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Thermodynamic Temperature Scale Derived from Measurements of ^{3}He Latent Heat*

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The latent heat of ^{3}He has been measured along the melting curve from 23 to 1 mK. With these results an absolute temperature scale is established which depends only on measurements of heat, volume, and pressure and the assumption that the entropy of solid ^{3}He approaches $R\ln 2$ at high temperatures.

It has been recognized for some time that the low-temperature ^{3}He melting curve presents the same essential ingredients[1-3] of a thermometric reference standard as have been successfully exploited in the case of the vapor-pressure–temperature relations of ^{3}He and ^{4}He. The region below 3 mK is of considerable current interest following the discoveries of superfluid pairing phenomena in liquid ^{3}He,[4] and a nuclear magnetic condensation in solid ^{3}He.[5] The measurement of melting-curve pressure provides an intermediate variable which permits comparison of different thermometers used in various laboratories. These are usually calibrated in some way that has its basis in the vapor-pressure standards T_{L55} and T_{62}.[6] One exception is the Johnson-noise thermometer,[7] another is the thermodynamic temperature of a paramagnetic salt as determined from its demagnetization isentropes.[8] In this work we propose and develop a method that determines absolute thermodynamic temperatures of melting ^{3}He, in particular in the low-temperature region 23 to 1 mK. With ^{3}He itself the subject of intense experimental interest it is important to have a fundamental determination of the P-T relation on the melting curve complementary to its use as a transfer standard.

Our approach depends on measurement of the latent heat of converting liquid to solid. This was performed on a ^{3}He sample that was self-cooled by adiabatic compression. A detailed description of apparatus and techniques is given elsewhere.[9] Determination of the latent heat required precise measurement of heat, pressure, and volume. Pressure and volume gauges had absolute accuracies of 0.01% and 1%, respectively. The heat was pulsed into the ^{3}He through a heater that was uniformly distributed throughout the sample region. The heater was thermally isolated from the chamber walls and the heater power was measured with a four-terminal technique to an accuracy of 0.3%. The ^{3}He sample volume was 4.34 cm^3 with 0.169 mole of ^{3}He liquid just pressurized to the melting curve at an initial temperature of 19 mK.

At the beginning of a compressional-cooling experiment the Pomeranchuk cell was thermally isolated from the dilution refrigerator by actuating a tin thermal switch, following which the ^{3}He was cooled by compression to a particular melting-curve pressure. Then measurements of the latent heat were obtained using an electrically activated ^{4}He pressurizer.[10] This device consisted of a stainless steel, 5-cm^3 bomb containing ^{4}He placed in the 4.2-K helium bath. It was connected at low temperatures to the ^{4}He side of the compression cell. The bomb was virtually filled with heater wire, such that when current was increased (or decreased) the ^{3}He sample was given a compression (or decompression) pulse. The characteristic response time for these pulses was 0.3 sec. This pressurizer was electrically connected to the output of the ^{3}He-pressure-gauge capacitance bridge which permitted the regulation of the ^{3}He pressure at any selected value. In our cell[5] the liquid ^{3}He and the liquid-solid interface have a thermal equilibrium time of 6 sec at 20 mK which decreases approximately as T^2 to less than 0.3 sec at 3 mK. Regulation of the pressure on the melting curve, then, regulates the temperature. When a heat pulse ΔQ is applied to the ^{3}He heater, the feedback system containing the ^{4}He pressurizer changes the ^{3}He volume ΔV in order to maintain constant pressure and temperature. During this process Δn moles of ^{3}He are converted to solid such that $\Delta n = \Delta V/(v_s - v_l)$. The latent heat of this conversion, $\Delta n T(s_s - s_l)$, just balances the heat input ΔQ. s_s, s_l and v_s, v_l are molar entropies and molar volumes of solid and liquid. Combining the above with the Clausius-Clapeyron equation $(dP/dT)_{\rm mc} = (s_s - s_l)/(v_s - v_l)$, we obtain

