some of the assumptions we have used are too crude. It is clear from the foregoing that the next step should be a calculation similar to those in Refs. 13–16 for a rare-earth ion with the addition of a nearby electron, or a calculation similar to that of Wendin,[20] but with the inclusion of multiplet structure.

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Parametric Coupling in an Optically Excited Plasma in Ge

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Recent measurements of apparent absorption saturation and ultrafast relaxation in Ge have been reinterpreted as a parametric coupling between optical beams due to an index grating produced by a high-density electron-hole plasma. A sharp spike in probe-beam transmission of width 2 psec observed near zero time delay between a strong pumping and a weak probing beam has been shown to be the autocorrelation of the optical electric fields. In addition, evidence of a true saturation effect with a much longer recovery time is observed.

Recently the optical properties of germanium have been the subject of investigation on a picosecond time scale.[1,2] Kennedy *et al.*[1] have reported measurements of the nonlinear absorption of Ge from which they infer an ultrafast carrier relaxation time. They have observed an apparent saturation of the absorption of 1.06-μm picosecond pulses in thin Ge crystals at high optical intensities. The recovery time of the absorption was observed to be less than the optical pulse width of 5 psec. They have interpreted this result to be a saturation of band-to-band transitions with an apparent relaxation time of less than 5 psec. In this paper we provide new experimental evidence and calculation which allow us to reinterpret the experiments of Kennedy *et al.* as a parametric scattering in an optically generated electron-hole plasma.

A Ge crystal was mounted on a glass substrate and polished to a 5-μm thickness. A Syton polish was used to minimize surface damage. With the same configuration as that used by Kennedy *et al.*, the experiment was performed with a single pulse selected from a train of mode-locked pulses from a Nd:glass laser. A small probing pulse was obtained by splitting off 5% of the selected pulse. Both the weak and the strong pulses were focused onto the crystal surface at an angle of approximately 20 deg. This angle was sufficient to separate the beams conveniently upon passage through the crystal. The energy transmission of the probe pulse was measured as a function of relative path difference or time delay between the strong and weak pulse. The experiments were performed at room temperature.

The experimentally measured transmission data are plotted in Fig. 1 as a function of time delay. We interpret the data in terms of two mechanisms. We attribute the sharp spike near $\tau = 0$ to a parametric scattering mechanism, and the gradual rise and fall of the induced transmission to band filling.

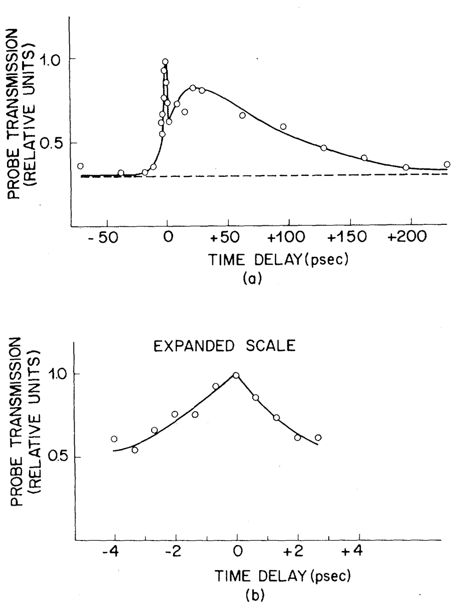

FIG. 1. (a) Plot of experimentally measured transmission as a function of relative time delay between pump and probe beam. (b) Expanded scale near $\tau = 0$ showing coherence spike.

The features of the sharp spike are shown in more detail on the expanded scale in the figure. The width of the spike is approximately 2 psec, much narrower than the pulse width which is estimated to be 7 psec. We interpret this spike as a parametric coupling between the pump and the probe beam in the electron-hole plasma formed by absorption of the two beams. With the pumping density used in this experiment a peak density of approximately 3×10^{20} cm^{-3} electrons is expected near the surface.

One can think of the coupling as being the result of scattering of the strong beam by a phase grating formed by a spatially modulated, electron-hole–plasma induced, index of refraction. This modulation is created by the spatial interference between the strong and weak beams. The period and phase of this grating are such that the strong beam is scattered into the direction of the weak beam. One would expect this coupling to peak at $\tau = 0$ and fall off as the delay exceeds the coherence length of the pulse.

