Precise Determination of $E2$ and $E4$ Moments in ^{165}Ho from Muonic X Rays*

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The quadrupole and hexadecapole moments of ^{165}Ho were derived from the hfs of the $3d$ muonic states. The resulting values are $Q_0 = 7.44 \pm 0.07$ b and $\Pi_0 = 0.52 \pm 0.10$ b^2. The parameters of the nuclear charge distribution were determined. An indication for an angular variation of the skin thickness was found.

Although the radial nuclear charge distribution has been extensively studied for years by using muonic x rays and electron scattering,[1] little information has emerged concerning the angular dependence of the charge distribution in the body-fixed nuclear coordinate system. Even such an elementary quantity as the ground-state deformation as characterized by the spectroscopic quadrupole moment q (or by the intrinsic quadrupole moment Q_0) is subject to great uncertainty.

In Table I the measured values[2,3] of Q_0 for ^{165}Ho, determined by various methods, are compared. Although each method yields precise (± 2%) experimental observables, the interpretation of some of the experimental data appears to give rise to discrepancies as large as 30%. The determination of the intrinsic hexadecapole moment Π_0 involves even more uncertainties.

We wish to report a precision determination of the quadrupole moment in ^{165}Ho as well as the first direct determination of Π_0 using muonic x rays. Our data also indicate an angular variation of the skin thickness t besides the usual variation of the half-density radius c.

Traditionally[4] the hyperfine structure of the muonic $2p$ level has been used to determine q. The actual observable quantity is the quadrupole interaction energy. In general, the electric interaction energy of multipolarity λ is given by

$$[\epsilon_\lambda(j,j')]_{nl} = \frac{1}{10}\left(\frac{16\pi}{2\lambda+1}\right)^{1/2} \times \langle nlj|\left[\frac{1}{r^{\lambda+1}}\int_0^r \rho(r',\theta')r'^\lambda Y_{\lambda 0}(\theta')r'^2\,dr'\,d\Omega' + r^\lambda \int_r^\infty \rho(r',\theta')Y_{\lambda 0}(\theta')\frac{r'^2\,dr'\,d\Omega'}{r'^{\lambda+1}}\right]|nlj'\rangle, \quad (1)$$

where $\rho(r,\theta)$ is the nuclear charge distribution. The quadrupole moment is

$$Q_0 = (\tfrac{16}{5}\pi)^{1/2}\int_0^\infty \rho(r',\theta')Y_{20}(\theta')r'^4\,dr'\,d\Omega'. \quad (2)$$

For spin-$\frac{7}{2}$ rotational nuclei, q and Q_0 are related by $q = \frac{7}{15}Q_0$.

The quadrupole interaction energy of the muonic $2p$ level is changed by as much as 50% by the effect of nuclear size in high-Z nuclei. Clearly

TABLE I. Charge moments of ^{165}Ho.

	Q_0 (b)	Π_0 (b^2)
Other measurements	7.48 ± 0.14 [a]	12.6 [b]
	5.85 ± 0.13 [b]	0.4 [e]
	7.62 ± 0.12 [c]	
	7.4 ± 0.4 [d]	
Present experiment	7.44 ± 0.07	0.52 ± 0.10

[a] Average of all values given in Ref. 2.
[b] Atomic beam (hfs), Ref. 3.
[c] Coulomb excitation, Ref. 2.
[d] Giant resonance, Ref. 2.
[e] Theory quoted in Ref. 3.

such an effect can reveal interesting information about the quadrupole charge density, but unless some nuclear-charge model is invoked, a value of q cannot be deduced from the $2p$ hfs. On the other hand, the nuclear-size effects are only about 5% in the $3d$ muonic levels. With the nuclear charge distribution deduced from our absolute muonic x-ray energies, the finite-size corrections can be easily calculated with an accuracy better than 20%, thus allowing the extraction of q (or Q_0). This question is treated in more detail elsewhere.[5] In ^{165}Ho the hfs of the $3d$ level amounts to 7 keV which can be measured with 1% accuracy.

One byproduct of a careful analysis of the hfs of the muonic $3d$ levels is the possibility of directly observing the static hexadecapole ($E4$) effects. Note that only the $3d_{5/2}$ state may be affected by the $E4$ operator, while neither the $3d_{3/2}$ nor $2p$ states have diagonal $E4$ matrix elements. The intrinsic nuclear hexadecapole moment Π_0 is given by

$$\Pi_0 = (\tfrac{4}{9}\pi)^{1/2}\int_0^\infty \rho(r',\theta')Y_{40}(\theta')r'^6 dr'\,d\Omega' \qquad (3)$$

and the spectroscopic hexadecapole moment $\Pi = \frac{20}{7}\Pi_0$. In the ^{165}Ho $3d_{5/2}$ level the $E4$ splittings are approximately 1% of the $E2$ splittings and comparable with the $M1$ splittings. The $M1$ effects were included in our analysis.