$$T(dP/dT)_{\rm mc} = \Delta Q/\Delta V. \qquad (1)$$

This quantity was measured as a function of melting pressure from 33.513 to 34.394 bar, corresponding to a temperature range of 23.2 to 1.1 mK. It is convenient to refer pressure measurements on the melting curve to the A transition, where the specific-heat discontinuity at P_A gives a reproducible and clearly defined signature in a pressurization trace. The pressure-temperature relation along the melting curve can be obtained in terms of the temperature of the A transition T_A,

$$T/T_A = \exp \int_0^{P-P_A} \{T(dP/dT_{\rm mc})\}^{-1} dP. \qquad (2)$$

The integration variable P is the pressure difference from the A transition. Using the notation $T^* \equiv T/T_A$ we can determine $(dP/dT^*)_{\rm mc}$ from the $\Delta Q/\Delta V$ data, where $(dP/dT^*)_{\rm mc} T_A$ is the melting-curve slope. The integral in (2) was evaluated with more than 200 data points from which the smoothed values given in Table I were obtained. From (2) it can be shown that relative temperatures measured in this way have an accuracy that decreases as $\delta \ln T^*$, where $\delta = 2\%$ is the maximum error in the measurement of $\Delta Q/\Delta V$. A negligible inaccuracy is introduced from the measurement of pressure itself since a linear gauge is used to determine small differences from the fiducial pressure P_A.

To complete a thermodynamic determination of the P-T relation of ^{3}He it is necessary to find T_A on the Kelvin temperature scale. With this in mind we rewrite the Clausius-Clapeyron equation in the framework of the T^* scale: $(dP/dT^*)_{\rm mc} = (s_s^* - s_l^*)/(v_s - v_l)$, where $s_s^* \equiv s_s T_A$ and $s_l^* \equiv s_l T_A$. In its high-temperature-series expansion[11] the entropy of solid ^{3}He approaches $R \ln 2$ according to $s_s^* = T_A R \ln 2 - \overline{C}/T^{*2}$, where $\overline{C}$ is a constant and when T^* is sufficiently large that $T^{*2} \gg \overline{C}/T_A R \ln 2$. Substituting s_s^* in the Clausius-Clapeyron equation we find

$$T_A = (R \ln 2)^{-1} \times [s_l^* + \overline{C}/T^{*2} - (v_l - v_s)(dP/dT^*)_{\rm mc}]. \qquad (3)$$

The molar volume difference $v_l - v_s = 1.314$ cm^3 mole^{-1} was measured at 20 mK.[9] With some theoretical bias as to the nature of the temperature dependence of s_l^* such as $s_l^* = a_1 T^* + a_2 T^{*2} + \ldots$, the above equation represents a multiparameter fit using the data in the table for the coefficients T_A, $\overline{C}$, a_1, a_2 .... In practice it is more accurate to determine s_l^* separately from liquid-specific-heat experiments. Measurements[9] of $c_l^* \equiv c_l T_A$, performed in the range $0.51 < T^* < 8.22$, were used to integrate $c_l^* T^{*-1}$ yielding $s_l^* = (2.72 \times 10^3 T^* - 9.5 T^{*2})$ erg mole^{-1}, for $T^* > 1$. Substitution of s_l^* in (3) permits calculation of both T_A and $\overline{C}$. The results are plotted in Fig. 1 at each T^* for which a latent-heat measurement was obtained. From the figure it is apparent that the high-temperature region in which Eq. (3) holds corresponds to $T^* > 3$. We find that $T_A = 2.75 \pm 0.11$ mK and $\overline{C}/R = 0.6$ mK fit the data best. It is important to emphasize that the first two terms in (3), the liquid entropy and the deviation of the solid entropy from $R \ln 2$, have only a

TABLE I. Smoothed values of the P-T relation for melting ^{3}He are expressed relative to the melting-curve fixed point (P_A, T_A), where T_A was found to be 2.75 ± 0.11 mK and $T^* \equiv T/T_A$.

