Freeman, A. J. Toepfer, and M. J. Clauser, to be published; L. I. Rudakov and A. A. Samarsky, in *Proceedings of the Sixth European Conference on Controlled Fusion and Plasma Physics, Moscow, U. S. S. R, 1973* (U.S.S.R. Academy of Sciences, Moscow, 1973), p. 487.

[2] J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, Nature 239, 139 (1972); K. A. Brueckner and S. Jorna, Rev. Mod. Phys. 46, 325 (1974).

[3] G. Cooperstein, J. J. Condon, and J. R. Boller, J. Vac. Sci. Technol. 10, 961 (1973).

[4] G. Cooperstein, J. G. Siambis, and J. J. Condon, Bull. Amer. Phys. Soc. 19, 532 (1974).

[5] G. Yonas, K. R. Prestwich, J. W. Poukey, and J. R. Freeman, Phys. Rev. Lett. 30, 164 (1973); P. A. Miller, J. Chang, and G. W. Kuswa, Appl. Phys. Lett. 23, 423 (1973); K. M. Glibert, J. Chang, and L. P. Mix, Bull. Amer. Phys. Soc. 19, 869 (1974).

[6] G. Cooperstein, J. J. Condon, and J. R. Boller, Bull. Amer. Phys. Soc. 18, 1310 (1973); G. Cooperstein and J. J. Condon, to be published.

[7] S. A. Goldstein and J. G. Siambis, private communication.

[8] L. S. Levine and I. M. Vitkovitsky, IEEE Trans. Nucl. Sci. 18, No. 4, 225 (1971).

Measurement of Ion-Acoustic Plasma Turbulence by Cross-Power Spectra*

Dragan B. Ilić,
Institute for Plasma Research, Stanford University, Stanford, California 94305

(Received 21 October 1974)

The propagation of current-driven ion-acoustic waves in a positive column is studied by measuring the cross-power spectral density up to 30 MHz. Dispersion characteristics are verified in the weakly turbulent state, and the measurements of instability correlation times and lengths are illustrated for higher levels of turbulence, where the dispersion relation no longer applies.

Statistical methods are very useful for analyzing the results of experiments in weakly turbulent plasmas, where many interacting waves are present: The cross-correlation function is used in the time domain, while its Fourier transform, the cross-power spectral density, is used in the frequency domain.[1] The currently available cross correlators can process signals of at most a few megahertz,[1,2] unless sampling techniques are used,[3] while the application of well-developed signal-processing techniques in the frequency domain[4] can, in principle, lead to analog instruments for measuring the cross-power spectra in the gigahertz range.[2]

This note reports the results of studying ion-acoustic waves in a weakly turbulent plasma, obtained by measuring the cross-power spectral density up to 30 MHz. While measurements of the cross-power spectra below 1 MHz have been used to deduce the ambipolar diffusion coefficient[5] and electron density[6] in thermal plasmas, and the spectral index in turbulent plasmas,[7] we believe this to be the first application of the cross-power spectra to the study of propagating plasma waves up to 30 MHz. It will be demonstrated that important information about the turbulent characteristics of saturated instabilities can be obtained in this way.

For a given pressure, current-driven ion-acoustic waves are excited in the positive column above a critical value of discharge current, in the form of a band of frequencies around the frequency f_0 for which the convective growth rate

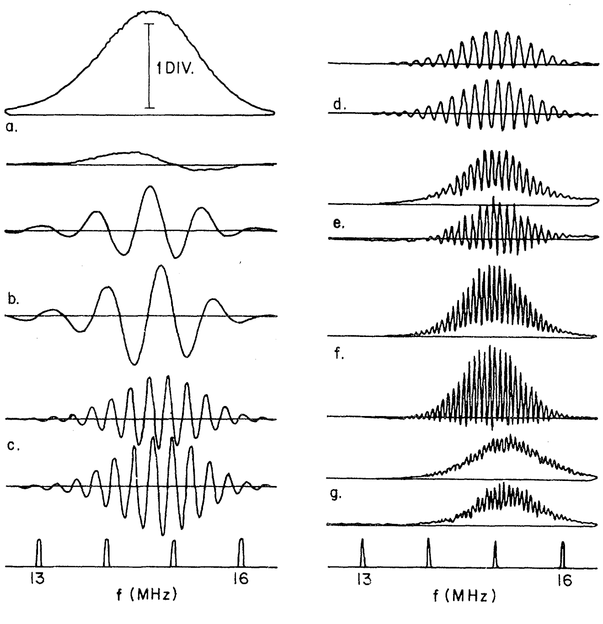

FIG. 1. The cross-power spectra at $pa = 0.48$ Torr cm, $I = 2.5$ A, and the following probe separations Δz (mm), and vertical sensitivities (mV/div): (a) 0, 20; (b) 10, 50; (c) 30, 50; (d) 50, 50; (e) 80, 10; (f) 110, 20; (g) 130, 10.

