Orbital Excitations in Charmonium*

Barry J. Harrington, Soo Yong Park, and Asim Yildiz
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
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We use a linear potential to analyze the orbital excitations of a bound charmed quark-antiquark system. The model predicts that a 235-MeV γ ray should be emitted in the $E1$ transition from the $2\,^3S$ to be $1\,^3P_1$ state.

In a previous Letter,[1] we considered the recently discovered[2] narrow resonances at 3.105 and 2.695 GeV as bound states of a charmed quark and its antiquark[3] in a linear potential. An exact solution of the nonrelativistic Schrödinger equation for the S states ($l=0$) was used to predict the existence of higher radial excitations, such as the $3\,^3S_1$ state at 4.18 GeV. Values for the total decay widths of the known resonances were calculated and are consistent with the experimental results.

Here we extend the spectrum of the linear potential to the lower-lying $l\neq 0$ states. Eschewing an exact solution, we revert to the WKB approximation in which the quantization condition is specified by

$$I=\int_{r_1}^{r_2} dr\left(E-\frac{(l+\frac{1}{2})^2}{m_{\mathcal{P}'}r^2}-Kr\right)^{1/2}$$

$$=(n'+\tfrac{1}{2})\frac{\pi}{(m_{\mathcal{P}'})^{1/2}}, \qquad (1)$$

where r_1 and r_2 are the classical turning points, $m_{\mathcal{P}'}$ is the mass of the charmed quark, E is the binding energy in the potential Kr ($K=0.211$ GeV^2 while $m_{\mathcal{P}'}=1.16$ GeV),[1] and n' is one less than the principal quantum number. An evaluation of Eq. (1) in terms of the complete elliptic integrals[4] of the first $[K(m)]$, second $[E(m)]$ and third $[\pi(m,n)]$ kinds yields

$$I=\left(\frac{Kr_0^3}{t_2-t_0}\right)^{1/2}\left[(1+t_0)K(m)+(t_2-t_0)E(m)+\frac{2\gamma-1}{1+t_2}\pi(n,m)\right] \qquad (2)$$

where $t_2>t_1>t_0$ are the solutions of the cubic equation $t^3+\frac{3}{2}t^2-\gamma=0$, with $t=(r-r_0)/r_0$,

$$\gamma=\tfrac{1}{2}-(l+\tfrac{1}{2})^2/m_{\mathcal{P}'}Kr_0^3, \quad m=(t_2-t_1)/(t_2-t_0),$$
$$n=-(t_2-t_1)/(1+t_2)$$

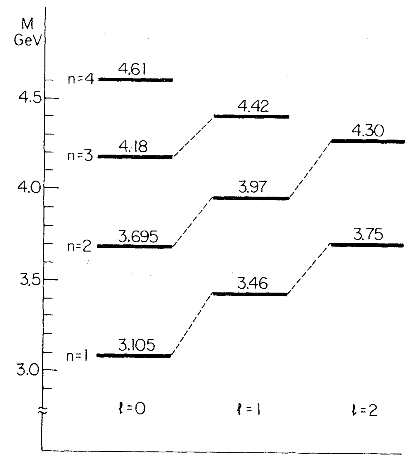

FIG. 1. Charmonium energy levels. Here, n is the principal quantum number and l is the orbital quantum number.

while the mean radius of the S states $\langle r\rangle\equiv r_0=\frac{2}{3}E/K$. The validity of Eq. (2) for the lower-lying S states can be checked with the known exact results and the agreement is excellent.[5] A straightforward numerical evaluation generates the $l\neq 0$ spectrum, the first few states of which are presented in Fig. 1.

The two most interesting results[6] are the positions of the $1\,^3P_1$ and $1\,^3D_1$ states. The former state, at 3.46 GeV, could indicate the existence of a non-Coulomb force law since the first Coulomb P state would normally be degenerate with the $2\,^3S_1$ state. In the $E1$ transition from the $2S$ to the $1P$ state, a photon of momentum 235 MeV should be ejected. The latter state, $1\,^3D_1$, is slightly above the $2S$ resonance which raises the possibility of D-S mixing.