Our findings are based on muonic x-ray data taken at the muon channels of the 600-MeV synchrocyclotron of the Space Radiation Effects Laboratory in Newport News, Virginia, and the Clinton P. Anderson Meson Physics Facility in Los Alamos, New Mexico.

In our analysis the nine observables listed in Table II were used in order to determine the best-fit charge parameters. The absolute energies of the indicated transitions refer to the levels unperturbed by hyperfine effects. In the analysis of $2p$ levels we took into account the $E2$ mixing of the first four nuclear rotational levels. In the $3p$ and $3d$ levels only the first three nuclear levels were included as the largest first-excited-state amplitude was no more than 0.01. $M1$ effects were included in all levels with $n<4$. The effects of weak inner transitions such as $4d \to 3p$ were also included.

TABLE II. Observables in muonic ^{165}Ho. All quantities are in keV. $[\epsilon_\lambda(j,j')]_{nl}$ is defined in Eq. (1), Δ is the fine-structure splitting, and the transition energies are explained in the text.

Observable	Experimental value	Fitted values: Variable skin thickness	Fitted values: Constant skin thickness
$[\epsilon_2(\frac{1}{2},\frac{3}{2})]_{2p}$	45.97 ± 0.10	45.94	45.92
$[\epsilon_2(\frac{1}{2},\frac{3}{2})]_{3p}$	12.24 ± 0.21	12.14	12.10
$[\epsilon_2(\frac{3}{2},\frac{3}{2})]_{3d}$	5.53 ± 0.05	5.53	5.63
$[\epsilon_4(\frac{5}{2},\frac{5}{2})]_{3d}$	0.157 ± 0.03	0.157	0.134
Δ_{3d}	19.19 ± 0.11	19.29	19.29
$2s_{1/2} \to 2p_{1/2}$	703.67 ± 0.41	703.70	703.77
$3p_{3/2} \to 2s_{1/2}$	1141.09 ± 0.60	1141.64	1141.62
$3d_{5/2} \to 2p_{3/2}$	1727.69 ± 0.49	1727.94	1728.00
$2p_{3/2} \to 1s_{1/2}$	4848.11 ± 1.60	4847.31	4847.49

The assumed form of the nuclear charge distribution was the modified Fermi distribution

$$\rho(r,\theta) = \rho_0\{1 + \exp[4.4(r-c)/t]\}^{-1}, \qquad (4)$$

where

$$c = c_0[1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta)],$$
$$t = t_0[1 + \beta_2 \eta Y_{20}(\theta)]. \qquad (5)$$

Two fits to our data were made: a five-parameter fit allowing β_2, β_4, c_0, t_0, and η to vary and a four-parameter fit with t not a function of angle, i.e., $\eta = 0$. Theoretical values for our observables were found by solving the Dirac equation as described in Ref. 5. Corrections were made for first- and higher-order vacuum polarization, electron screening, nuclear polarization, Lamb shift, relativistic recoil, and the quadrupole component of the vacuum polarization. The theo-

TABLE III. Parameters of the charge distributions.

	c_0 (fm)	β_2	t_0 (fm)	β_4	η	χ^2 per degree of freedom
Constant skin thickness	6.146 ± 0.027	0.3218 ± 0.0056	2.148 ± 0.074	0.040 ± 0.020	0	7.2/5
Variable skin thickness	6.093 ± 0.035	0.3437 ± 0.0144	2.276 ± 0.089	0.057 ± 0.021	-1.04 ± 0.56	3.2/4

retical predictions for the two charge distributions considered are shown in Table II. The indicated error includes the experimental error as well as the theoretical uncertainties in Lamb shift (30%) and nuclear polarization (20%).

The χ^2 per degree of freedom for each fit is acceptable although there seems to be a preference for the model which allows t to vary with angle. The main difference between the two fits is that the variable-skin-thickness model is more successful in simultaneously fitting the observed hfs in the $2p$ and $3d$ states. The inferred charge parameters are given in Table III.

Our values for Q_0 and Π_0 are taken exclusively from the $3d$-state data. Both models tested yield values of Q_0 which differ only by 0.02 b. On the other hand, the angular charge distributions differ sizably as demonstrated in Fig. 1. It is interesting to note that a recent analysis[6] of electron-scattering data in ^{176}Yb indicates a similar variation in skin thickness which is greater at the nuclear equator than at the poles.

