tion to theorists.

The authors thank their colleagues at Cornell University and Harvard University for discussion. Particular thanks are due K. Wilson for aid with the computer programming and T. Appelquist for a discussion of Eq. (9).

*Work supported in part by the National Science Foundation.

†Work supported in part by the National Science Foundation under Grant No. GP-38863.

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[9]ψ is a continuum wave function and is therefore normalized to unit flux at spatial infinity.

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[11]The reader should note that a nonrelativistic description of the high-energy end of Figs. 1(a) and 1(b) is very crude.

ψ Particles, SU(4), and Anomalous Currents

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(Received 24 December 1974)

We propose assignment of the new ψ particles in an SU(4) symmetry scheme, incorporating SU(3) and a new additive quantum number, paracharge Z. Anomalous currents are introduced to account for the production and decays of these particles. Two more rather broad resonances at around 4.3 and 5.0 (± 0.2) GeV are expected in the e^+e^- annihilation, one of which has recently been reported (at 4.15 GeV).

We describe in the present note a phenomenological scheme for the new narrow ψ particles[1] at 3.105 and 3.695 GeV and the (possible) resonance[2] at 4.15 GeV. We shall regard them as hadrons. The decay width of a typical hadron having a mass of a few GeV is generally expected to be quite sizable, unless forbidden by some selection rule. We, therefore, introduce a new additive quantum number, called[3] the "paracharge" Z. While all the earlier known particles will be assumed to have $Z=0$, the $\psi(3.1)$ and the $\psi(3.7)$ will be assigned $Z\neq 0$. Only strong interactions are supposed to conserve Z. The observed small width of the $\psi(3.1)$ will thus be accounted for. The strong decays $\psi(3.7)\to\psi(3.1)+$ hadrons ($Z=0$, e.g., 2π) are allowed by quantum numbers but are suppressed for reasons to be discussed below.

The observation of the $\psi(3.1)$ and the $\psi(3.7)$ in Bhabha scattering will be assumed to proceed directly through the electromagnetic current ($e^+e^-\to$ virtual $\gamma\to\psi\to$ virtual $\gamma\to e^+e^-$). Thus the ψ particles will be assumed to have $J^P=1^-$ and charge conjugation $C=-1$. This naturally leads us to postulate new additional Z-changing anomalous (nonminimal) pieces in the phenomenological *hadronic* electromagnetic current *(but not in the well-established electromagnetic current of the charged leptons)*.

We incorporate $U(1)_Z$ along with the conventional SU(3) into a larger SU(4) symmetry group of strong interactions, regularly broken according to the chain $SU(4)\supset SU(3)\otimes U(1)_Z\supset SU(2)_I\otimes U(1)_Y\otimes U(1)_Z$.

The $\psi(3.1)$ has $I=Y=0$ and is the $C=-1$ mixture of $Z=\pm 1$ states of a near ideally mixed $\underline{15}\oplus\underline{1}$ multiplet containing the vector mesons of the ρ family. A new vector state, the P, with $I=Y=Z=0$ and $C=-1$ is also expected in this family (according to the broken SU(4) mass formula) at around 4.3 GeV. This is identified with the recently discovered (possible) resonance[2] at 4.15 GeV. The $\psi(3.7)$ has the same quantum numbers as the $\psi(3.1)$, but belongs to another mixed $\underline{15}\oplus\underline{1}$ multiplet which contains the $\rho'(1600\text{ MeV})$. Correspondingly, another state, the P', with the same quantum numbers as the P, is predicted in the multiplet at around 5.0 GeV (from the evidence of the P, this prediction may well be off by $\simeq 0.2$ GeV). Observation of the P' in e^+e^- annihilation is thus a crucial test of this scheme.

Emission of energetic photons (~ 1 GeV) and final states of η, η', φ, f' among the decay products of the ψ particles are other prominent features of our scheme. This helps in resolving the so-called "energy crisis" in $e^+e^- \to$ hadrons.

