11] Depending on the assumptions one makes concerning the behavior of vortices, a number of velocity fields satisfy Eq. (2). If the vortices rotate with the substrate, those inside the radius r $[N(r,\omega)]$ will contribute to the phase slippage by an amount $\kappa\omega N/2\pi$. If the velocity at r is due only to the presence of the N vortices inside r, then $v_s = N\kappa/2\pi r$ and we can calculate the number $N(r,\omega)$ and the corresponding velocity field by solving Eq. (2) using the experimentally observed profile. These results are shown in Fig. 3 by the solid circles. The translational critical velocity is 28 cm/sec. We note that the superfluid velocity is less than ωr (the substrate velocity) by an amount which appears to correspond to the critical velocity (assuming that it varies inversely with thickness)[12]

$$v(r) = |v_{\text{substrate}}| - v_c(d). \qquad (3)$$

If the superfluid slips behind the substrate by v_c then $N(r,\omega) = (2\pi/\kappa)(r\omega - v_c)r$. Assuming $v_c = 8.3 \times 10^3/d$, where d is in angstroms ($v_c = 28$ cm/sec

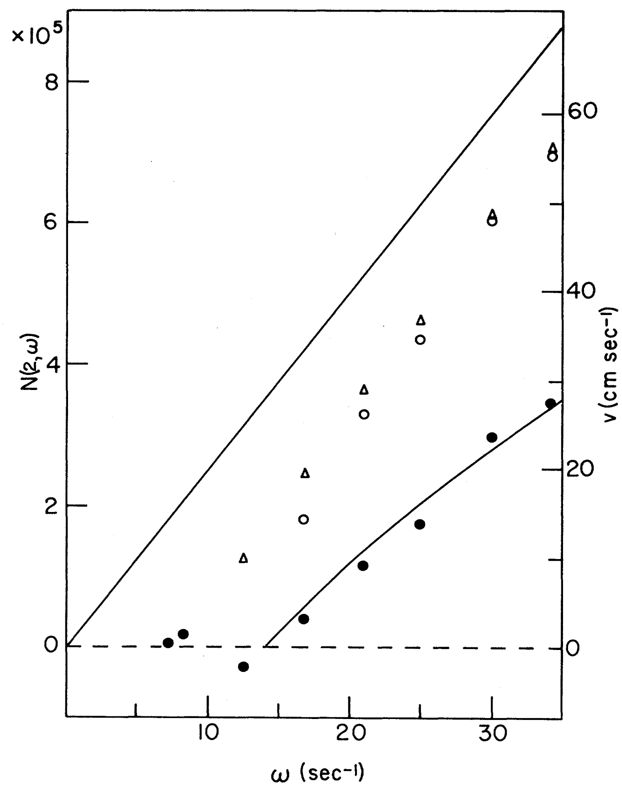

FIG. 3. The left-hand ordinate represents the total number of vortices enclosed by a circle of 2 cm radius, and the right-hand ordinate the velocity at this radius. The solid line is for the uniform distribution $2\omega/\kappa$, as occurs in bulk helium. The solution of Eq. (2) on the assumption that the vortices are pinned to the substrate is given by the solid circles, and for vortices moving with the local velocity by the open circles. The curved solid line is a fit to the data if one takes the superfluid velocity to be less than that of the substrate by the critical velocity. The triangles refer only to the number of vortices if one takes $v=\omega r$ for $r > r_c$.

at $d \sim 300$ Å), we have plotted $N(2, \omega)$ in Fig. 3 (curved solid line). The agreement with the experimental data is seen to be good.

We have considered two other possibilities. The first assumes that the vortices move with the local superfluid velocity, and the results obtained from Eq. (2) are shown in Fig. 3. In the other, the superfluid is taken to flow with the substrate outside the critical radius r_c. It is then necessary to postulate a mechanism to contribute to the velocity field without contributing to the phase slippage in Eq. (2), or else $\Delta d_{20}(\omega)$ for $r_c < 2$ will simply be the same as that at high temperatures. The N for this assumption is also plotted in Fig. 3. The critical velocity is lower (~17 cm/sec). The thickness difference calculated using this model, shown in Fig. 2, exhibits a negative region between $r = v_c/\omega$ and $r = (2)^{1/2} v_c/\omega$. The difference $\Delta d_{20}(\omega)$ tends to be negative in this region but it cannot be studied satisfactorily with the present apparatus. Moreover there are a number of other possible explanations. For example, if the film is not completely homogeneous in thickness, the revolving substrate may create some flow patterns in the film even before ω_c, which would result in a larger thickness decrease on the outside in accordance with Bernoulli's equation.[13]

In summary we have shown that the normal fluid in a He II film rotates with the substrate, whereas the superfluid remains at rest until the linear velocity of the substrate exceeds the translational critical velocity of the film. The most plausible velocity field is one in which the superfluid velocity is less than the velocity of the substrate by the critical velocity.

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[10]There is no contradiction implicit in this. The geometry of the can outside the capacitance cell is very complex and there the superfluid is pushed into rotation, just like in the simple case of superfluid in an elliptical bucket. Furthermore as there may be some bulk liquid at the bottom of the can, its rotation would be achieved at very low ω. In any case, the surface of the film is an equipotential surface, and so the film in the capacitance cell is expected to thin, though it is not rotating when equilibrium is achieved.

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Second-Harmonic Generation in an Inhomogeneous Laser-Produced Plasma

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It is experimentally shown that specular second-harmonic emission from a dense, inhomogeneous plasma produced by laser irradiation of plane solid targets occurs on oblique reflection of a p-polarized light wave, in agreement with theoretical predictions.

Oblique reflection of a p-polarized electromagnetic wave from an overdense, inhomogeneous plasma[1-3] should, according to theoretical predictions,[4,5] be accompanied by second-harmonic (SH) emission with the following characteristic features: (i) The intensity of SH radiation increases quadratically with the intensity of the incident electromagnetic wave. (ii) Resonant behavior of the electric field at the critical layer and strong SH generation occur only for a p-polar-