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Electron-Positron Annihilation at and above the Threshold for Production of Charmed Hadrons

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Charmed-quark models of the new narrow resonances at 3.1 and 3.7 GeV suggest that a virtual S state lies slightly above the charmed-hadron threshold. This state causes a large enhancement of $R=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ just above the threshold, followed by a dip and several broad but smaller enhancements at higher energy. Estimates of this intriguing structure in R are presented.

It has been proposed that the new narrow resonances[1] are S-wave bound states of heavy charmed quarks.[2,3] This physical picture and the idea that strong interactions are described by an asymptotically free gauge theory provide a good first-order understanding of some of the striking properties of these states. Pursuing this suggestion further several authors[4] have constructed simple models of these bound states in order to understand their structure and decay modes in more detail.

These investigations have dealt most vigorously with either the new resonances themselves or the approach of the annihilation cross section to the asymptotic region. However, between these two regions lies the threshold for the production of charmed hadrons. This kinematic region and the behavior of R there is the subject of this note. On the basis of the ideas mentioned in the first paragraph we suggest that R has considerable interesting structure in the energy region $E=3.9$–5.5 GeV. Calculations done in simple models described below suggest that just above threshold there is a *sharp enhancement* in R followed by a dip and *several smaller broad enhancements.* These calculations strengthen a conjecture of Eichten *et al.*[4] that the presence of a 3S state above threshold should be visible in R. The height and width of the enhancement just above threshold (at 4.0 GeV, say) are about 6.2 and 300–400 MeV, respectively. The position of the first dip above threshold occurs at about 4.5 GeV. The height of the second enhanced region is estimated at 4.5. A second dip occurs at 5.5 GeV and a smaller enhancement follows [see Figs. 1(a) and 1(b)].

First we discuss the theory behind these remarks. Consider a simple approach to bound states of heavy charmed quarks (let c denote the charmed quark) in a theory containing ordinary quarks and Yang-Mills gauge fields. Under certain circumstances the interaction between a c and a $\bar{c}$ can be described by a nonrelativistic potential $V(r)$. These conditions are similar to those which determine the validity of the Born-Oppenheimer approximation[5] in molecular physics. In particular, if the c quarks are sufficiently heavy and slow moving, then the ordinary quark and vector fields can be treated in an adiabatic approximation.[5] To define the adiabatic approximation we first consider a fictitious problem in which the degrees of freedom are the Yang-Mills field $A_{\alpha,\mu}$, the ordinary, light-quark fields ψ, and a pair of static sources of the Yang-Mills field at positions r_1 and r_2. The static sources consist of

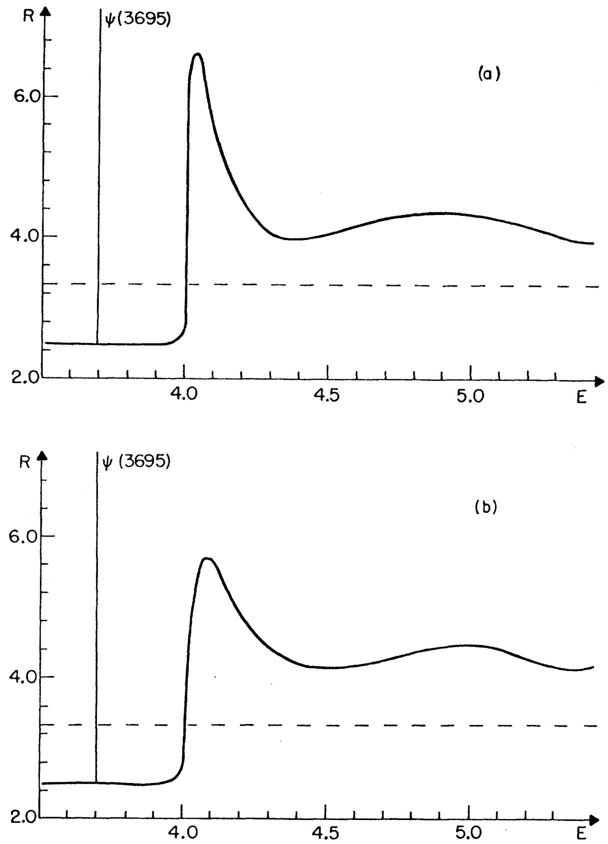

FIG. 1. (a) R as a function of c.m. energy. The virtual S-wave bound state lies ≈95 MeV above the threshold at 4.0 GeV. (b) Same as (a) except virtual state lies ≈125 MeV above threshold.

