These results have a bearing on the solution of the two-dimensional classical XY model proposed by Kosterlitz and Thouless.[12] Very briefly, they consider a piece of the Hamiltonian that refers to states called vortices, which have a logarithmic interaction potential. Treated separately, these vortices lead to an unusual phase transition with non-power-law singularities. However, consider the correlation length ξ. It has contributions from the vortex (v) and non-vortex (nv) parts of the Hamiltonian, giving

$$\xi^{-1} = \xi_{\mathrm{v}}^{-1} + \xi_{\mathrm{nv}}^{-1}, \tag{14}$$

where ξ_{v} diverges exponentially at T_{KT} and ξ_{nv} diverges as a power at T_c. It appears[13] that T_c is below T_{KT}, and therefore the system will go critical ($\xi^{-1} \to 0$) at T_c, with power-law singularities, so that the vortices are not the essential driving mechanism of the phase transition in the XY model.

In conclusion, I have presented an approximate calculation of the critical properties of the classical XY model on a triangular lattice. The question of the nature of the low-temperature phase is under investigation.

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Targets for Electron-Beam Fusion*

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I have performed computer simulations of high-density, spherical shells containing DT gas, irradiated by pulsed electron beams. For these targets, (1) the optimal shells are thick enough that few electrons reach their inner surface, (2) the low-level bremsstrahlung which penetrates the shell significantly affects the target performance, (3) breakeven requires about 10^{15} W deposited for 5 nsec. This represents a considerable improvement over the behavior of bare DT spheres; further improvements may reduce the power to 10^{14} W.

Recently the possibility of using focused relativistic electron beams to initiate thermonuclear reactions has aroused considerable interest.[1-3] We now have some experimental evidence that two beams can be focused onto a spherical target of a few millimeters diameter, producing nearly spherical irradiation of the target.[4] A variety of schemes have been proposed for the targets, ranging from the direct heating of solid DT to the implosion of high-density metal shells containing DT.[2,5,6] Rudakov and Samarsky recently presented[2] a few calculations for targets with shells thin enough to allow the electrons to pass entirely through the target, uniformly heating the shell. This paper summarizes results of an extensive series of computer simulations of fusion targets with metal shells, irradiated by constant-power (unshaped) electron beam pulses. My study concentrates on the breakeven regime where beam energy and thermonuclear energy are comparable. I believe these are the first published calculations of breakeven and thermonuclear energy gain for electron-beam targets of this type.

Conceptually, the targets consist of three parts: (1) the ablator, the outer part of the metal shell where most of the beam energy is absorbed; (2) the pusher, the inner part of the metal shell, which is imploded by the high pressure in the ablator; (3) the fuel, a DT mixture which is compressed and heated by the imploding pusher to achieve thermonuclear ignition. During the burn the pusher serves as a tamper, keeping the fuel compressed longer.

The calculations have been done with CLYDE,[7] a one-dimensional (spherical) Lagrangian hydrocode. It has ion, electron, and radiation temperatures and uses a Thomas-Fermi-Dirac equation of state. It includes flux-limited radiation diffusion, electron thermal conduction, and equilibration between the three temperatures. The α-particle energy from the DT reactions is transported within the fuel region and deposited as appropriate. The electron-beam deposition profile used in the calculations is obtained from a Monte Carlo electron-photon transport calculation[8] wherein 1-MeV electrons with a cosinusoidal angular distribution were incident on gold. The calculation includes relevent collisional interactions but not beam-plasma collective effects since the latter do not appear to be important here.[3]

The calculated behavior of a breakeven target is shown in Fig. 1. As the beam energy is deposited, a shock wave is formed in the gold shell which then enters the fuel region and reflects several times while the pusher compresses the fuel. Aside from radiation losses to the colder pusher, the final part of the compression is nearly adiabatic. Losses to the pusher by thermal conduction, which have concerned several authors,[2,5,9] were generally small compared to radiation losses. The thermonuclear burn takes place at the time of maximum compression, producing 5 MJ of thermonuclear energy. With a 5-nsec beam pulse, breakeven is obtained with 4 MJ. At maximum compression, the fuel density is 44 g/cm^3, about 1850 times larger than the initial density. If the poles of the sphere implode with a velocity 8% ($=1850^{-1/3}$) faster than the equator, they would meet at the center when the equator reached the calculated turn-around radius, presumably destroying the implosion. A 16% shell-thickness asymmetry or a 24% irradiation power asymmetry would produce an 8% velocity asymmetry. Thus if the asymmetries are limited to about 5%, a reasonable implosion should occur.

