Comparable calculations have been performed at KMS Fusion and incorporated into the KMS Fusion hydrodynamics code TRHYD; the results will be published later, including more detailed simulations of the experimental conditions. The present comparison is perhaps even more compelling because it is a contrast between theory and experiment independently arrived at. Henderson concludes that "when true thermonuclear neutrons are observed from small microshells, they will serve as proof of compression, an important milestone to laser fusion." The compression is demonstrated by the x-ray pinhole-camera pictures. The agreement between the experimental and theoretical determinations of neutron production confirms the "true thermonuclear" origin of the neutrons in the Los Alamos definition.

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Comment on Double-Scattering Effect in Deuteron Breakup Reactions

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The effect of double scattering on the spectator distribution is re-examined for the process $K^+d \to K^+\pi^- pp$, integrating over the spectator angles and making a comparison with experimental data.

Some time ago Dean[1] showed, using Glauber theory,[2] that the double-scattering effect on the high-momentum part of the spectator distribution is constructive and huge: In this way one could find an easy explanation of the experimental fact that spectator distributions[3] for various hadronic reactions present a high-momentum tail which is *not* reproduced by the spectator model even with the most sophisticated deuteron wave functions.[3]

Assuming that the spectator is unambiguously identified as in the process $\pi d \to K\Lambda p$, Dean considered the Glauber amplitude for the deuteron breakup,[4]

$$F(\vec{\Delta}, \vec{p}) = \varphi(p)F(\Delta) + (i/2\pi k)\int d^2q\, F_1(\vec{\Delta} - \vec{q})F_e(\vec{q})\varphi(\vec{p} + \vec{q}), \tag{1}$$

and calculated the double-differential cross section,

$$d\sigma/dp_l\, dp_t \cong 2\pi p_t \int |F(\vec{\Delta}, \vec{p})|^2 d^2\Delta/k^2, \tag{2}$$

using a simple S-wave Gaussian wave function and Gaussian amplitudes.

To evaluate the effect of double scattering on the above distribution, we define a function C, the ratio between the cross section (2) and the cross section without double scattering. C turns out to be a simple analytical expression of p_t only,

$$C = 1 - \frac{\sigma}{2\pi(a+2b+2c)} \exp\left(\frac{2c^2 p_t^{\,2}}{a+2b+2c}\right) + \frac{\sigma^2}{(8\pi)^2} \frac{\exp[2c^2 p_t^{\,2}/(b+c)]}{(b+c)(a+b+c)}. \tag{3}$$

This function of p_t is several orders of magnitude larger than 1, for $p_t = 400$–500 MeV/c, and this is the main argument in Dean's paper, which favors the interpretation of the high-momentum tail as a double-scattering effect.

At this point, three comments should be made: (1) The effect on p_t is over evaluated because in that region the D wave of the deuteron is important; (2) the effect on $d\sigma/dp$ is even more suppressed, as a result of integration over the spectator angles; (3) the actual comparison with a typical experimental distribution does not confirm the above explanation and requires the presence of another effect.

To calculate the effect of the D wave we use a multi-Gaussian representation of the Reid[5] soft-core wave function[6]:

$$\psi_0(p)=\sum_i A_i \exp(-\alpha_i p^2); \quad \psi_2(p)=p^2\sum_i B_i \exp(-\beta_i p^2), \tag{4}$$

with ψ_0 and ψ_2 defined by

$$\psi(p)=\psi_0(p)-(1/\sqrt{2})[3(\vec{J}\cdot\vec{p})^2/p^2-2]\psi_2(p),$$

where $\vec{J}$ is the spin operator of the deuteron.

