Anomalous dc Resistivity and Turbulent Ion Heating in Isothermal Plasmas*

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Anomalous resistivity and intense ion heating are observed in isothermal plasmas where ion sound is damped, concomitant with onset of resonant discrete and nonresonant continuous spectra of ion-cyclotron drift waves. These results are explained by a theory based on statistically averaged instability amplitudes and frequencies.

Plasma transport processes are greatly affected by plasma instabilities, as has been recognized for some time. For nonisothermal plasmas, where $T_e/T_i \gg 1$, the occurrence of anomalous resistivity together with enhanced ion heating is well documented[1-3] at applied electric fields approaching or larger than the runaway field, $E_r = \nu_{ei} m_e v_{\text{th}e}/e$, and is generally analyzed in terms of current-driven ion sound (for $E/E_r < 1$) and two-stream (Buneman-Budker) instability (for $E/E_r > 1$). However, for isothermal plasmas, where $T_e/T_i \simeq 1$, with $E/E_r < 1$, ion sound is ion Landau damped and electron runaway cannot occur. A theory of instability-generated resistivity for this regime, based on nonresonant and low-frequency drift modes, has been described.[4] It was conceived to explain stellarator and tokamak resistivity results (where $T_e/T_i > 1$), but no instability data were available for comparison with the predictions.

The significant results of this work are measurements of both total and local resistivity and of ion heating for small fields ($E/E_r = \frac{1}{30}-1$) in isothermal plasmas where ion sound is damped, $T_e/T_i \simeq 1$, together with detailed local parallel-drift-velocity determinations and amplitude and frequency measurements for the instabilities causing the anomalies. The instability has been identified as an ion-cyclotron (diamagnetic) drift wave[5] (ICDW) with both discrete and continuous spectra, and has not heretofore been connected with anomalous resistivity and ion heating. The electron drift velocity u saturates at $v_{\text{th}e}/3$; thus the resistivity η is proportional to E, since $I \simeq \text{const}$. Instability-caused ion heating is intense, on a time scale $\tau \simeq 1/\gamma \ll \tau_{\text{Ohmic}}$. It is stronger in the presence of the continuous, turbulent ICDW ($\omega \gg \omega_{ci}$) than in the presence of the discrete, coherent ICDW ($\omega \simeq \omega_{ci}$). A novel consideration derives from the fact that the continuous ICDW is driven by ∇n_e and ∇T_e: Electron momentum is transferred to the wave in the perpendicular direction, different from the resonant, parallel-electron-current mechanism generally connected with inverse Landau damping. Experimental results are explained by a theory[6] based on statistically averaged instability amplitudes and frequencies.

The experiments were performed on the Princeton Q-1 thermally ionized alkali-metal plasma, 126 cm long and 3 cm in diameter, with confining field $B = 1-7$ kG and $\omega_{ce}/\omega_{pe} \gg 1$. The plasma is fully ionized. Current is applied through the end plates, parallel to $\vec{B}$. Plane Langmuir probes can be turned into or away from the electron drift to produce precise local measurements of u, $v_{\text{th}e}$, n, and j simultaneously. Perpendicular ion temperatures ($T_{i0} = 0.4$ eV) are determined by using (Langmuir) probes with a mechanical limiter prohibiting electron collection.[7] An immersed, electrodeless rf conductivity probe[8] ($f = 0.5$ MHz) measures local resistivity based on the dissipation seen by a marginal-oscillator test coil. Since $\omega_{\text{probe}} \ll \nu_e$, this probe determines dc conductivity. Electron-ion relaxation times are ~50 times longer than ion confinement times (~5 msec), so that Ohmic heating of ions can be disregarded.

Measured resistivities, for small currents, agree with classical (Spitzer-Härm) values, when sheath resistances at the end plates can be neglected[9] ($n > 10^{10}$ cm^{-3}). For $u = (5-10)v_{\text{th}i}$, collisional DW's set in (~3 kHz). This instability and the radial transport caused by it are well understood.[10] No observable resistivity change occurs, as expected for the low-frequency DW. For $u = v_{\text{th}e}/10$, an ICW ($\omega = 1.15\omega_{ci}$) destabilizes. This