Figure 2 shows the variation of thermonuclear

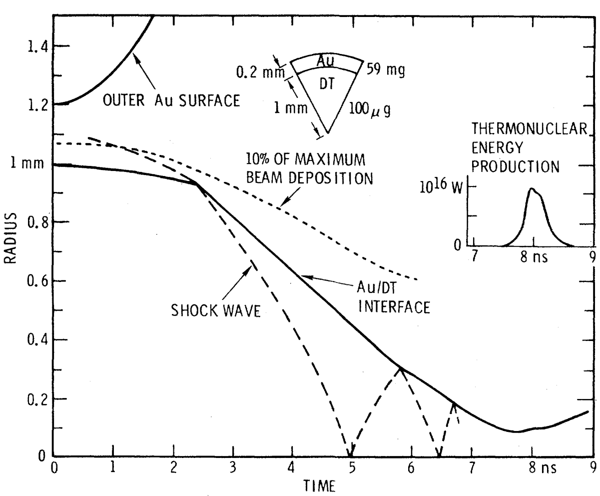

FIG. 1. Position of the gold and DT interfaces and of the main shock wave as a function of time. The electron-beam power is 8×10^{14} W. The dashed line shows where the electron-beam deposition drops to 10% of its peak value. The initial configuration of the target is shown in the pie diagram.

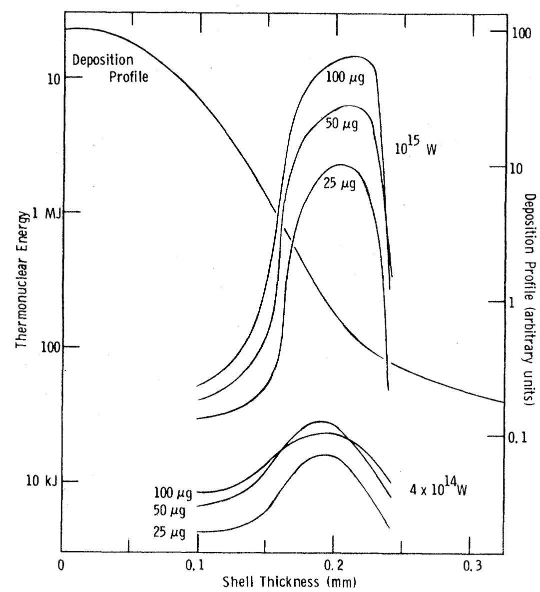

FIG. 2. Thermonuclear energy produced as a function of gold shell thickness. The shell inner radius is 1 mm in all cases. The DT fuel mass is indicated for each curve. The upper set of curves are for electron-beam powers of 10^{15} W, the lower curves for 4×10^{14} W. Also shown is the relative amount of electron-beam energy deposited as a function of distance from the outside surface of a gold thick shell.

output with shell thickness and the beam energy deposition profile. As can be seen, very little energy penetrates further than 0.15–0.2 mm. For thin shells, around 0.1 mm thick, the deposition is nearly uniform throughout the target, and the gold simply ablates inward, so that the pusher is hot, low-density gold. For thicker shells, little energy is deposited in the pusher, and it is driven inward by high pressures in the ablator. As a result it is colder and denser, and consequently works more effectively, both as a pusher and as a tamper during the burn. Beyond 0.2 mm the inner surface heating levels off somewhat but the increased pusher mass decreases its velocity. The optimum shell thickness depends only on the electrons' penetration depth.

As can be seen, the deposition at the inner surface of a 0.2-mm-thick shell is about 1% of the deposition near the outer surface. The tail of the deposition profile extending beyond 0.2 mm is due primarily to bremsstrahlung from beam electrons in the outer part of the shell. Even though this tail is more than 2 orders of magnitude below the deposition near the outer surface, its effect on the performance of thick-shell targets is very important. Previous calculations in which this tail was omitted produced breakeven for beam powers about 3 times lower. As in the case of thin-shell targets, the deleterious effects are evidently the result of early-time ablation of the inner surface, resulting in a lower-density tamper.[10]

The thermonuclear output as a function of beam power is shown in Fig. 3 for different targets. The targets of curves C and F differ only in their

