How Well Can the Chew-Low Theory Reproduce Pion-Nucleon Scattering?

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When the Chew-Low theory for the πN interaction is used as a basis for studying the π-nucleus interaction, it is required to fit the free πN scattering data well. In this respect many calculations so far done are unsatisfactory. We show that an excellent fit is obtained within the existing Chew-Low framework if the inelasticity together with recoil and the crossing term are taken into account, and we comment on a recent work by Dover *et al.*

The pion-nucleon (πN) interaction is a basic ingredient in any theory for the pion-nucleus interaction. In the medium-energy range, the πN interaction is dominated by the 33 resonance, corresponding to $\Delta(1236)$. Besides a variety of *ad hoc* separable interactions, a commonly accepted model for the πN interaction in this energy range is the well-known Chew-Low (CL) theory,[1] which has successfully explained main features of πN scattering including the existence of the 33 resonance. Since the CL theory is an approximate one, we would not expect that it reproduce all the details of the πN data. However, if we are trying to derive the π-nucleus interaction from a πN interaction model, the latter should reproduce the free πN data as well as possible. Otherwise, for example, one could not determine how the πN interaction is modified when the nucleon is bound.

In the CL theory, if the πN coupling constant is fixed to its experimental value ($f^2 \approx 0.08$), we are left with only one adjustable element, i.e., the form factor v_k or, more simply, the cutoff energy. As is probably well known, the CL theory, in its static form with only one adjustable parameter, cannot fit πN scattering data very well. For example, Eisenberg and Weber[2] used the CL theory in their π-nucleus calculations. Although they fitted the 33-resonance energy, the calculated width was much too large. Also they had to use a large cutoff energy, $\sim 0.4\mu$ (μ is the pion mass), which is not very satisfactory because the CL theory is a nonrelativistic one. Dover and Lemmer's calculation[3] is very similar in this respect.

Recently Dover *et al.*[4] proposed to replace the energy-independent coupling constant λ_α (defined later) in the CL theory by $\lambda_\alpha\gamma_\alpha(\omega)$, where $\gamma_\alpha(\omega)$ is a complex function of the pion energy ω, and showed how $\gamma_\alpha(\omega)$ and the form factor v_k can be determined from given πN scattering data. In other words one can find $\gamma_\alpha(\omega)$ such that the πN data are exactly reproduced. However, the relation between $\gamma_\alpha(\omega)$ and more basic πN interactions such as the CL interaction is not clear.

The purpose of this note is to show that the CL theory can in fact fit the πN data including inelasticity within its existing framework, without introducing such an energy-dependent, complex coupling constant. We also point out that effects of recoil and the crossing term are instrumental in fitting the data with a reasonable cutoff energy.

We start with the static model. The Hamiltonian is given, in standard notation, by[5]

$$H = \sum_k \omega a_k^\dagger a_k + \sum_k (V_k a_k + V_k^\dagger a_k^\dagger), \tag{1}$$

$$V_k = i(4\pi)^{1/2}(f_0/\mu)\vec{\sigma}\cdot\vec{k}\tau_k v_k/(2\omega)^{1/2}. \tag{2}$$

Here f_0 is the unrenormalized coupling constant, and $\omega = (\mu^2+k^2)^{1/2}$ is the pion energy, etc. We confine ourselves to the 33 state throughout. A solution of the Low equation for $h_3(\omega) = \exp(i\delta_3) \times \sin\delta_3/(k^3 v_k^2)$ is given in the form[6,7]

$$h_3(\omega) = \lambda_3/\omega g_3(\omega), \tag{3}$$

with

$$g_3(\omega) = 1 - \frac{\lambda_3}{\pi}\omega\int_\mu^\infty d\omega' \frac{k'^3 v_{k'}^2}{\omega'^2}\left(\frac{F_3(\omega')}{\omega'-\omega-i\epsilon} + \frac{G_3(\omega')}{\omega'+\omega}\right). \tag{4}$$

Here the subscript 3 refers to the 33 state, and $\lambda_3 = \frac{4}{3}f^2$, f being the renormalized coupling constant. The function F_3 is determined by

$$F_3(\omega) = [\sigma_{tot}(\omega)/\sigma_{el}(\omega)]_3 = 1 + (\sigma_{inel}/\sigma_{el})_3. \tag{5}$$