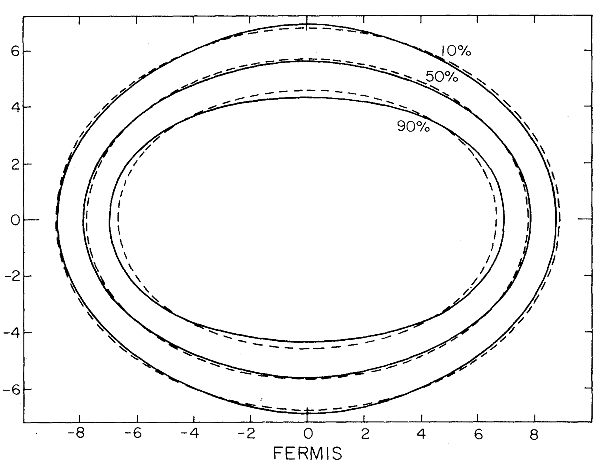

FIG. 1. Contour plots of the angular distribution $\rho(r,\theta)$ of ^{165}Ho, Eq. (4), showing a cut through the poles. The dashed contours correspond to the best-fit parameters assuming $\eta = 0$. The solid curves correspond to the fit where the skin-thickness deformation parameter η was allowed to vary. Each set of three curves denotes the 90%, 50%, and 10% profiles relative to the central density.

Our results agree very well with the Coulomb-excitation data.[2] Such a comparison is possible, however, only if ^{165}Ho is well described by the rotational model. Our data yield independent confirmation that this is indeed the case. From the $2p$ hfs we determined that the transitional and static $E2$ matrix elements agree with the rotational-model predictions to within 0.2%. It is interesting to note that the experimental Q_0 value agrees extremely well with the calculations of Hassan, Skaldanowski, and Szymanski.[7]

On the other hand there is poor agreement between our value of Q_0 and that determined by another spectroscopic technique, the atomic-beam method (see Table I). But in this case the Sternheimer[8] corrections for the effects due to the deformation of the atomic core were not made.

We have found but one other measurement of Π_0 for ^{165}Ho—an atomic-beam measurement.[3] This value (Table I), corrected for the Sternheimer effect,[9] is an order of magnitude larger than our value. On the other hand, our value is in reasonable agreement with the theoretical prediction[3] of 0.4 b^2. It corresponds to the value of the hexadecapole deformation parameter β_4 similar to those found by the Coulomb-excitation experiments in this region of the periodic table. Direct comparison with Coulomb-excitation work is, however, impossible, since the parameters β_2 and β_4 are model dependent.

A more detailed description of our work will be published elsewhere.

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Energy Spectra and Charge States of H, He, and Heavy Ions Observed in the Earth's Magnetosheath and Magnetotail

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H, He, and heavy ions of energies $\geqslant 0.12$ MeV/charge were detected in the magnetotail and in the magnetosheath. It was found that their relative abundances were $\sim 9:1:4\times10^{-2}$, their differential energy spectra were $\sim 1/E^4$, and their atomic electrons were almost completely stripped. These results led us to suggest that they were the low-energy "quiet-time" cosmic rays accelerated within the magnetotail and the magnetosheath to the observed energies.

Pulses of energetic electrons are frequently observed in the magnetotail.[1-6] The observation of energetic protons (10–30 keV) in the tail region was reported only recently by Hones *et al.*[7] In this paper, we report the observation of pulses of ions in the magnetosheath and in the tail with an electrostatic deflection spectrometer on the IMP-7 satellite. We have for the first time identified three species of ions, H, He, and heavier, measured the energy spectra, and inferred the charge states of the ions. From the consideration of the degree of ionization of these particles and the total energy content involved, we suggest that they are low-energy "quiet-time" cosmic rays accelerated locally to the observed energies. The possibility of accelerating electrons and protons in these regions has been suggested previously.[2-7]

The IMP-7 satellite was launched into space on 22 September 1972. The orbit has an apogee of 38 and a perigee 34 earth radii with an inclination of about 15° with respect to the geomagnetic equator. Among all the particle detectors on board the satellite, the electrostatic deflection spectrometer of the University of Maryland is designed for the measurement of low-energy particles. In this instrument, a particle is first selected according to its energy per charge by means of an electrostatic field, and then its energy T and charge Q are determined separately by an additional measurement of the energy deposit of the ion in an Au-Si solid state detector. The details of the system have been published elsewhere.[8] The characteristic of the system (abbreviated as