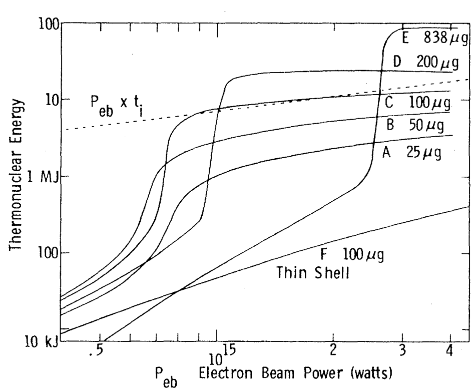

FIG. 3. Thermonuclear energy produced as a function of electron-beam power for various targets. The gold shell inner radius of all targets is 1 mm. The shell thickness for curves A–E is 0.2 mm, for curve F, 0.1 mm. The DT fuel mass is indicated for each curve.

shell thickness. These graphs emphasize the difference between the two kinds of targets: The thick-shell target (C) has a well-defined ignition threshold where the thermonuclear output increases by about 2 orders of magnitude for very little additional beam energy. The output of the thin-shell targets (F) tends to level off and the fuel does not ignite for beam powers up to 10^{16} W.

Below the ignition threshold the thermonuclear output is relatively independent of fuel mass, though there is an optimum around 50 μg. Above the ignition threshold, a more or less constant fraction (20–40%) of the fuel mass is burned, so that the output is approximately proportional to the fuel mass.

Breakeven occurs for a beam power around 8×10^{14} W for targets of this diameter. In these calculations the beam power was turned on instantaneously and left on indefinitely. The broken line shows the beam deposited in the thick-shell targets if the beam power were turned off at t_i, the time of implosion (maximum compression). Other calculations show that the power can be turned off at about $\frac{3}{4}$ of the implosion time without significantly affecting the thermonuclear output, since energy deposited during the final stages of implosion does not affect the fuel until after burn has occurred. The beam energy required for breakeven with these targets is around 4 MJ. This energy can be decreased somewhat by using shorter, higher power pulses. The effect of a finite pulse risetime has not yet been investigated for these targets, but it seems unlikely that a 1–2-nsec risetime would greatly affect the results reported here.

For other sphere sizes the optimum shell thickness is nearly constant, 0.2 mm, decreasing slightly for smaller targets. The beam power required for breakeven increases slowly with shell radius. For a shell inner radius of 2 mm, breakeven requires 10^{15} W and ~12 MJ. The compression ratio is also about 3 times larger (requiring a more symmetric implosion). The main advantage of the larger targets is that the implosion time increases about linearly with shell radius. This means that the pulse risetime and the pulse length can be longer.

In this study I have considered fusion targets with a single metal shell irradiated by 1-MeV electrons in a pulse of constant power. For such targets I find that (1) the targets producing the most thermonuclear energy have shells which are thick enough that few beam electrons reach the metal shell's inner surface; (2) the small amount of bremsstrahlung from the beam which does reach the inner surface has a significant deleterious effect on the target performance; (3) breakeven will require about 10^{15} W of beam power deposited in a pulse length of about 5 nsec. For comparison, the power required for breakeven with bare DT spheres[11] is about 100 times greater, unless some mechanism can be found which shortens the beam electrons' range so that a symmetric implosion can be produced.[3] However, 10^{15} W requires a beam current of 10^9 A at 1 MeV, focused to a current density of 5×10^9 A/cm^2 for the 2.4-mm-diam targets, with a risetime on the order of 1 nsec. This is well beyond the capabilities of present electron-beam technology[4] (10^{12} W, 10^6 A, 10^7 A/cm^2). Several improvements to our relatively crude target design are currently being studied. In order to ignite a thermonuclear burn, the pusher must attain an implosion velocity on the order of 10^7 cm/sec, with higher velocities required for less dense pushers. The use of a low-Z ablator surrounding a high-density pusher would reduce the bremsstrahlung heating of the pusher inner wall. By eliminating the consequent early-time ablation of the pusher wall, the pusher density is kept higher, reducing the beam power required to achieve ignition. Another method is to use two or more concentric shells to achieve a velocity multiplication.[6] When an outer shell collides with an inner, less massive shell, the inner shell bounces off with a higher velocity. Since the velocity of the outer shell need not be as high, the power required will be lower and the pulse length can be longer. At current densities of 10^9 A/cm^2, the electron range in low-Z ablators may be shortened by self-magnetic field effects.[3] This would allow us to use higher beam voltages for a given target, making it easier to produce the high powers required. It may be possible to decrease the energy required by tailoring the time dependence of the voltage or current pulse in a fashion similar to that suggested for laser fusion with bare DT spheres.[3,12]

By use of a combination of the techniques outlined above, it may be possible to reduce the breakeven power to 10^{14} W or even lower. However, extensive target design studies are required to verify this. A 10^{13}-W accelerator is now under development,[13] and a 10^{14}-W accelerator appears possible, but will require significant technological advances.