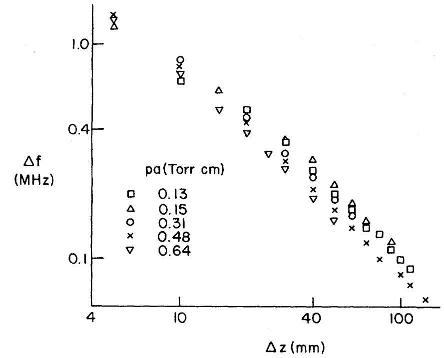

FIG. 2. The dependence of the frequency period Δf of wave forms such as Fig. 1 on probe separation Δz for several values of pressure. $I=2.5$ A (except 1.5 A at $pa=0.15$ Torr cm).

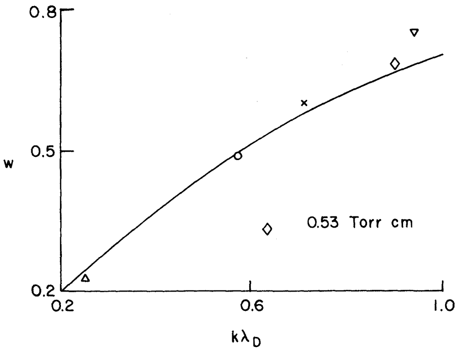

FIG. 3. The comparison of measured dispersion characteristics with the theory, Eq. (1). w is the frequency normalized to the ion plasma frequency.

maximizes.[8] Further increase in current leads to broadening of this frequency band, and to increases in f_0 and the power-spectrum amplitude. The waves propagate in the direction of electron drift and are strongly damped in the opposite direction. Assuming weakly excited, one-dimensional traveling waves, so that the modes interact only weakly, for each value of wave number k a single wave with frequency ω is excited, as given by the dispersion relation,[7]

$$v_{\rm ph}=\omega/k=v_s/(1+k^2\lambda_{\rm D}^2)^{1/2}, \tag{1}$$

where $v_s=(T_e/m_i)^{1/2}$ is the ion sound speed, $\lambda_{\rm D}$ the electron Debye length, T_e the electron temperature, and m_i the ion mass. The cross-power spectral density detected by two probes separated by distznce Δz is then[2]

$$H(\omega,\Delta z)\propto\varphi_k^2\exp[ik(\omega)\Delta z]. \tag{2}$$

At $\Delta z=0$, ${\rm Re}H\propto\varphi_k^2$, so that the amplitude factor, φ_k^2, has the shape of the power spectrum. The real and imaginary parts of H vary periodically in $\omega\Delta z/v_{\rm ph}$, amplitude-modulated by the shape of the instability power spectrum.

Cross-power spectra were measured using two plane ion-wave–detecting probes in a helium positive column of radius $a=2.5$ cm,[8] by use of the analyzer described in Ref. 2. A typical set of results is shown in Fig. 1, where pressure p and current I are kept constant, and the axial probe separation Δz is varied. Each pair of plots represents the real (upper trace) and the imaginary part of H, and the plots clearly demonstrate the features of Eq. (2) discussed above. Since H is periodic in frequency, the phase velocity is determined by plotting the frequency interval corresponding to one cycle of the wave forms Δf for different probe spacings Δz as shown in Fig. 2 for several sets of measurements. The points fall on the expected lines with slopes of -1, and for these weakly excited waves, the correlation lengths extend over many wavelengths (the wavelengths are typically ~0.5 mm). The resolving power of the analyzer (the i.f. bandwidth is 0.1 MHz) sets the limit for measuring very long correlation lengths, so that the actual correlation lengths for some runs, e.g., Fig. 1, may be longer than the ~10 cm implied by Figs. 1 and 2.

With the values of $v_{\rm ph}$ determined from Fig. 2, and the wavelength $\lambda=v_{\rm ph}/f_0$, $k\lambda_{\rm D}$ was calculated by using the discharge parameters for He.[9,10] The corresponding dispersion diagram is shown in Fig. 3, together with the theoretical curve, Eq. (1). Similar results obtained by using a tunable phasemeter have indicated coherence lengths of less than 5 mm,[11] perhaps because that method requires a larger amplitude level, and hence a more turbulent state, than the present one. The measured $v_{\rm ph}$ can also be substituted into Eq. (1) to determine v_s, and thus T_e. Figure 4 shows the resulting dependence of T_e on pa, together with the theoretical result,[10] where the discrepancies are due to the inaccuracies of the theory in

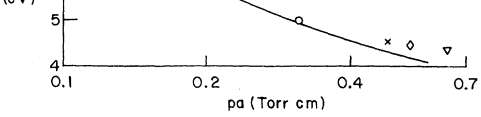

FIG. 4. The comparison of measured electron temperature T_e with the theory, Ref. 10.