The symmetry scheme.—We would like to accommodate the lower-lying new particle $\psi(3.1)$ in the regular fifteen-dimensional representation of SU(4) containing the SU(3) octet of vector mesons of the ρ family. This requirement together with the $Q = Y = 0$, $Z \neq 0$ assignment for the $\psi(3.1)$ *uniquely* specifies the expressions for Q, I_3, Y, and Z in terms of the generators of SU(4). We construct physical representations by combining the basic quartets $\xi \equiv (\mathcal{P}, \mathfrak{N}, \lambda, \chi)$ transforming under the $\underline{4}$ representation and $\bar{\xi}$ (which transforms under the $\underline{4}^*$). Here $(\mathcal{P}, \mathfrak{N}, \lambda)$ is the conventional SU(3) triplet of quarks assigned $Z = 0$, and χ is a new paracharged SU(3)-singlet quark with $Z = 1$. The baryons are constructed[4] as $\xi \otimes \xi \otimes \xi$ and the mesons as $\xi \otimes \bar{\xi}$. For the quarks we must then make the following assignments[5]: electric charge $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$, $I_3 = \mathrm{diag}(+\frac{1}{2}, -\frac{1}{2}, 0, 0)$, hypercharge $Y \equiv 2Q - 2I_3 = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3})$, $Z = \mathrm{diag}(0, 0, 0, 1)$, baryon number $B = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

Note that Q, Y, and Z are not traceless and are thus not themselves among the generators of SU(4). The three diagonal generators of SU(4) in the basic representation are [in an obvious extension of the conventional SU(3) notation] $F_3 = I_3 = \frac{1}{2}\lambda_3 = \mathrm{diag}(+\frac{1}{2}, -\frac{1}{2}, 0, 0)$, $F_8 = (1/\sqrt{3})\lambda_8 = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0)$, and $F_{15} \equiv (\sqrt{\frac{3}{8}})\lambda_{15} \equiv \mathrm{diag}(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4})$. The remaining twelve nondiagonal "shift" generators may be constructed in the well-known manner. We also introduce $\lambda_0 \equiv (1/\sqrt{2})\mathrm{diag}(1, 1, 1, 1)$. Then the physical "charges" in terms of the generators are (valid for all representations)

$$Q = F_3 + \tfrac{1}{2}F_8 + \tfrac{1}{3}F_{15} - \tfrac{1}{4}B,$$

$$Y = F_8 + \tfrac{2}{3}F_{15} - \tfrac{1}{2}B, \quad Z = \tfrac{3}{4}B - F_{15}.$$

We combine the $\psi(3.1)$ with the old U(3) nonet $(\rho, \omega_8, k^*, \bar{k}^*) \oplus \omega_1$ into a $\underline{15} \oplus \underline{1}$ representation of SU(4), thus forming a $\underline{16}$-plet of U(4). We identify the ω_1 with the SU(4) singlet. The SU(3) $\otimes$ U(1)$_Z$ content of the 15-plet is as follows:

$$\underline{15} \to \underline{8}(Z=0) \oplus \underline{3}(Z=-1) \oplus \underline{3}^*(Z=+1) \oplus \underline{1}(Z=0).$$

The new SU(3) multiplets have the (I, Y, Z) contents

$$\underline{3}(Z=-1) \to \begin{cases} (D^+, D^0), & I = \frac{1}{2}\ (Q = 1, 0), \quad Y = 1, \\ & \qquad\qquad\qquad\qquad Z = -1; \\ S, \quad I = 0\ (Q = 0), & Y = 0, \quad Z = -1; \end{cases}$$

with $\underline{3}^*(Z=+1)$ containing the antiparticles $(\bar{D}^0, D^-)$, $\bar{S}$; and $\underline{1}(Z=0) \to P_1$ $(Q = I = Y = Z = 0)$.

The electromagnetic current.—Since it is the $C = -1$ combination[6] $S_-{}^0 = (S - \bar{S})/\sqrt{2}$ that is being produced in the annihilation of e^+e^- via a virtual (massive) photon, it is clear that the hadronic electromagnetic current must be modified by the addition of an anomalous piece that has $\Delta Z = \pm 1$, $\Delta I = 0$, $\Delta Y = 0$, and $C = -1$. But the charge of the conserved electromagnetic current is still given by $Q = I_3 + \frac{1}{2}Y$; and since Q represents a superselection operator, the charge carried by the anomalous piece *must* vanish identically.

