guarantee $|\eta_{+-}| \sim |\eta_{00}|$, hence dynamical suppression of the $|\Delta \vec{I}| = \frac{3}{2}$ component must be invoked. It is not ruled out that the source of CP nonconservation in $K_L{}^0 \to 2\pi$ is indeed dominantly superweak with $|\eta_{+-}| = |\eta_{00}|$ and that the electromagnetic αf effects represent an *additional* source of CP nonconservation which contribute <10% of $|\eta|$ for $f \lesssim \frac{1}{20}$.

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Theorem on the Scalar Form Factor in K_{l3} Decay

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A current-algebra result is derived for the slope of the scalar form factor in K_{l3} decay which is valid at all momentum transfers. This theorem is compared with the previous results of Dashen and Weinstein and Dashen, Li, Pagels, and Weinstein. Our analysis indicates that there are large corrections to these previous results at $t = m_K{}^2 + m_\pi{}^2$.

The semileptonic decays of the K meson have provided an important testing ground for basic theoretical ideas regarding the symmetry properties and the dynamics of the strong interaction. In particular, considerable effort has been devoted to understanding the momentum-transfer dependence of the form factors in K_{l3} decays. A theorem regarding the slope of the scalar form factor in such decays was obtained some time ago by Dashen and Weinstein[1] and was later modified by Dashen, Li, Pagels, and Weinstein[2] (DLPW). In its amended form the theorem determined the slope at the unphysical point $t = m_K{}^2 + m_\pi{}^2$ to be $\frac{1}{2}(F_K/F_\pi - F_\pi/F_K) + O(\epsilon)$, where the parameter ϵ sets the scale of chiral symmetry breaking. The leading term, $\frac{1}{2}(F_K/F_\pi - F_\pi/F_K)$, is of order $\epsilon \ln\epsilon$. In this paper we generalize the DLPW analysis to yield a theorem valid for all t. We then make a simple estimate which indicates that the corrections to the DLPW result are of order 50% at $t = m_K{}^2 + m_\pi{}^2$. Finally, we give an exact evaluation of the theorem based on a previous analysis[3] we have made of three-point functions within the framework of the $(\underline{3}, \underline{3}^*)$ model of chiral symmetry breaking. We again find corrections on the order of 50% at $t = m_K{}^2 + m_\pi{}^2$. The reason the corrections are so large is that they are of order $2m_K{}^2/m_\kappa{}^2$ relative to the leading term.[4] The exact predictions of the theorem are in good agreement with the experimental data.[5]

First we derive the generalized theorem. We define the standard K_{l3} form factors $f_\pm(t)$ by

$$\langle M_a(q)| V_b{}^\mu(0)| M_c(k)\rangle = i f_{abc}[f_+(t)(k+q)^\mu + f_-(t)(k-q)^\mu], \tag{1}$$

where $t = (k-q)^2$. Our interest is in the function $D_{abc}(t)$ defined by

$$D_{abc}(t) \equiv i f_{abc} \tilde{D}_{abc}(t) \equiv i\langle M_a(q)| \partial_\mu V_b{}^\mu(0)| M_c(k)\rangle = i f_{abc}[(m_c{}^2 - m_a{}^2) f_+(t) + t f_-(t)]. \tag{2}$$

We begin by studying the function

$$S(q^2, k^2, t') \equiv \int d^4x\, d^4y\, e^{iqx} e^{-iky} \langle 0| T[\partial_\mu A_a{}^\mu(x) \partial_\sigma V_b{}^\sigma(0) \partial_\nu A_c{}^\nu(y)]|0\rangle, \tag{3}$$

which we consider as a function of the variables q^2, k^2, and $t' \equiv (k-q)^2 - \sigma(k^2 + q^2)$. σ is an arbitrary constant. For $\sigma = 1$ we reproduce the DLPW analysis. It is useful to isolate the meson poles appearing in Eq. (3) since this will enable us to identify the on-shell amplitude D_{abc}. Thus, following DLPW, we introduce currents $\hat{A}^\mu$ in which the meson poles have been removed,

$$\partial_\mu \hat{A}^\mu(p) = \partial_\mu A^\mu(p) - (p^2 - m^2)^{-1} \lim_{p^2 \to m^2} (p^2 - m^2) \partial_\mu A^\mu(p). \tag{4}$$

We can then rewrite Eq. (3) as

$$S(q^2, k^2, t') = \frac{(im_a^2F_a)(im_c^2F_c)}{(q^2-m_a^2)(k^2-m_c^2)}D_{abc}(t') + \frac{im_a^2F_a}{q^2-m_a^2}\langle M_a|T[\partial V_b(0)\partial\hat{A}_c(k)]|0\rangle$$
$$+\frac{im_c^2F_c}{k^2-m_c^2}\langle 0|T[\partial\hat{A}_a(q)\partial V_b(0)]|M_c\rangle + \langle 0|T[\partial\hat{A}_a(q)\partial V_b(0)\partial\hat{A}_c(k)]|0\rangle. \tag{5}$$