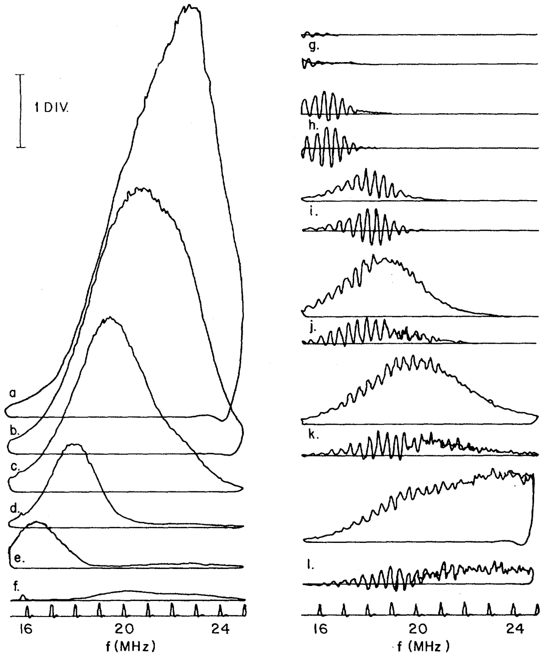

FIG. 5. Spectra at $\Delta z = 20$ mm, $pa = 0.53$ Torr cm. Power (self) spectra taken at the following values of discharge current (vertical sensitivity 50 mV/div): (a) 4.0 A; (b) 3.5 A; (c) 3.0 A; (d) 2.5 A; (e) 2.0 A; (f) 1.5 A. Corresponding cross-power spectra at the following discharge currents (amperes) and vertical sensitivities (mV/div): (g) 1.5, 20; (h) 2.0, 50; (i) 2.5, 50; (j) 3.0, 20; (k) 3.5, 20; (l) 4.0, 20.

Ref. 10 for predicting density, and thus λ_D. It is seen that the dispersion characteristics of ion-acoustic waves close to the instability boundary can be deduced by the simple theory of Eq. (2).

When the discharge current is further increased, the instability becomes so strong that the dispersion relation, Eq. (1), no longer holds, and the detected spectra change considerably. This is illustrated in Fig. 5, where the probe spacing is set at 20 mm and the level of instability is varied by varying the current. Figures 5(a)–5(f) show the power spectra (ReH for $\Delta z = 0$) from just below instability onset to much above it. These results are consistent with our earlier measurements with a commercial spectrum analyzer.[8] The corresponding cross-power spectra [5(g)–5(l)] show increased complexity, loss of correlation (note the sensitivity settings), and imply increased phase mixing as the level of wave turbulence increases. In this case, it would be necessary to perform a spatial Fourier transform to obtain the wave spectral density.[2] While this is a task which requires the use of computer data-processing techniques for our experimental conditions, spectra such as Fig. 5 can still be used to study the transition from weakly to strongly turbulent regimes. For example, the loss of coherence demonstrated by the decrease of cross-power spectral amplitudes, the decrease in correlation times corresponding to the broadening of the power spectra in Figs. 5(a)–5(f), and the decrease in correlation length demonstrated by the degradation of the periodicity of $H(\omega)$ in Figs. 5(g)–5(l) may all be interpreted as both qualitative and quantitative characteristics of turbulent states. Correlation lengths and times are fundamental parameters of nonlinear plasma theory, and their direct measurements are very important for further development of realistic models of turbulent plasma states.

Discussions with Professor F. W. Crawford and Dr. K. J. Harker are gratefully acknowledged.

---

*Work supported by the National Science Foundation.

[1] E. B. Hooper, Jr., Plasma Phys. 13, 1 (1971).

[2] K. J. Harker and D. B. Ilić, Rev. Sci. Instrum. 45, 1315 (1974).

[3] I. Alexeff, R. V. Neidigh, and W. R. Wing, Int. J. Eng. Sci. 7, 531 (1969).

[4] J. S. Bendat and A. G. Piersol, *Measurement and Analysis of Random Data* (Wiley, New York, 1966), Chap. 6.

[5] G. S. Mills, C. K. McLane, and T. Tsukishima, Phys. Fluids 13, 2135 (1970).

[6] K. H. Geissler, R. A. Greenwald, and W. Calvert, Phys. Fluids 15, 96 (1972).

[7] D. E. Smith and E. J. Powers, Phys. Fluids 16, 1373 (1973).

[8] D. B. Ilić, G. M. Wheeler, F. W. Crawford, and S. A. Self, J. Plasma Phys. 12, 433 (1974).

[9] N. A. Karelina, J. Phys. (U.S.S.R.) 6, 218 (1942).

[10] D. B. Ilić, J. Appl. Phys. 44, 3993 (1973).

[11] D. B. Fenneman, M. Raether, and M. Yamada, Phys. Fluids 16, 871 (1973).