$P-P_A$ (bar)	dP/dT^* (bar)	T^*	$P-P_A$ (bar)	dP/dT^* (bar)	T^*	$P-P_A$ (bar)	dP/dT^* (bar)	T^*
0.050	−0.075 02	0.432	0.015	−0.094 24	0.844	−0.100	−0.109 04	1.954
0.045	−0.077 91	0.497	0.010	−0.095 88	0.896	−0.200	−0.111 06	2.860
0.040	−0.081 13	0.560	0.005	−0.097 15	0.948	−0.300	−0.110 84	3.761
0.035	−0.084 30	0.621	0.000	−0.098 15	1.000	−0.400	−0.109 86	4.667
0.030	−0.087 36	0.679	−0.005	−0.099 14	1.050	−0.500	−0.108 49	5.583
0.025	−0.090 10	0.735	−0.010	−0.100 01	1.100	−0.600	−0.106 93	6.511
0.020	−0.092 38	0.790	−0.020	−0.101 54	1.200	−0.700	−0.105 40	7.453

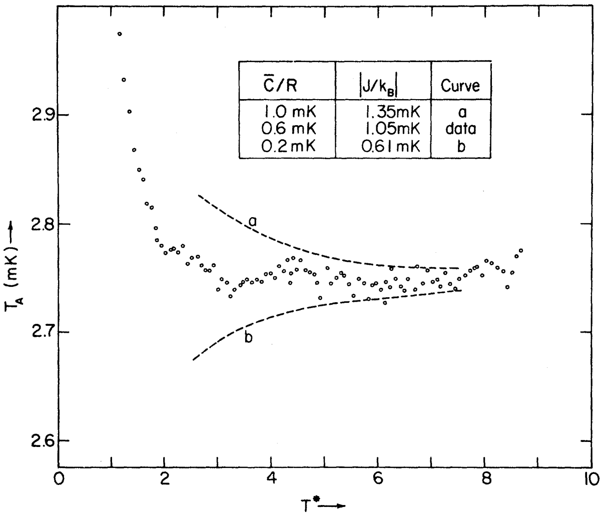

FIG. 1. Latent-heat and specific-heat data are used to calculate T_A according to Eq. (3). The high-temperature region where this equation is appropriate corresponds to the portion of the graph which is independent of $T^* \equiv T/T_A$. The dashed lines show the effect on the calculation of T_A of allowing for a wide range in solid-^{3}He exchange interaction J.

small effect on the calculation, contributing 10% to T_A. In particular the calculation of T_A is essentially independent of the latter when $T^* > 6$. This can be seen in the figure where alternative calculations are shown as dashed lines with $\bar{C}/R$ arbitrarily set equal to 0.2 and 1 mK. The precision of the method is displayed by directly introducing the raw data into (3) in the form $(v_l - v_s) \times (dP/dT^*)_{\rm mc} = (v_l - v_s)\Delta Q/\Delta V T^*$. In the simple one-parameter formulation of the Heisenberg model for ^{3}He, $\bar{C}$ is related to the exchange energy J by $\bar{C}/R = \frac{3}{2} T_A^{-1}(J/k_B)^2$. $\bar{C}/R = 0.6$ mK implies that $|J/k_B| = 1.05$ mK is somewhat higher than previously reported values[11] 0.7 and 0.8 mK.

In this work the melting-curve fixed points in zero magnetic field are found to be the minimum, $P_{\rm min} = 29.316$ bar; the A transition, $P_A = 34.342$ bar, $T_A = 2.75 \pm 0.11$ mK; the B transition, $P_B - P_A = 19.9 \pm 0.1$ mbar, $T_B = 2.18 \pm 0.10$ mK; and the solid ordering transition, $P_S - P_A = 52.3 \pm 0.2$ mbar, $T_S = 1.10 \pm 0.06$ mK. Analysis of preliminary latent-heat measurements yielding $T_S = 1.17$ mK was reported by Halperin *et al.*[5] This value reflects a small systematic error corrected in the present work and discussed elsewhere.[9]