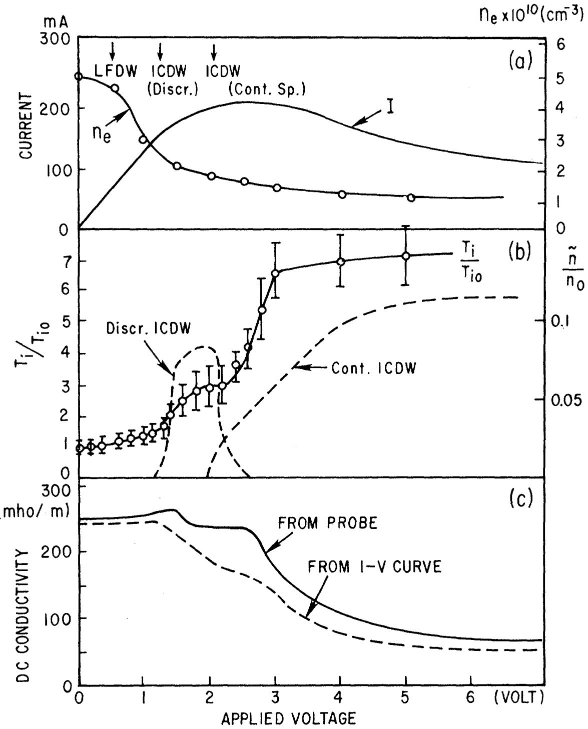

FIG. 1. Evolution of plasma parameters as a function of applied voltage. (a) Current and density; arrows indicate onsets. (b) Normalized T_i and amplitude of ICDW. (c) Conductivity; solid line: conductivity probe $(\sigma_\parallel{}^2+\sigma_\perp{}^2)^{1/2}$; broken line: I-V curve $(\sigma_\parallel)$. $B=6$ kG, $T_{e0}=0.25$ eV, $T_{i0}=0.4$ eV. Q device is double ended.

instability was identified as an ICDW by measurements of ω and $\vec{k}$, and explained by a theory for both the discrete and the continuous ($\omega \simeq 0$–$\omega_{pi}/2$) spectrum regimes.[5] Coincident with onset of the discrete ICDW, moderate increases of η and T_i are observed. As the continuous ICDW becomes dominant, η and T_i increase greatly and current inhibition sets in. Figure 1 indicates the correlation of resistivity anomalies with wave amplitudes, both for the conductivity measured directly and for that calculated from the I-V plot. The conductivity decreases by a factor of 4–5 and the ion temperature increases by an order of magnitude. Ion heating due to the electrostatic ICW excited by filamentary currents in the center of the plasma column (where $\nabla n/n=0$) has been reported previously.[11] The T_e increase measured is small ($T_e/T_{e0} \lesssim 3$); electrons are well connected thermally to the end-plate heat sinks. It was observed that the radial amplitude distribution leads to a resistivity increase in the ∇n region, thus redistributing η radially to produce $\eta(r) \simeq$ const; before the instability onset η_{center} is greater than

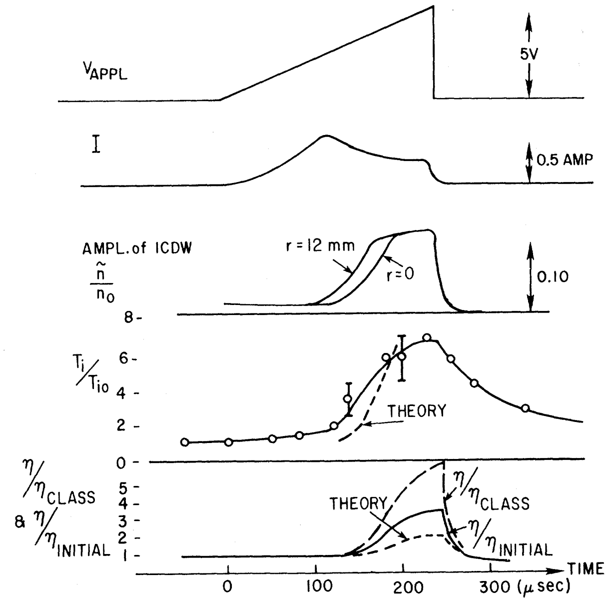

FIG. 2. Time evolution of plasma parameters in pulsed operation. In η plot, solid line is $\eta_\parallel$ from I-V curve. η/η_{class} is normalized to classical resistivity. $\eta_{\text{initial}}=5\times10^{-3}$ Ω m is the Spitzer-Härm value. $B=2$ kG. $n_0=1.1\times10^{11}$ cm^{-3}.

η_{edge}, since $T_{e,\text{center}}$ is less than $T_{e,\text{edge}}$.