We postulate that this anomalous current transforms as a member of a 15-plet $\mathcal{G}_\mu{}^i(x)$ $(i = 1, \ldots, 15)$ of SU(4). With the normal currents $\mathfrak{F}_\mu{}^i(x)$ $(i = 0, 1, \ldots, 15)$, whose charges $F_i \equiv \int \mathfrak{F}_0{}^i\, d^3x$ are the generators of U(4), our hadronic electromagnetic current has the form,

$$J_\mu{}^{\mathrm{em}}(x) = \mathfrak{F}_\mu{}^3(x) + \tfrac{1}{2}\mathfrak{F}_\mu{}^Y(x) + \mathcal{G}_\mu{}^{\mathrm{em}}(x);$$
$$\mathfrak{F}_\mu{}^Y \equiv \mathfrak{F}_\mu{}^8 + \tfrac{2}{3}\mathfrak{F}_\mu{}^{15} - \tfrac{1}{2}\mathfrak{F}_\mu{}^B,$$

where $\mathfrak{F}_\mu{}^B = (\sqrt{2}/3)\mathfrak{F}_\mu{}^0$ is the baryonic current. The current $\mathcal{G}_\mu{}^{\mathrm{em}}$ has $C = -1$, and contains in general the components $\mathcal{G}_\mu{}^i$, $i = 0, 3, 8, 14, 15$. The piece $\mathcal{G}_\mu{}^{14}$ transforms like the $S_-{}^0$ and makes possible transitions with $\Delta Z = \pm 1$, $\Delta I = \Delta Y = 0$. The strengths of the various pieces must, of course, be fixed phenomenologically. A possible form for $\mathcal{G}_\mu{}^i$ in terms of the quark fields is

$$\mathcal{G}_\mu{}^i(x) = \beta(i/2M)\partial_\nu[\bar{\xi}(x)(\lambda^i/2)\sigma_{\mu\nu}\xi(x)].$$

We shall also admit a corresponding addition of anomalous pieces to the weak-interaction currents.

The breaking of SU(4) and mass formulas.—We start with a degenerate U(4) 16-plet and calculate the mass splittings taking into account the maximal (or ideal) mixing[7] of the neutral mesons ω_8, P_1, and ω_1. Describing the symmetry breakings

U(4) → SU(3) by means of the effective Hamiltonian $H = H_0 + \mu_\chi \bar{\chi}\chi$, and diagonalizing the mass-squared matrix, we obtain, in terms of the physical orthonormal eigenstates ω, P, and φ, the mass relation $2m_S^2 = m_\rho^2 + m_P^2$, with the quark contents as follows: $S \sim |\lambda\bar{\chi}\rangle$, $D^+ \sim |\mathcal{P}\bar{\chi}\rangle$, $D^0 \sim |\mathfrak{N}\bar{\chi}\rangle$, $P \sim |\chi\bar{\chi}\rangle, \ldots$. We follow Okubo[7] and take $\omega \sim (1/\sqrt{2})(|\mathcal{P}\bar{\mathcal{P}}\rangle + |\mathfrak{N}\bar{\mathfrak{N}}\rangle)$ and $\varphi \sim |\lambda\bar{\lambda}\rangle$. Similarly, we form another $\underline{15} \oplus \underline{1}$ multiplet of the ρ'(1600 MeV) family containing the $S_-^{0\prime} \equiv \psi(3.7)$ and the P'.

With $m_S = 3.1$ GeV, we obtain $m_P \simeq 4.3$ GeV. With $m_{S'} = 3.7$ GeV, we get $m_{P'} \simeq 5$ GeV. It is to be noted, of course, that ideal mixing is expected to be only approximate.

With the unfreezing of paracharge at the energies now available we expect enlargements of all the well-known SU(3) multiplets. The baryons, e.g., will be obtained as $\xi \otimes \xi \otimes \xi$: $\underline{4} \otimes \underline{4} \otimes \underline{4} = \underline{20}_S \oplus \underline{20} \oplus \underline{20} \oplus \underline{4}^*$. The octet baryons belong to $\underline{20} \to \underline{8}(Z=0) \oplus \underline{6}(Z=1) \oplus \underline{3}^*(Z=1) \oplus \underline{3}(Z=2)$, and the decimet to $\underline{20}_S \to \underline{10}(Z=0) \oplus \underline{6}(Z=1) \oplus \underline{3}(Z=2) \oplus \underline{1}(Z=3)$. The general mass formula (assuming that the effective symmetry breaking transforms as $\sim F_{15}$) in terms of the highest weight notation (p, q) for SU(3) multiplets is $M = M_0 + M_1 Z + M_2[p^2 + q^2 + pq + 3(p+q) - Z^2]$. At this stage, of course, we cannot use this formula for estimating the masses of new particles like the pseudoscalar (PS) mesons S_{PS}, etc. If we use, for illustration only, the spin-SU(4)-spin independence relation[8] $m^2(S) - m^2(S_{\mathrm{PS}}) \simeq m_\rho^2 - m_\pi^2 \simeq 0.57\ \mathrm{GeV}^2$, we obtain $m(S_{\mathrm{PS}}) \simeq 2.9$ GeV.