Since the discovery of superfluidity in liquid ^{3}He the A transition has been used as a reference point in melting-curve measurements. Johnson-noise temperatures at P_A of 2.66 and 2.81 mK have been reported by Johnson *et al.*,[3] and Wheatley.[12] They have suggested that their values are too large because of small heat leaks into the thermometers. Webb, Giffard, and Wheatley[7] have used a thermodynamic cerium magnesium nitrate temperature scale proposed by Fisher *et al.*[8] to deduce that $T_A = 2.58$ mK. Measurements with a platinum-powder NMR thermometer by Ahonen *et al.*[13] give results consistent with our value of 2.75 mK. Alvesalo *et al.*[14] have extended melting-curve calculations to low temperatures with a thermometer based on viscosity measurements of liquid ^{3}He. They find $T_A = 2.57$ mK. This value reflects the choice by Alvesalo *et al.*[14] of a high-temperature melting curve.[1,2] Alternatively, if the viscosity data are analyzed with either the high-temperature melting curve of Grilly[15] or that given below in the present work it is found that the corresponding value of T_A increases to 2.67 mK in reasonable agreement with the results reported here.

Calculation of the melting-curve slope from the Clausius-Clapeyron equation and experimentally or theoretically derived molar entropies and volumes has been reported by Thompson and Meyer,[16] Goldstein,[17] and Grilly.[15] Our measurements have been extended to higher temperatures in this way and are analytically expressed as $P(T) - P_A = 0.6359 \times 10^{-7}/T^2 - 0.1044 \times 10^{-3}/T + 0.1492 - 43.86T + 137.0T^2 - 65.38T^3 - 1590.0T^4 + 6294.0T^5$ (bar), $0.003\ \mathrm{K} < T < 0.100$ K. The corresponding melting-curve slopes agree well with those calculated by Grilly[15] for $T > 0.02$ K.

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Indications of Strongly Flux-Limited Electron Thermal Conduction in Laser-Target Experiments*

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It is shown by comparison with calculations that anomalies in the results of intense laser irradiation of solid targets, including two-humped ion distributions, indicate a reduction of electron thermal conduction to considerably below classical values. This reduction is interpreted as a flux limit and appears to be sufficiently restrictive to modify significantly the design of laser fusion targets.

Most of the recent interest in laser-produced plasmas has focused on conditions under which electron thermal conduction is expected to play a central role in the transport of energy in the plasma. In particular, in laser fusion applications, the ablation-driven compression and thermal heating of the target core are strongly influenced by the manner in which the laser energy deposited near the critical density ($n_c = 10^{21}$ cm^{-3} for Nd-glass and $n_c = 10^{19}$ cm^{-3} for CO$_2$ lasers) is transported into the target interior. It is the purpose of this Letter to point out that certain anomalies seen recently in high-power-density laser-plasma experiments indicate that electron thermal fluxes near critical density are limited to much less than the classical or mildly flux-limited values that have previously been assumed. This more stringent flux limiting would constitute an energy decoupling mechanism which would significantly modify design of fusion targets.

Previous predictions of experimental results were based on calculations of coupled hydrodynamics and heat flow in which the electron thermal flux was given by

$$F = \begin{cases} F_c, & F_c < F_l, \\ F_l, & F_l < F_c, \end{cases} \qquad F_c = -K_c \nabla T_e, \qquad F_l = f n_e k T_e (kT_e/m_e)^{1/2}, \tag{1}$$

where K_c is the classical conductivity of Landshoff and Spitzer[1] and F_l is an upper limit imposed to approximate the free-streaming, high-flux failure of the perturbation derivation of K_c.[2] Here m_e, n_e, and T_e are the electron mass, number density, and temperature, respectively, and f is a dimensionless number which is calculated to be between 0.5 and 1.0, depending on assumptions about collisionless processes, but is commonly taken to be about 0.6.[3,4] Here we show, by detailed experimental-calculational comparisons, that $f = 0.6$ is highly inconsistent with experimental results, but that the discrepancies are largely removed and certain new experimentally observed effects are given a plausible physical explanation by using the same form, Eq. (1),