The temporal development of instability-caused effects is determined from pulsed measurements (Fig. 2); only the continuous ICDW destabilizes,[5] since $\gamma_{\text{cont}} \gg \gamma_{\text{discr}}$. Coincident with instability onset and current inhibition both η and T_i increase, on a time scale similar to the ICDW growth time, as expected. Normalized resistivity results are compared with those reported for stellarator and tokamak[12] in Fig. 3. The anomalous resistivity onset in our plasma at higher E/E_r is assumed to be due to the greater electron drift velocity necessary to destabilize the ICDW in comparison to

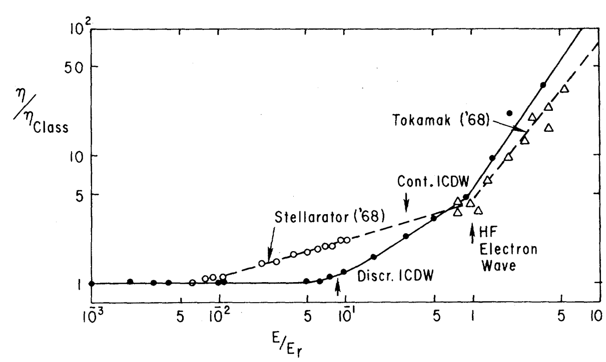

FIG. 3. Normalized resistivity versus E/E. Present experiment: $B=6$ kG, $n_0=6\times10^{10}$ cm^{-3}.

sound-wave destabilization in the stellarator. A high-frequency instability (~ 100 MHz) occurring in our experiment for $E/E_r \geq 1$ has not yet been identified. Impurities and their effects on η are negligible in our plasma.

To calculate resistivity and heating due to the instability we consider a plasma slab with $B = B_z =$ const, density gradient $\partial n/\partial x$, and parallel electron drift velocity u. Anomalous resistivity $\eta_{\rm anom}$ and ion-heating rate $d(nT_i)/dt$ may be expressed[6] as

$$\eta_{\rm anom} = (4\pi n^2 e^2 u)^{-1} \sum_{\vec{k}} \int d\omega\, k_z \,{\rm Im}\,[\chi_e(\vec{k},\omega)] \langle |E^2|(\vec{k},\omega)\rangle, \tag{1}$$

$$d(nT_i)/dt = (6\pi)^{-1} \sum_{\vec{k}} \int d\omega\, (\omega - \omega_i^*)\, {\rm Im}\,[\chi_i(\vec{k},\omega)] \langle |E^2|(\vec{k},\omega)\rangle, \tag{2}$$

with $\omega_{i(e)}^* = (1/n)(dn/dx)(cT_{i(e)}/q_{i(e)}B)k_y$, $\langle |E^2|(\vec{k},\omega)\rangle$ the spectral function of electric field fluctuations, and $\chi_e(\vec{k},\omega)$ and $\chi_i(\vec{k},\omega)$ the polarizabilities of electrons and ions. $\langle |E^2|(\vec{k},\omega)\rangle$ is related to the mean square of the measured n_e fluctuations with the aid of Poisson's law and Parseval's theorem as

$$(\delta n^2)_{\rm ave} = \sum_{\vec{k}} \int d\omega\, [(k/4\pi e)\, {\rm Re}\chi_e(\vec{k},\omega)]^2 \langle |E^2|(\vec{k},\omega)\rangle, \tag{3}$$

where $k^2 = k_z^2 + k_y^2$; $k_y^2 \gg k_z^2$. Angular brackets imply an average with respect to spectral distribution $|n^2(\vec{k},\omega)|$ of n_e fluctuations, evaluated by substituting values of the physical parameters measured at the peak of the wave spectrum.

The explicit expressions[5] for ${\rm Re}\chi_e(\vec{k},\omega)$ and ${\rm Im}\chi_e(\vec{k},\omega)$ depend on the relative magnitudes of $\omega - k_z u$ and $k_z v_{{\rm th}e}$. For the discrete, current-driven ICDW, where $\omega - k_z u \ll k_z v_{{\rm th}e}$ and $|\omega_e^*| < \omega$, ${\rm Re}\chi_e$, ${\rm Im}\chi_e$, and ${\rm Im}\chi_i$ have been given before.[5]