We can also apply standard current-algebra techniques to Eq. (3) [i.e., the left-hand side of Eq. (5)] and pull the derivatives through the time-ordering instruction. The lengthy sum of terms that results is given by DLPW.[2] We multiply these terms and Eq. (5) by $(q^2-m_a^2)(k^2-m_c^2)/m_a^2m_c^2F_aF_c$, differentiate both sides with respect to t' with k^2 and q^2 held fixed, and evaluate the derivatives at $t'=0$. Further, we set $k^2=q^2=0$ *wherever they appear off shell.* We then obtain the generalized DLPW theorem:

$$\frac{d}{dt'}D_{abc}(t')|_{t'=0} = (if_{abc})\tfrac{1}{2}\left(\frac{F_c}{F_a}-\frac{F_a}{F_c}\right) + (F_aF_c)^{-1}\frac{d}{dt'}\{\langle 0|T[\partial V_b(0)\Sigma_{ac}(q-k)]|0\rangle$$
$$+S(0,0,t') - \lim_{q^2\to m_a^2}\lim_{k^2\to m_c^2}(m_a^2m_c^2)^{-1}(q^2-m_a^2)(k^2-m_c^2)S(q^2,k^2,t')\}_{t'=0}, \tag{6}$$

where we have defined

$$\Sigma_{ac}(q-k) \equiv \int d^4x\, d^4y\, e^{iky}e^{-iky}\delta(x_0-y_0)\tfrac{1}{2}\{[A_c^0(x), \partial_\lambda A_a^\lambda(y)] + [A_a^0(x), \partial_\lambda A_c^\lambda(y)]\}. \tag{7}$$

In deriving Eq. (6), we have reintroduced $\partial_\mu A^\mu$ instead of $\partial_\mu\hat{A}^\mu$ and employed Eq. (3). For the on-shell amplitude D_{abc}, $t'=t-\sigma(m_a^2+m_c^2)$. Thus $(d/dt')D_{abc}(t')|_{t'=0}$ is the same as $(d/dt)D_{abc}(t)|_{t=\sigma(m_a^2+m_c^2)}$. For $\sigma=1$, Eq. (6) reduces to the DLPW theorem.[2] But since σ is arbitrary, we now have a theorem for the slope of $D_{abc}(t)$ valid for all t.

The leading term in Eq. (6), $\frac{1}{2}(F_c/F_a - F_a/F_c)$, is of order $\epsilon\ln\epsilon$, while the other terms are of order ϵ or higher. DLPW[2] estimated these corrections on the basis of threshold dominance to be of order 10%. On the other hand, a simple estimate can also be obtained using current algebra and the $(\underline{3},\underline{3}^*)$ model of chiral symmetry breaking.[6] Following the usual hard-meson treatment of vertex functions,[7-9] we parametrize $S(q^2,k^2,t)$ as

$$S(q^2,k^2,t) = if_{abc}m_a^2F_am_b^2F_bm_c^2F_c\frac{(\alpha+\beta k^2+\gamma q^2+\delta t)}{(q^2-m_a^2)(k^2-m_c^2)(t-m_b^2)}. \tag{8}$$

Assuming partial conservation of axial-vector current and vector current and using the $(\underline{3},\underline{3}^*)$ commutation relations,[10] we find for the $\pi\kappa K$ vertex that the constants α, β, γ, and δ are given by

$$\alpha = [m_K^2(Z_\pi/Z_K)^{1/2} - m_\pi^2(Z_K/Z_\pi)^{1/2}]/F_\kappa, \quad \beta+\delta = [(Z_K/Z_\kappa)^{1/2} - (Z_\kappa/Z_K)^{1/2}]/F_\pi,$$
$$\beta+\gamma = [(Z_K/Z_\pi)^{1/2} - (Z_\pi/Z_K)^{1/2}]/F_\kappa, \quad \gamma+\delta = [(Z_\pi/Z_\kappa)^{1/2} - (Z_\kappa/Z_\pi)^{1/2}]/F_K. \tag{9}$$

Using the parametrization of Eq. (8) for S and evaluating the $\Sigma_{\pi K}$ term, we can express Eq. (6) in the form

$$\frac{d}{dt}\tilde{D}_{\pi\kappa K}(t)|_{t=\sigma(m_K^2+m_\pi^2)} = \frac{1}{2}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\right) + \frac{F_\kappa\sqrt{Z_\kappa}}{2m_\kappa^2F_\pi F_K}\left(\frac{m_K^2F_K}{\sqrt{Z_K}}+\frac{m_\pi^2F_\pi}{\sqrt{Z_\pi}}\right)$$
$$+m_\kappa^2F_\kappa\left(\frac{(\alpha+\beta m_K^2+\gamma m_\pi^2+\delta m_\kappa^2)}{[m_\kappa^2-\sigma(m_K^2+m_\pi^2)]^2} - \frac{(\alpha+\delta m_\kappa^2)}{m_\kappa^4}\right). \tag{10}$$