Decays and e^+e^- annihilation.—It is important to note that the Stanford Linear Accelerator Center (SLAC) experimental setup detects only the decay modes involving at least two charged particles. The modes in which all the detected decay products are neutral are unknown. The partial decay widths extracted from experiments are thus to be taken only as lower bounds.

(a) $\psi(3.1)$ decays: The $\psi(3.1)$ (the S_-^0) cannot decay through (Z-conserving) strong interactions into the usual $Z=0$ hadrons. The narrow width does not permit paracharged pseudoscalar mesons to be much lower in mass (see also the estimate above). The strong decays will be taken then to be negligible.

The dominant decay modes of the $\psi(3.1)$ are electromagnetic. The main two-body modes, using selection rules and the nonet *Ansatz*[7] for the tensor mesons, will be $S_-^0 \to \eta + \gamma$, $\eta' + \gamma$, $f' + \gamma$ (via the anomalous current); $S_-^0 \to S_{\mathrm{PS}+}{}^0 + \gamma$ (via the normal current). As an illustration consider the decays of the type $S_-^0 \to A + \gamma$, where A is a $J^P = 0^-$ meson. The relevant matrix element is

$$\langle A(k) | eJ_\mu^{\mathrm{em}}(0) | S_-^0; p, \epsilon\rangle = e\frac{G_A(q^2=0)}{m}\frac{\epsilon_{\mu\nu\rho\sigma}\epsilon_\nu p_\rho k_\sigma}{(4p_0 k_0 V^2)^{1/2}},$$

where $q \equiv p - k$, and m is a mass parameter so that G_A is a dimensionless form factor. The decay rate is accordingly

$$\Gamma(S_-^0 \to A + \gamma) = \frac{\alpha G_A^2(0)}{24}\left(\frac{m_S}{m}\right)^2\left(1 - \frac{m_A^2}{m_S^2}\right)^3 m_S.$$

For $A = S_{\mathrm{PS}+}{}^0$, assuming $m(S_{\mathrm{PS}}) \simeq 2.9$ GeV, $m \simeq 1$ GeV (characteristic of the normal current), and $G_A^2(0) \simeq 1$, we obtain

$$\Gamma(S_-^0 \to S_{\mathrm{PS}+}{}^0 + \gamma) \simeq 20\ \mathrm{keV}.$$

For $A = \eta$ (or η'), the decay is via the *anomalous current.* Since the latter must have an identically zero "charge," the effective current of the particles involved has the form $\partial_\lambda T_{\lambda\mu}$, with an antisymmetric $T_{\lambda\mu}$ built out of the phenomenological fields of the particles. Then the effective form factor has the form $G_\eta(q^2) = (\beta m/M)(q\cdot p/m_s^2) \times F_\eta(q^2)$. The factor $q\cdot p$ takes care of the requirement of the vanishing "charge," and $F_\eta(q^2)$ is assumed to be a relatively smooth function. Taking $M \simeq m_S$ (characteristic of anomalous currents) and $F_\eta(0) \simeq 1$, we obtain $\Gamma(S_-^0 \to \eta\gamma + \eta'\gamma) \simeq 180\beta^2$ keV. To make a very rough estimate of β we take the anomalous current decay width $\Gamma(S_-^0 \to e^+e^-)$ to be of the order of $\beta^2(m_S/m_\rho)$ compared to the typical normal-current decay width $\Gamma(\rho^0 \to e^+e^-)$. Thus $\beta^2 \simeq \frac{1}{4}$. This gives $\Gamma(S_-^0 \to \eta\gamma + \eta'\gamma) \simeq 45$ keV. In a similar way, we make the rough estimate $\Gamma(S_-^0 \to f' + \gamma) \simeq 150$ keV. All of these estimates are very preliminary, especially since the relevant form factors are unknown. Also the parameters β and M may need revision in the future.