Many aspects of the behavior of these electron-beam fusion targets can be tested using machines producing $10^{12}-10^{13}$ W at voltages of 1–3 MeV with pulse lengths of a few tens of nanoseconds. In this range, 10^6-10^{10} neutrons should be produced; however, the number of neutrons produced is not as important as the information they will provide about the target behavior.

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Formation of a Non-Neutral Relativistic-Electron-Beam Ring in a Toroidal Magnetic Field

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A quiescent equilibrium state of a non-neutral beam ring may have been realized when a relativistic electron beam (450 keV, 16 kA, 25 nsec) was injected parallel to a toroidal magnetic field in vacuum. The longest lifetime obtained was 20 μsec which corresponded to 3000 revolutions of the electrons around the torus. The lifetime was limited by the appearance of an ion-resonance instability.

In recent years, the injection of high-current relativistic electron beams into toroidal systems has been a subject of absorbing interest in connection with an ion accelerator as proposed by Budker,[1] a heavy-ion accelerator,[2,3] and plasma confinement[4,5] and/or heating.[6] The electron-ring accelerator is also under investigation at many laboratories.[7] Until now, toroidal rings of relativistic electron beams have been realized in the following cases: (1) neutralized beams in toroidal magnetic fields,[8,9] (2) neutralized beams in the Astron experiments with[10] and without[11] (small) toroidal field at relatively high gas pressure, and (3) non-neutral beams in the electron-ring-accelerator experiments. However, non-neutral–beam equilibrium in a toroidal field, the application of which has been discussed by Rostoker,[3] has not yet been observed. This paper presents experimental results on the formation of the non-neutral relativistic-electron–beam ring in a strong toroidal field.

Figure 1 shows a schematic drawing of the experimental setup which mainly consists of an axisymmetric toroidal device (SPAC-II) and a relativistic-electron-beam source (Phoebus-I). The toroidal device has an aluminum shell 1.5 cm thick and a liner made of a stainless-steel bellows with a thickness of 0.3 mm. The major radius of the torus is 28 cm and the inner radius of a molybdenum diaphragm limiter is 5.5 cm. The toroidal magnetic field of 20 kG maximum is generated with a 250-kJ capacitor bank. An important feature is that a quasisteady vertical magnetic field of up to 200 G is available. This device can be operated as a tokamak if an iron-core transformer is excited to drive the plasma current.[12] In the experiment described here, the Ohmic-heating transformer and its circuit were isolated from the electron-beam current by shortening the longitudinal gap of the shell. The electron-beam source consists of a two-stage Marx generator (100 kV, 5 kJ), an air-core step-up transformer with a step-up ratio of 9, and a 4.6-Ω, coaxial, pulse-forming line of water dielectric, the length of which corresponds to a 25-nsec pulse width. The impedance of the diode gun is somewhat higher than the 4.6 Ω of the coaxial line, so that an impedance-transforming line is inserted as shown in the figure. Through this section, the pulse height becomes 3 times higher. The inner conductor of the line in the vacuum region is so small in diameter that the breakdown of the line is protected by the strong self mag-

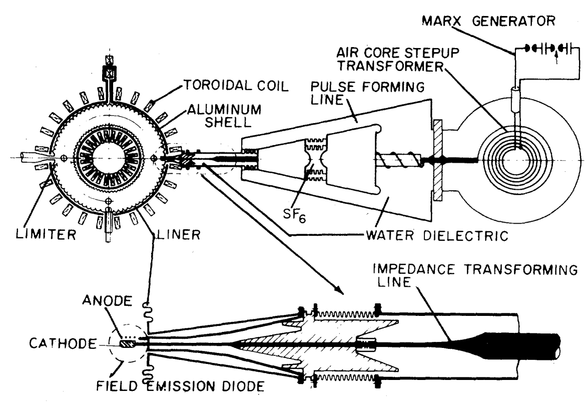

FIG. 1. Schematic view of the experimental setup.