To obtain a rough estimate of the correction terms in Eq. (10), we make the approximation $\sqrt{Z_\pi}\cong\sqrt{Z_K}\cong\sqrt{Z_\kappa}$. This gives $\beta\cong\gamma\cong\delta\cong 0$. Using the expression for α in Eq. (9) and neglecting m_π^2 relative to m_K^2, we can write the correction terms as

$$\frac{m_K^2}{m_\kappa^2}\left[\frac{F_\kappa}{2F_\pi} + \frac{\sigma(2m_K^2m_\kappa^2-\sigma m_K^4)}{(m_\kappa^2-\sigma m_K^2)^2}\right]. \tag{11}$$

From the Glashow-Weinberg[7] sum rules $F_\pi\sqrt{Z_\pi} = F_K\sqrt{Z_K}+F_\kappa\sqrt{Z_\kappa}$ and $m_\pi^2F_\pi/\sqrt{Z_\pi} = m_K^2F_K/\sqrt{Z_K} + m_\kappa^2F_\kappa/\sqrt{Z_\kappa}$, we find in the present approximation $F_K/F_\pi \cong m_\kappa^2/(m_\kappa^2-m_K^2)$ and $F_\kappa/F_\pi \cong -m_K^2/(m_\kappa^2-m_K^2)$. For m_κ/m_K in the range 2 to 3 (i.e., F_K/F_π between 1.33 and 1.12), the ratio of Eq. (11) to $\frac{1}{2}(F_K/F_\pi - F_\pi/F_K)$ varies from 0.67 to 0.25 at $t=m_K^2+m_\pi^2$ but only from 0.14 to 0.06 at $t=0$.[11]

It is apparent that the corrections to the DLPW result at $t=m_K^2+m_\pi^2$ are small only when $m_\kappa^2 \gg m_K^2$, in which case a threshold-dominance approximation to the background dispersion integral would give an accurate estimate.[12]

The rough estimates made above can be checked by giving a complete evaluation of Eq. (10). This requires detailed knowledge of the parameters in Eq. (9). By combining the Wilson short-distance expansion with low-energy theorems, we have derived previously a set of sum rules[3] which determine these parameters in terms of the value of $F_K/F_\pi f_+(0)$. In particular, we found

$$F_K/F_\pi=1.26,\quad F_\kappa/F_\pi=-0.23,$$
$$(Z_K/Z_\pi)^{1/2}=0.94,\quad (Z_\kappa/Z_\pi)^{1/2}=0.76.$$

Using these values to compute α, β, γ, and δ, we find from Eq. (10) that the slope at $t=m_K^2+m_\pi^2$ is

$$(d/dt)\tilde{D}_{\pi KK}(t)\,|_{t=m_K^2+m_\pi^2}=0.35.$$

In contrast, the term $\frac{1}{2}(F_K/F_\pi - F_\pi/F_K)$ contributes only 0.23. Hence, as in our simple estimate above, the corrections to the DLPW estimate are of order 50% at $t=m_K^2+m_\pi^2$ At $t=0$, Eq. (10) yields

$$(d/dt)\tilde{D}_{\pi KK}(t)\,|_{t=0}=0.19,$$

so that at this point the correction terms are only of order 20% and are of opposite sign, again in agreement with our rough estimate.

Our results thus indicate that the slope of the scalar form factor varies considerably (~80%) between $t=0$ and the unphysical point $t=m_K^2+m_\pi^2$. In terms of the usual slope parameter λ_0 defined by $\lambda_0 \equiv [m_\pi^2/(m_K^2-m_\pi^2)](d/dt)\tilde{D}_{\pi KK}(t)$, we find $\lambda_0=0.016$ at $t=0$ and $\lambda_0=0.029$ at $t=m_K^2+m_\pi^2$. At the limit of the physical region, $t=(m_K-m_\pi)^2$, we find $\lambda_0=0.021$. It is thus apparent that in principle the scalar form factor should not be parametrized by simply a linear function of t. A linear fit will yield only an average for the slope over the physical region. From such a fit Donaldson *et al.*[5] find $\lambda_0=0.019\pm0.004$. This value for the average slope is in excellent agreement with our predictions.

The large corrections that we have found to the DLPW value for the slope at $t=m_K^2+m_\pi^2$ imply that arguments based on the DLPW theorem may be suspect. Thus Baluni and Broadhurst[13] have obtained bounds on the slope of the scalar form factor which are badly violated by the DLPW result. From this violation Broadhurst[14] concludes that the dimension Δ of the chiral-symmetry-breaking Hamiltonian density must be greater than or equal to 3. But our value of $\lambda_0=0.029$ at $t=m_K^2+m_\pi^2$ is consistent with the bounds, and so the conclusion that $\Delta \geqslant 3$ appears to be vitiated.

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