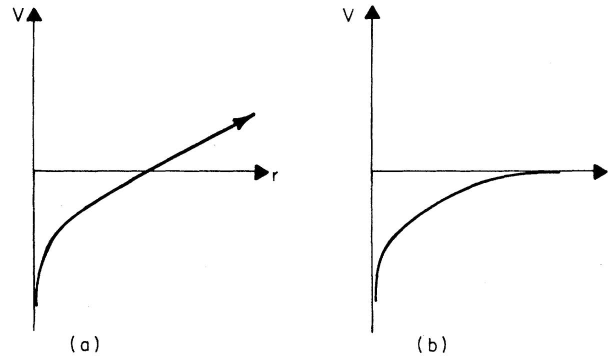

FIG. 2. (a) Unscreened potential between quarks. (b) Screened potential. For small (large) r the potential acts between charmed quarks (mesons).

a pair of color spins represented by the SU(3) color λ_α matrices. The interaction between the field $A_{\alpha,\mu}$ and the static sources is given by

$$H_{\text{int}} = A_{\alpha,0}(r_1)\lambda_1^{\alpha} + A_{\alpha,0}(r_2)\lambda_2^{\alpha}, \tag{1}$$

where α is a color index, $A_{\alpha,0}$ is the time component of $A_{\alpha,\mu}$, and $\lambda_{1,2}$ are the color-spin degrees of freedom of the two static sources. Suppose that the ground-state eigenvectors and eigenvalues of this static-sorce Hamiltonian are

$$|\chi\{r_1,r_2\}\rangle,\ \ E(r_1,r_2), \tag{2}$$

where r_1 and r_2 are thought of as parameters in the Hamiltonian. The adiabatic approximation to this problem consists in assuming that at any instant when the heavy quarks are located at r_1 and r_2, the other degreees of freedom are described by the state $|\chi\{r_1,r_2\}\rangle$. Roughly speaking, the adiabatic approximation assumes that the ordinary-quark degrees of freedom (i.e., the $\mathfrak{N}$ and $\mathcal{P}$) have time to continually readjust to the slowly moving sources and remain in the ground state.

If the adiabatic approximation is good, then it follows that at every instant the potential energy stored in the Yang-Mills and light-quark fields is just $E(r_1,r_2)$ which is defined as

$$E(r_1,r_2) = V(r_1 - r_2). \tag{3}$$

Then the system is described by the Hamiltonian

$$H = m_1 + m_2 + p_1^2/2m_1 + p_2^2/2m_2 + V(r_1 - r_2). \tag{4}$$

For the moment ignore the light quarks and consider only the contribution of the Yang-Mills field to the potential V. For small r, asymptotic freedom[6] gives

$$V(r) \sim \frac{g^2}{r\ln(r_0/r)},\ \ r \ll r_0. \tag{5}$$

It is hoped that as $r \to \infty$ the Yang-Mills field energy will grow and confine quarks. In particular, theories of this phenomenon have been advanced in which the Yang-Mills electric field between the pair is distorted to a tubelike configuration when r becomes large.[7] In this event the energy will grow linearly with distance as in Fig. 2(a).

Next consider the effects of the light quarks. As r and the field energy increase, it will eventually become energetically favorable to lower the energy by producing ordinary quark pairs from the vacuum. The point is that the long-range force can be screened by creating a color-neutralizing light quark near each heavy quark. In fact, when $r \to \infty$ the state of lowest mass will consist of a pair of color-neutral charmed mesons. Thus, as $r \to \infty$ the energy of a c-$\bar{c}$ pair plus screening cloud becomes the sum of the masses of two charmed mesons. Accordingly the final potential will resemble that in Fig. 2(b).

In this adiabatic picture the potential at short distances represents the c-$\bar{c}$ potential, but at larger distances it represents the potential between charmed mesons. Thus, the finite ionization potential of Fig. 2(b) is associated with charmed-meson production and not with charmed quarks. Likewise, at small distances the system's wave function describes two half-spin quarks, and at large distances two mesons. In e^+e^- annihilation the virtual photon directly decays into a c-$\bar{c}$ pair in a ^3S wave. As the quarks separate they evolve into a pair of mesons in a P wave. Therefore, the calculation of R must in principle confront some angular-momentum questions. However, we have found that as long as the P-wave mesons are produced at a distance so large that their centrifugal repulsion is insignificant, then R can be calculated just in terms of the c-$\bar{c}$ wave function at the origin and quasifree heavy-quark kinematics. A fuller justification of this scheme will be reported in the near future.

We shall sketch now the calculation of R. For simplicity we approximated the potential of Fig. 2(b) by

$$V(r) = ar - b, \quad r < b/a, \qquad V(r) = 0, \quad r > b/a, \tag{6}$$

where the constant a is determined (essentially) by fitting the energy difference between the $\psi(3100)$ and the $\psi(3695)$. The constant b is adjusted by requiring the charmed-meson threshold to be at about 4.0 GeV as suggested by several authors.[2] In fact, the numerical analysis of the model was performed for several choices[8] of b. Typical values of a and b are

$$a = 0.25, \quad b = +1.72, \tag{7}$$

in GeV units.

Before solving the Schrödinger equation with the potential of Eq. (6), it is instructive to consider the potential of Fig. 2(a). Given the parameters of Eq. (7), one then finds a bound 3S state at $E = 4.2$ GeV. Thus, for the screened potential of Eq. (6), this state must lie in the continuum and become a *virtual bound state.* The presence of this virtual state causes nearby S waves to be highly distorted and concentrated at the origin. This physical effect lies behind the large enhancements in R discussed below.