For the nonresonant continuous ICDW,[5] where $\omega - k_z u > k_z v_{{\rm th}e}$, $\omega\nu_{\rm eff} > k_z^2 v_{{\rm th}e}^2$ ($\nu_{\rm eff}$ is the effective collision frequency),[13] and strong plasma inhomogeneities exist ($\omega \simeq \omega_i^*$, $k_y\rho_i \gg 1$, $\mu_e = \partial \ln T_e/\partial \ln n_e < 0$, $|\mu_e| > 2$), we find ${\rm Re}\chi_e = (k_{{\rm D}e}^2/k^2)\omega_e^*/\omega$,

$${\rm Im}\chi_e(\vec{k},\omega) = 1.96\, \frac{k_{{\rm D}e}^2}{k^2} \frac{k_z^2 v_{{\rm th}e}^2}{\omega \nu_{\rm eff}} \left[1 - \frac{\omega_e^*}{\omega}(1 + 1.71\mu_e)\right], \quad {\rm Im}\chi_i(\vec{k},\omega) = \sqrt{\pi}\, \frac{k_{{\rm D}i}^2}{k^2} \frac{\omega - \omega_i^*}{k v_{{\rm th}i}} \exp\left(-\frac{\omega^2}{k^2 v_{{\rm th}i}}\right). \tag{4}$$

We note that although the instability may be highly turbulent, the calculation is applicable only to the linearly growing stage of the instability, since χ_e and χ_i were calculated from nonperturbed distribution functions.

The mechanism generating anomalous resistivity for the continous ICDW in steady state is different from that of the current-driven discrete ICDW or of ion sound, i.e., from inverse Landau damping, where $u > \omega/k_z$. The continuous ICDW is nonresonant, $u \ll \omega/k_z$, and thus not driven mainly by the parallel electron current but by the radial density inhomogeneity. A strong T_e gradient develops as a result of electron heating by the current near the edge of the plasma column. When the ∇T_e-determined diamagnetic electron drift velocity, $(1/n)(dn/dx)T_e/m\omega_{ci}(1.71|\mu_e| - 1)$, exceeds the perpendicular phase velocity, $\omega/k_y \simeq v_i^*$, electron momentum is transferred to the wave. Thus, anomalous resistivity can occur in an inhomogeneous plasma even when $u < \omega/k_z$.

With substitution of experimental values into Eqs. (1) and (2), anomalous resistivity and ion heating can be calculated (Fig. 2). Considering the limited accuracy of the measurements, the agreement with ion-heating predictions is good. Anomalous-resistivity calculations for the discrete ICDW ($\eta/\eta_{\rm class} = 1.1$–$1.3$) also agree well with the experiment (Fig. 1). In steady state, however, the discrepancy between observed ($\eta/\eta_{\rm class} \simeq 5$–$10$) and calculated ($\eta/\eta_{\rm class} \simeq 2.0$) resistivity for the continuous ICDW may indicate the presence of highly nonlinear mechanisms. Note that Eq. (1) gives parallel resistivity for comparison with the I-V plots which provide parallel resistivity only. The anomaly in the perpendicular resistivity is expected to be stronger, but was not measured explicitly. The theory predicts higher resistivity and ion heating for the continuous ICDW than for the discrete ICDW (since $\omega_{\rm cont} \gg \omega_{ci}$ and $\gamma_{\rm cont} \gg \gamma_{ci}$), in agreement with the experiment (Fig. 1). We note that end-plate sheaths do not play any important role, since the resistivity measured by I-V plots agrees reasonably well with results from local measurements by the rf conductivity probe (Fig. 1). The effect of ion heating due to sheath acceleration was negligible: Pulsed experiments show that the ion heating time is much shorter than the ion transit time. Radial sheath acceleration should scale as $1/B$, which is not observed.

Several important conclusions arise from this work. The ICDW constitutes a new, previously not invoked mechanism for generation of anomalous resistivity and ion heating, independent of the current (since the ICDW can be resonantly driven, $\omega/k_z<u$, and nonresonantly driven, $\omega/k_z>u$). Current, however, may be instrumental in preparing plasma parameters which allow instability growth (∇n and ∇T_e). In an isothermal plasma where ion sound is damped the ICDW produces anomalous effects similar to those of ion sound in nonisothermal plasmas. ICDW effects may explain anomalous resistivity and ion heating (currently discussed on the basis of ICW)[14] in the topside ionosphere.

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[13]A. A. Rukhadze and V. P. Silin, Usp. Fiz. Nauk 96, 87 (1968) [Sov. Phys. Usp. 11, 659 (1969)]. From Eq. (4), $\eta/\eta_{\rm class}=\nu_{\rm eff}/\nu_{e-i}=1+ak_z^2v_e^2/\omega\nu_{\rm eff}$, $a=$const; thus $\eta/\eta_{\rm class}=1/2[(1+4ak_z^2v_e^2/\omega\nu_{e-i})^{1/2}+1]$.

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