To estimate the electromagnetic decay width for $S_-^0 \to$ hadrons, we assume that the decay proceeds according to $S_-^0 \to$ virtual $\gamma \to$ hadrons. Then

$$\frac{\Gamma(S_-^0 \to \mathrm{hadrons})}{\Gamma(S_-^0 \to \mu^+\mu^-)} \simeq \left[\frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\right]_{\simeq 3\ \mathrm{GeV}} \simeq 3.$$

As for the modes $S_-^0 \to e^+e^-$ or $\mu^+\mu^-$, the partial widths (assuming e-μ universality) are determined by the Bhabha scattering cross section at the $\psi(3.1)$ peak. Thus the total width of $\psi(3.1)$ is of the order of 200–250 keV, which (in view of the uncertain parameters) is consistent with the value of $\sigma(e^+e^- \to$ "charged" modes) at $\sqrt{s} = 3.105$ GeV as measured at SLAC.

(b) $\psi(3.7)$ decays: The $\psi(3.7)$ (the $S_-^{0\prime}$) can decay strongly into the $\psi(3.1)$ and pions. The most prominent such mode $\psi(3.7)\to\psi(3.1)+2\pi$ (all S waves) will, however, suffer a sizable suppression[9] through the Adler zero due to the partial conservation of axial-vector current. Other strong modes could be (i) $S'\to S_{PS}+\eta$, $D_{PS}+\bar{K}$, $\bar{D}_{PS}+K$, or (ii) $S'\to S+\eta$, $D+\bar{K}$, $\bar{D}+K$. By the SU(4) symmetry the set (i) is related to $\rho'\to\pi\pi$ (only f coupling allowed) and the set (ii) to $\rho'\to K^*\bar{K}$ (only d coupling). From the absence of these ρ' decay modes, we conclude that all these modes are forbidden by SU(4).

We shall also have the electromagnetic modes analogous to those for the $\psi(3.1)$ as well as the mode $\psi(3.7)\to S_+{}^0(3.1)+\gamma$. These radiative modes together could well be comparable to the (suppressed strong) mode $\psi(3.7)\to\psi(3.1)+2\pi$.

(c) P,P' decays: The P (at 4.15 GeV) and the P' (expected at $\simeq 5$ GeV) both have $I=Y=Z=0$. These are expected to have large, typical strong-interaction widths in the few-hundred MeV range, since no selection rule will strictly forbid them. The expected departures from ideal mixing ($P\sim|\chi\bar{\chi}\rangle$), even if small, can lead to sizable widths.

(d) $e^+e^-\to$ hadrons: Every one of the radiative decay modes of each of the expected new particles also implies (by crossing) a new channel in e^+e^- annihilation. Thus a very large number of new channels—about fifteen—open in the energy range $\simeq 3.5$ to 4 GeV. It also turns out that the decays of each of the relevant new particles are into a high-energy (~1 GeV) photon plus hadrons and that the final resulting hadrons (left after decays) either are mostly neutral or are K mesons. Thus the so-called "energy crisis"[10] has a possible resolution.

A detailed account of all aspects of our scheme will be given elsewhere.

We thank Dr. V. Gupta, Dr. G. Rajasekaran, and Dr. P. V. Ramanamurthy for informative correspondence, and colleagues at the Tata Institute of Fundamental Research, especially Dr. J. Pasupathy, for stimulating discussions.

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[2]J.-E. Augustin *et al.*, this issue [Phys. Rev. Lett. 34, 764 (1975)].

[3]The name emphasizes essential differences from previous schemes using an additional additive quantum number. For older SU(4) schemes see, e.g. M. K. Gaillard, B. W. Lee, and J. Rosner, Fermilab Report No. Pub. 74/86 (to be published).

[4]The usual modifications (like adding color, or considering parastatistics for the quarks, etc.) can be easily incorporated, but will not be gone into here.

[5]Note that the values of Q and Y are quite different from any earlier scheme employing the SU(4) group.

[6]We denote the $C=\pm1$ eigenstates by $S_\pm=(S\pm\bar{S})/\sqrt{2}$.

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