Substituting this expression into (1), but only for single scattering, we obtain for the double-differential cross section, using well-known trace properties of the tensor operator,[7]

$$d\sigma/dp_l\,dp_t=2\pi p_t\sigma_1\{\sum_{ij}A_iA_j \exp[-(\alpha_i+\alpha_j)p^2]C_{ij}(p_t)+\psi_2{}^2(p)\}, \tag{5}$$

where

$$C_{ij}(p_t)=1-\frac{\sigma}{2\pi}\frac{1}{\gamma_i}\exp(y_ip_t{}^2)+\frac{\sigma^2}{32\pi^2}\frac{\epsilon_i\epsilon_j\exp(z_{ij}p_t{}^2)}{(b+\alpha_i)\epsilon_i+(b+\alpha_j)\epsilon_j},$$

and

$$\gamma_i=a+2b+2\alpha_i,\quad y_i=2\alpha_i{}^2/\gamma_i,\quad \epsilon_i=(a+b+\alpha_i)^{-1},\quad z_{ij}=\alpha_i{}^2\epsilon_i+\alpha_j{}^2\epsilon_j+a(\alpha_i\epsilon_i+\alpha_j\epsilon_j)^2/[2-a(\epsilon_i+\epsilon_j)],$$

and we obtain the ratio

$$C=\frac{\sum_{ij}A_iA_j\exp[-(\alpha_i+\alpha_j)p^2]C_{ij}(p_t)+\psi_2{}^2(p)}{\psi_0{}^2(p)+\psi_2{}^2(p)}. \tag{6}$$

Comparing the above expression with (3) we realize that while Dean's result is independent of p_l, ours is not. So, to compare our result with his, we consider the behavior of C as a function of p_t for p_l fixed and equal to 0; in this way p_t coincides with p. However, the factor C with D wave depends on p_l and this dependence must be explored. A clear way of doing this is to transform the double-differential cross section $d\sigma/dp_l\,dp_t$ into $d\sigma/dp\,d\cos\omega=(d\sigma/dp_l\,dp_t)p/\sin\omega$, where ω is the angle of the spectator momentum with respect to the direction of the beam. Keeping ω fixed, we can plot the ratio C as a function of p, and for $\omega=\pi/2$ we can compare our result with the result of Dean; this is done in Fig. 1, where it can be seen that the D wave of the deuteron strongly suppresses the values of C around 400 MeV/c. Moreover, from the plot of C at $\omega=2\pi/5$ and $\omega=\pi/10$, we realize that the effect is strongly angle dependent with a maximum at $\omega=\pi/2$ and a negative minimum at $\omega=0$. We expect that, once the integration on ω is made, the effect is smaller than at exactly $\pi/2$.

Fortunately, this integration can be made analytically,

$$\frac{d\sigma}{dp}=\int_{-1}^{+1}\frac{d\sigma}{dp\,d\cos\omega}\,d\cos\omega=p^2\int_{-1}^{+1}\frac{1}{p_t}\frac{d\sigma}{dp_l\,dp_t}\,d\cos\omega, \tag{7}$$

and the result is

$$\frac{d\sigma}{dp}=4\pi p^2\sigma_1\{\sum_{ij}A_iA_j\exp[-(\alpha_i+\alpha_j)p^2]D_{ij}(p)+\psi_2{}^2(p)\}, \tag{8}$$

where

$$D_{ij}(p)=1-\frac{\sigma}{2\pi}\frac{1}{\gamma_i}\exp(y_ip^2)\frac{\sqrt{\pi}}{2\sqrt{y_i}p}\operatorname{erf}(\sqrt{y_i}p)+\frac{\sigma^2}{32\pi^2}\frac{\epsilon_i\epsilon_j}{(b+\alpha_i)\epsilon_i+(b+\alpha_j)\epsilon_j}\exp(z_{ij}p^2)\frac{\sqrt{\pi}}{2\sqrt{z_{ij}}\,p}\operatorname{erf}(\sqrt{z_{ij}}\,p),$$