The phase shift δ_3 becomes complex when $F_3 > 1$. In the one-meson approximation, $F_3 = 1$ at all energies. The term with G_3 is referred to as the crossing term. The function G_3 is not well known, but in the one-meson approximation it may be approximated by $G_3 \approx 2$.[7] We assume that

$$G_3(\omega) = 2F_3(\omega). \tag{6}$$

This is the only ambiguity in determining $h_3(\omega)$ in the static CL theory.[8] Dover *et al.*[4] use the one-meson approximation, i.e., $F_3 = 1$. Then, for a real coupling constant, the phase shift δ_3 remains real beyond the π-production threshold. They take account of inelastic channels by replacing λ_3 by an energy-dependent, complex $\lambda_3\gamma_3(\omega)$. Also they ignore the crossing term, i.e., they put $G_3 = 0$. What we want to emphasize in the following is that inelasticity is simply taken care of by F_3, which can be determined by putting experimental data for $\sigma_{\rm inel}/\sigma_{\rm el}$ into Eq. (5). Also the effect of the crossing term will be shown to be very important.

Since we are mainly interested in the medium-energy range where the 33 resonance dominates, let us put $h_3(\omega)$ in the form of the Breit-Wigner formula. First the resonance energy ω_r is determined by $\mathrm{Re}g_3(\omega_r) = 0$. Then we expand $\mathrm{Re}g_3(\omega)$ around ω_r:

$$g_3(\omega) \approx (\omega - \omega_r)\,\mathrm{Re}[g'(\omega_r)] + i\,\mathrm{Im}[g(\omega)] = \mathrm{Re}[g'(\omega_r)]\{\omega - \omega_r + \tfrac{1}{2}\Gamma(\omega)\}, \tag{7}$$

where $g'(\omega) = dg(\omega)/d\omega$ and $\Gamma(\omega) = 2\,\mathrm{Im}[g(\omega)]/\mathrm{Re}[g'(\omega_r)]$.

Next, let us consider the nucleon recoil effect. This is approximately treated by replacing ω in Eq. (1), but not in Eq. (2), by the center-of-mass energy $\tilde\omega = \omega + (m^2 + k^2)^{1/2} - m$, where m is the nucleon mass. Then ω in Eqs. (3)–(7) is replaced by $\tilde\omega$, while Eq. (4) is modified as

$$g_3(\tilde\omega) = 1 - \frac{\lambda_3}{\pi}\,\tilde\omega \int_\mu^\infty d\omega' \frac{k'^3 v_{k'}^2}{\tilde\omega'^2}\left(\frac{F_3(\tilde\omega')}{\tilde\omega' - \tilde\omega - i\epsilon} + \frac{G_3(\tilde\omega')}{\tilde\omega' + \tilde\omega}\right). \tag{8}$$

Note that ω' for $d\omega'$ has not been changed.

We have tried to fit the resonance energy $\tilde\omega_r$ and the width $\Gamma(\tilde\omega_r)$. For the experimental values, we took $\tilde\omega_r = 1233\ \mathrm{MeV} - m = 2.13\mu$, and $\Gamma = 116\ \mathrm{MeV} = 0.84\mu$. For the masses we took $\mu = 138.0$ MeV and $m = 6.80\mu$. We considered three types of form factor v_k: (i) square cutoff, $v_k = \theta(k_c - k)$, (ii) Gaussian cutoff, $v_k = \exp(-k^2/2k_c^2)$, and (iii) Yukawa cutoff, $v_k = k_c^2/(k^2 + k_c^2)$. Since the results are not sensitive to the type of the form factor, we show only the results for the square cutoff. For the inelasticity we assumed that

$$\frac{\sigma_{\rm tot}}{\sigma_{\rm el}} = \begin{cases} 1 \text{ for } \tilde\omega < 4\mu \\ 2.5 \text{ for } \tilde\omega > 4\mu \end{cases}. \tag{9}$$

Note that this $\sigma_{\rm tot}/\sigma_{\rm el}$ corresponds to Dover *et al.*'s $1/\hat\eta$. They plotted experimental values of $\hat\eta$ in their Fig. 1, from which one can see that $1/\hat\eta = 1$ for $\tilde\omega \lesssim 4\mu$ and $1/\hat\eta$ increases rapidly for $4\mu \lesssim \tilde\omega \lesssim 5\mu$, and then it comes down to $1/\hat\eta \approx 2.5$. Equation (9) is a reasonable assumption for $\sigma_{\rm tot}/\sigma_{\rm el}$, although it may somewhat underestimate the inelasticity for $4\mu \lesssim \tilde\omega \lesssim 7\mu$. We have considered cases with and without taking account of effects of recoil, the crossing term, and inelasticity.