The calculation of the e^+e^- annihilation cross section in this model consists of two parts. First, there is the continuum wave function $\psi(r)$ of the c-$\bar{c}$ pair. The annihilation process samples the wave function at zero c-$\bar{c}$ separation. Second, there is the cross section for the production of a quasifree heavy c-$\bar{c}$ pair. This factor reads

$$\sigma = (4\pi\alpha^2/3s)3(\tfrac{2}{3})^2(1 - s_0/s)^{1/2}(1 + s_0/2s), \tag{8}$$

where s_0 is the position of the charmed-meson threshold. If we put these factors together, the c contribution to R (call it R_c) reads

$$R_c = 1.33(1 - s_0/s)^{1/2}(1 + s_0/2s)|\psi(0)|^2. \tag{9}$$

The continuum wave function[9] evaluated at the origin, $\psi(0)$, can have strong dependence on s.

The Schrödinger equation with the potential of Eq. (6) was solved numerically and $|\psi(0)|^2$ was tabulated as a function of energy. The qualitative features of $\psi(0)$ are as follows: Very near threshold it is large but as s increases it falls rapidly having additional smaller enhancements. It is crucial to note that the expression for R_c also contains the phase-space factor $(1 - s_0/s)^{1/2}$ which vanishes at $s = s_0$ and approaches unity as s grows. When these two factors are multiplied, one obtains a curve with oscillations. The amplitudes of these oscillations diminish as s grows [see Figs. 1(a) and 1(b)]. These calculations were repeated for several choices of the relative position of the threshold and the virtual bound state. In Fig. 1(a) the virtual state lies roughly 95 MeV above threshold. In Fig. 1(b) the difference is about 125 MeV. In these figures we have plotted the *total* R,[10]

$$R \simeq R_c + 2.5, \tag{10}$$

where R_c is defined above, and 2.5 is the height of the continuum contribution to R below the threshold.[1]

We conclude with several remarks. The presence of the first dip after threshold is due to the competition of a rising kinematic factor and a falling dynamical factor. The first enhancement in R beyond 4.0 GeV is, therefore, not indicative of a resonance. Instead it is an S-wave enhancement due to the virtual 3S state. Needless to say, the experimental verification of these structures should be important evidence for the existence of charmed quarks.

Finally, we believe that the general features of Figs. 1(a) and 1(b) are common to many models. The quantitative details are, however, difficult to calculate in a truly convincing, fundamental fashion.[11] Our model calculation should serve as a guide to experimenters and as a crude suggestion to theorists.

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[11]The reader should note that a nonrelativistic description of the high-energy end of Figs. 1(a) and 1(b) is very crude.

ψ Particles, SU(4), and Anomalous Currents

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We propose assignment of the new ψ particles in an SU(4) symmetry scheme, incorporating SU(3) and a new additive quantum number, paracharge Z. Anomalous currents are introduced to account for the production and decays of these particles. Two more rather broad resonances at around 4.3 and 5.0 ($\pm$0.2) GeV are expected in the e^+e^- annihilation, one of which has recently been reported (at 4.15 GeV).

We describe in the present note a phenomenological scheme for the new narrow ψ particles[1] at 3.105 and 3.695 GeV and the (possible) resonance[2] at 4.15 GeV. We shall regard them as hadrons. The decay width of a typical hadron having a mass of a few GeV is generally expected to be quite sizable, unless forbidden by some selection rule. We, therefore, introduce a new additive quantum number, called[3] the "paracharge" Z. While all the earlier known particles will be assumed to have $Z=0$, the $\psi(3.1)$ and the $\psi(3.7)$ will be assigned $Z\neq 0$. Only strong interactions are supposed to conserve Z. The observed small width of the $\psi(3.1)$ will thus be accounted for. The strong decays $\psi(3.7)\rightarrow\psi(3.1)+$ hadrons ($Z=0$, e.g., 2π) are allowed by quantum numbers but are suppressed for reasons to be discussed below.

The observation of the $\psi(3.1)$ and the $\psi(3.7)$ in Bhabha scattering will be assumed to proceed directly through the electromagnetic current ($e^+e^-\rightarrow$ virtual $\gamma\rightarrow\psi\rightarrow$ virtual $\gamma\rightarrow e^+e^-$). Thus the ψ particles will be assumed to have $J^P=1^-$ and charge conjugation $C=-1$. This naturally leads us to postulate new additional Z-changing anomalous (nonminimal) pieces in the phenomenological *hadronic* electromagnetic current *(but not in the well-established electromagnetic current of the charged leptons)*.

We incorporate $U(1)_Z$ along with the conventional SU(3) into a larger SU(4) symmetry group of strong interactions, regularly broken according to the chain $SU(4)\supset SU(3)\otimes U(1)_Z\supset SU(2)_I\otimes U(1)_Y\otimes U(1)_Z$.

The $\psi(3.1)$ has $I=Y=0$ and is the $C=-1$ mixture of $Z=\pm 1$ states of a near ideally mixed $\underline{15}\oplus\underline{1}$ multiplet containing the vector mesons of the ρ family. A new vector state, the P, with $I=Y=Z=0$ and $C=-1$ is also expected in this family (ac-