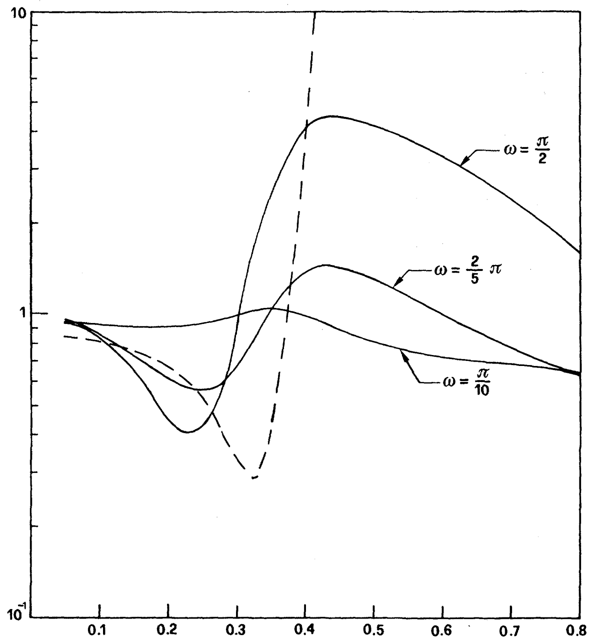

FIG. 1. The ratio C for the double-differential cross section $d\sigma/dp\,d\cos\omega$. Dashed line is Dean's result as a function of $p_t = p$ and solid lines are our results for different values of the spectator angle.

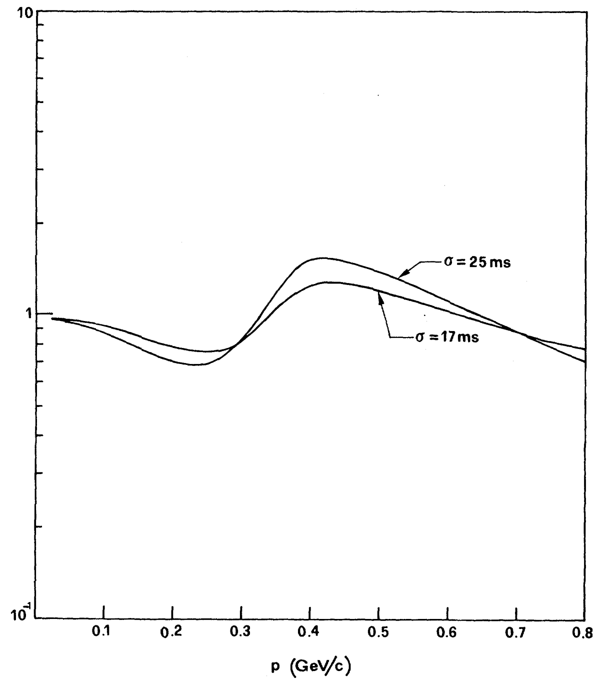

FIG. 2. The ratio C for the differential cross section $d\sigma/dp$, for two different values of σ.

with the notations of formula (5). We obtain the ratio C for the integrated cross section by substituting D_{ij} for C_{ij} in (6). In Fig. 2 we plot C for the integrated cross section for $\sigma = 25$ and 17 mb. The effect is again of the same type, i.e., destructive in the low-momentum part and constructive at about 400 MeV/c; however, the effect is reduced to 50% and 30% in the two cases considered.