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[4]V. M. Belyakov, P. I. Kravtsova, and M. G. Rappoport, *Tables of Elliptical Integrals* (Pergamon, New York, 1965), Pt. 1.
[5]Compared to the exact values for the total energy of the S states, the WKB result is accurate to within 1%.
[6]While this work was being completed, we received a preprint of E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975), in which a superposition of the Coulomb and linear potentials is used in a nonrelativistic Schrödinger equation. Their numerical solution leads to a $^3P_1$ state at 3.465 GeV and a $^3D_1$ state at 3.765 GeV.

Comment on Nonleptonic Decays of Charmed Hadrons*

A. Pais and V. Rittenberg
The Rockefeller University, New York, New York 10021
(Received 3 February 1975)

Within the framework of a U(4) symmetry, a discussion of the nonleptonic decays of charmed hadrons under the assumption of pentadecaplet dominance leads to conclusions which may be relevant to impending charmed-particle searches.

The most commonly held view about the newly discovered particles at 3095 MeV [1] and 3684 MeV is that a new quantum number is necessary for their interpretation. Various suggestions in this direction have been advanced recently. Among these, one proposal made some time ago is receiving particular attention. This is the charm scheme: a strong-interaction group U(4) with a new additive quantum number C broken only by weak interactions.[3] The basic sets of quarks (q) are $\mathcal{P},\mathfrak{N},\lambda,\mathcal{P}'$ with respective charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, $\frac{2}{3}$. The $\underline{15}$ representation of U(4) for mesons contains a $C=1$, $S=0$ isodoublet D^+,D^0, a $C=1$, $S=1$ isosinglet F^+, and their conjugates, and also a new self-conjugate isosinglet. In this context, it is conjectured that the 3095 is an extremely pure $\overline{\mathcal{P}}'\mathcal{P}'$ state with $J^P=1^-$ which decays via dynamically suppressed strong interactions and via electromagnetic interactions.[4]

Experimental proof of the existence of those C-carrying hadrons which can only decay weakly would strongly indicate that this is the right track. Leading contenders for such particles are pseudoscalar mesons of the D and/or F type. The signature for such particles is not just dictated by the approximate strong U(4) (which leads to guesses for mass values) but also by the accompanying structure of the weak interactions. Here the principal candidate is surely the weak $|\Delta Q|=1$ hadronic current dictated by the Glashow-Iliopoulos-Maiani (GIM) suppression mechanism.[5] Its particle structure is given by $\overline{\mathcal{P}}\mathfrak{N}_c+\overline{\mathcal{P}}'\lambda_c$, $\mathfrak{N}_c=\mathfrak{N}\cos\theta+\lambda\sin\theta$, $\lambda_c=-\mathfrak{N}\sin\theta+\lambda\cos\theta$, where θ is the Cabibbo angle. Just prior to the recent developments, this particular U(4) version was carefully analyzed by Gaillard, Lee, and Rosner.[6] Unless further dynamical conditions are imposed, the main two-particle nonleptonic channels are those with rates proportional to $\cos^4\theta$: $D^0\to K^-\pi^+$, $D^+\to\overline{K}^0\pi^+$, $F^+\to\overline{K}^0K^+$, $\eta\pi^+$. Exotic decays, like $D^+\to K^-\pi^+\pi^+$, are in principle allowed. An SU(3) analysis of nonleptonic $|\Delta C|=1$ modes has recently been carried out.[7] Confirmation of these various signals would not only favor impressively the U(4) scheme but, more specifically, would strongly indicate the direction that gauge theories must follow in their further development.

Much remains to be learned about this last subject. Thus it is known that the suppression of $|\Delta S|=1$ neutral currents, for which purpose the GIM current is so aptly designed, can come about in alternative ways. Such alternatives may arise, for example,[8] if the underlying gauge theory contains more currents than is the case for SU(2) $\otimes$ U(1). This is neither the time nor the place to