The magnitude of this scattering process can be calculated by taking the optical polarization created by the electron-hole plasma and substituting into the wave equation. From the Drude formula for the polarizability of a free electron-hole gas combined with a time-varying density produced by the absorbed optical beams, we find the following expression for the optical polarization of the plasma mks units):

$$P(r,t) = -\left(\frac{\epsilon}{\mu}\right)^{1/2} \frac{4n\alpha e^2}{(n+1)^2 \hbar \omega^3 m^*} E(r,t) \times \int_{-\infty}^{t} E(\mathbf{r},t') E^*(\mathbf{r},t')\, dt', \qquad (1)$$

where $E(r,t)$ is the total complex optical electric field amplitude, m^* is the reduced mass, α is the absorption constant, and n is the refractive index.

Upon solving the wave equation for the transmission of the weak beam as a function of relative time delay we obtain

$$T(\tau) = e^{-\alpha l}[1 + aG^2(\tau)], \qquad (2)$$

where τ is the delay time,

$$G(\tau) = \frac{\int_{-\infty}^{\infty} E(t) E^*(t+\tau)\, dt}{\int_{-\infty}^{\infty} E(t) E^*(t)\, dt},$$

$$a = \frac{1}{3}\left[\frac{240\pi(1-e^{-\alpha l}) e^2 \mathcal{E}}{m^* \hbar \omega^2 (n+1)^2}\right]^2,$$

and $\mathcal{E}$ is the optical pulse energy of strong beam. The quantity $G(\tau)$ is the electric field autocorrelation function. Substituting the experimental parameters $m^* = 0.088 m_0$, $\mathcal{E} = 113$ J/m^2, $l = 5\times10^{-6}$ m^{-1}, we obtain $a = 2.5$. The peak-to-background ratio is then

$$T(\tau=0)/T(\tau=\infty) = 3.5.$$

This is somewhat larger than the experimental value of approximately 2.

We would expect the measured spike to be somewhat broader and reduced in amplitude from that predicted by the electric field autocorrelation function because of imperfect overlap of the two beams. A bandwidth of 10 cm^{-1} would produce a spike of width of approximately 1 psec.

We believe the slower feature in the experiment (Fig. 1) to be due to a saturation of the absorption by band filling, i.e., a filling of conduction-band states and depletion of valence-band states to the point where the separation between electron and hole quasi-Fermi levels approaches the photon energy. The buildup of this effect follows the integrated optical pulse energy since recombination is expected to be slow. The decay, however, is most likely due to a reduction in density by the diffusion of the electron and holes from a region approximately α^{-1} (1 μm) near the surface into the crystal (approximately 5 μm thick). While the dynamics of this decay are quite complex, we can see that the overall decay rate is comparable to previous measurements[2] of diffusion rates. We can make a rough estimate of the plasma density required to produce band filling by calculating the positions of the electron and hole quasi-Fermi levels assuming parabolic bands. Using the density-of-states effective masses, $m_e = 0.55 m_0$ and $m_h = 0.35 m_0$, we find that the quasi-Fermi levels are separated by the photon energy of 1.17 eV (compared to $E_g = 0.66$ eV) at a density of approximately 2×10^{20} cm^{-3}, slightly less than the estimated experimental value. Although band filling could also contribute to the parametric coupling responsible for the spike by an amplitude grating, this process is at least an order of magnitude smaller than the plasma index mechanism.

In conclusion we have demonstrated that parametric coupling in an electron-hole plasma can explain the results of Kennedy *et al.* without requiring an ultrafast relaxation process. Furthermore, we have observed a true saturation of the absorption which can be accounted for by band filling.

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Physical Realizations of $n \geq 4$ Vector Models*

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It is pointed out that certain phase transitions which involve a doubling of the unit cell are described by n-component vector models with $n \geq 4$. In particular, it is noted that the structural transition in NbO_2 is described by an $n=4$ component model with some tetragonal anisotropy. The critical behavior of this model is studied to order ϵ^2, by the exact renormalization group in $d = 4-\epsilon$ dimensions. It is found that the critical behavior is determined by a new, tetragonal, fixed point.

The critical behavior of the n-component vector model has been of considerable interest in recent years.[1-7] For $n = 1, 2, 3$ the model corresponds to physical systems which are Ising, X-Y, and Heisenberg-like, respectively. It has also been argued[8,9] that the limit $n\to0$ corresponds to amorphous Ising systems. In this work, it is pointed out that certain phase transitions, which involve a doubling of the unit cell in one or more directions, are described by n-component models with $n \geq 4$. As discussed by Landau,[10] the symmetry-breaking order parameter associated with a second-order phase transition transforms as an irreducible representation of the symmetry group of the high-symmetry phase. The number of independent components of the order parameter is,