Dean[1] considered a case where the spectator is unambiguously identified. In a more accessible case like $K^+d \to K^+\pi^-pp$, the identification of the spectator is done in an operational way, choosing as "spectator" the slower of the two protons. In this case one must use the ordered form of Glauber theory[2] for $F(\vec{\Delta}, \vec{p})$, which, contrary to (1), is fully symmetric for a change of the two nucleons and contains even the principal part of the propagator in the double-scattering term. Furthermore, to make a theoretical calculation which is comparable with the experimental "spectator" distribution, one must integrate $|F|^2$ over $\vec{\Delta}$ and $\cos\omega$, with $|\vec{p}|$ always being smaller than $|\vec{\Delta} - \vec{p}|$. In this way p is no longer the spectator, but simply the slower of the two nucleons; however theory and experiment are defined in a perfectly compatible way. This procedure has already been used in a previous paper,[8] where more details can be found. There the integrations over $\vec{\Delta}$ and $\cos\omega$ were not independent, as in the above derivation, but, since energy must be conserved, for small values of $\vec{\Delta}$ there is not always enough energy available to have both nucleons with high momentum and only part of the range for $\cos\omega$ is allowed. Therefore, we expect that the actual "spectator" distribution is suppressed at high momenta because of energy conservation. This effect can be visualized by plotting the ratio between the probability distribution $P(p) = 4\pi[\psi_0^2(p) + \psi_2^2(p)]p^2\sigma_c$ and the integral $T(S) = 2\pi p^2 \int (d^2\Delta/k^2) \times d(\cos\omega)\,|\psi(p) - \psi(\vec{p} - \vec{\Delta})|^2 |f_c|^2$, performed with the constraint that $|\vec{\Delta} - \vec{p}| > |\vec{p}|$ (σ_c and f_c are, respectively, the production rate and the amplitude for the process $K^+n \to K^+\pi^-p$). This ratio is plotted in Fig. 3 for the process $K^+d \to K^+\pi^-pp$ at 4.6 GeV/c, under the assumption that the system $K^+\pi^-$ is dominated by the $K^{0*}(890)$. As seen from the figure, the effect becomes noticeable at about 400 MeV/c and the reason is that this reaction is highly peaked in the forward direction.[9] Therefore, the energy available for the nucleons is not enough to allow for large momenta. On the other hand, for low energy loss, when the relative momentum $|\vec{\epsilon}|$ of the two nucleons is less than $\Delta/2$,

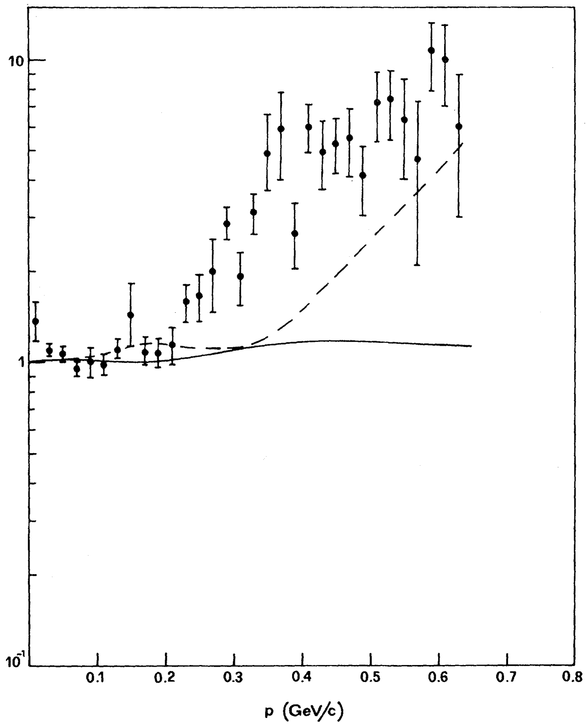

FIG. 3. The experimental points are the ratio between the "spectator" distribution for the experiment of Ref. 9 and $T(S)$ of the text. The dotted line is the ratio between the probability $P(p)$ and $T(S)$ and the solid line is the ratio between $d\sigma/dp$, calculated by using the ordered form of the Glauber theory, and $T(S)$.

and when the momentum of the slower nucleon is larger than $\Delta/2$, the kinematical configuration of $\vec{\epsilon} \parallel \vec{\Delta}$ is forbidden.

To visualize the effect of the double scattering, we study the ratio between $d\sigma/dp$, calculated by using the ordered form of Glauber theory, and $T(S)$, and compare it with the ratio between the experimental values $E \pm \Delta E$ and again $T(S)$. The Glauber amplitude is calculated by using again the above parametrization of Reid's[5] soft-core wave function, and the principal-part contribution to the double scattering is expressed by the Dawson function.[10] All the isospin complications of the problem are taken into account.[11]

The effect of double scattering shown in Fig. 3 is comparable with that of Fig. 2 and it is hardly enough to explain the discrepancy with the experimental data. Many other interesting effects could be important here; among these final-state interactions[8] and binding effects.[12]

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Note added.—After we submitted this work, a paper[13] appeared in this journal coming to similar conclusions. Their interpretation of the effect as coupling of the virtual t-exchange particle to the deuteron should be added to the above list of possible candidates.

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