We determined the cutoff energy $\omega_c = (\mu^2 + k_c^2)^{1/2}$ such that, together with $\lambda_3 = \frac{4}{3}\times 0.08$, the resonance energy $\tilde\omega_r = 2.13\mu$ is fitted, and then we calculated the width $\Gamma(\tilde\omega_r)$. When the nucleon recoil is not included, we fitted $\omega_r = 2.13\mu$. The results are summarized in Table I. Almost a perfect

TABLE I. The cutoff energy ω_c which, together with $\lambda_3 = \frac{4}{3}\times 0.08$, fits the 33-resonance energy $\tilde\omega_r = 2.13\mu$. Γ is the calculated width (at the resonance energy), which is to be compared with the experimental value $\Gamma = 0.84\mu$. In case C, for example, the recoil effect is taken account of, but the crossing term is not.

Case	Recoil	Crossing term	Without inelasticity ω_c/μ	Without inelasticity Γ/μ	With inelasticity ω_c/μ	With inelasticity Γ/μ
A	No	No	10.51	1.31	6.28	1.14
B	No	Yes	6.39	1.75	4.94	1.61
C	Yes	No	55.30	0.80	14.84	0.68
D	Yes	Yes	15.10	0.92	6.79	0.83

agreement with experimental values is obtained in case D when the inelasticity is taken into account. It is gratifying that the cutoff energy is reasonable in this case. Other cases are clearly inferior in the sense that Γ is much too large and/or ω_c is much larger than m. Case A corresponds to Eisenberg and Weber's calculation[2] except that they took a larger value for the coupling constant, $f^2 = 0.088$, and hence obtained a slightly smaller cutoff energy $\omega_c = 9.4\mu$. It is clear that effects of recoil, crossing term, and inelasticity are all important in improving the fit.

Before closing let us discuss a rather amusing, fictitious problem. Consider scattering of a pion from a nucleon which is bound in an external potential, e.g., a harmonic oscillator potential. If we increase the strength of the potential the nucleon becomes less and less mobile, and in the tight-binding limit the πN scattering amplitude is reduced to that in the static limit in which $m \to \infty$. Using the parameters of case D with inelasticity, we have examined how the 33 resonance varies as m varies from the actual nucleon mass to infinity. We have found that $\tilde{\omega}_r$ and Γ both decrease as m increases, and $\tilde{\omega}_r$ goes down to 1.12μ in the limit of $m \to \infty$. This exercise illustrates an effect of binding on the πN scattering amplitude.

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[5]Units are such that $c = \hbar = 1$.

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[7]E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962), Chap. 18.

[8]Although we feel that Eq. (6) is a reasonable assumption, we admit that it has no solid foundation. Equation (6) may be relaxed by replacing it by

$$\int d\omega' f(\omega, \omega') G_3(\omega') \approx 2 \int d\omega' f(\omega, \omega') F_3(\omega')$$

for not very large values of ω. Here $f(\omega', \omega) = k^3 v_k^2 / \omega^2(\omega + \omega')$.

Effect of Anomalous Neutral-Current Interactions in Threshold Pion Production by Neutrinos*

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Using the techniques of current algebra and soft-pion extrapolation, we have calculated the effect of weak neutral-current interactions involving scalar and pseudoscalar densities on the reaction $\nu + n \to \nu + p + \pi^-$ around the threshold region. We find that although the magnitude of the differential cross section (which depends on quark masses) can be of order 10^{-40} cm^2/(20 MeV), the general shape is different from the one observed experimentally.

Recent experimental observation of muonless events in neutrino inclusive reactions at CERN-Gargamelle[1] and the Fermi National Accelerator Laboratory[2] have established the existence of weak neutral-current interactions between hadrons and leptons. It is, however, too early to decide on the exact nature of these interactions and more detailed exclusive as well as inclusive experiments are clearly necessary for this purpose. From a theoretical point of view, if one believes in the gauge theories, then the nature of the neutral-current interaction regardless of the particular choice of the model is of the type $(\alpha V_\mu + \beta A_\mu) l_\mu$, where V_μ and A_μ are neutral $\Delta s = 0$ hadron currents and l_μ is the lepton current, with a coupling strength of order $G_{\rm F}$. In general, from Lorentz invariance, one could however get other combinations,[3] like SS, PP, TT, and $\tilde{T}\tilde{T}$ (and SP and $T\tilde{T}$ if CP invariance is not assumed), and their contribution to both exclusive and inclusive reactions must be estimated before arriving at the final conclusion regarding the exact structure of these interactions. The aim of the present note is to present an estimate of the contribution