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Measurement of the Thermal Boundary Resistance between Solid ^{3}He and Cerium Magnesium Nitrate

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The thermal boundary resistance between powdered cerium magnesium nitrate and solid ^{3}He was measured in the temperature range between 45 and 250 mK. The phonon bottleneck as well as the phonon-dependent boundary resistance was observed. At low temperatures we observed a magnetic Kapitza resistance in parallel with that due to the phonons. The magnetic Kapitza resistance is proportional to T^2 in agreement with theoretical predictions.

We have measured the thermal boundary resistance between powdered cerium magnesium nitrate (CMN) (average particle diameter ~50 μm) and solid ^{3}He (23.9 cm^3/mole, ~15 ppm ^{4}He) at several different applied magnetic field strengths in the temperature range between 45 and 250 mK. At temperatures below 70 mK and fields greater than ~55 G, the observed measurements of the magnetic Kapitza resistance are consistent with the T^2 dependence of that resistance predicted by Guyer.[1]

The observation of anomalously short time constants for thermal equilibrium between liquid ^{3}He and powdered CMN was first reported in 1966 by Able *et al.*[2] At a CMN magnetic temperature of 2 mK, Black *et al.*[3] observed that this time constant was about 100 times smaller than it would have been if the rate of energy transfer was limited by the usual phonon boundary resistance.

Leggett and Vuorio[4] proposed that the mechanism for these anomalously high thermal transfer rates was the electromagnetic dipole coupling between Ce^{3+} spins and ^{3}He nuclear spins. On the basis of this assumption, they calculated that this so-called "magnetic Kapitza resistance" would depend linearly on T. Guyer[1] rederived this T dependence using a different approach. Guyer went on to apply the assumption of nuclear-spin–electron-spin dipolar interaction to predict that an anomalous boundary resistance will also occur between CMN and solid ^{3}He and that this resistance will be proportional to T^2. Leggett is in agreement with these predictions,[5] which imply that at very low T, solid ^{3}He may be an even better medium for thermal contact to CMN than liquid ^{3}He.

The method used to measure the boundary resistance was similar to that of Abel *et al.*[2] The experimental chamber which was thermally tied to a dilution refrigerator contained 0.5 g of CMN and 2.7 cm^3 of ^{3}He. These quantities were chosen so that the specific heat of the ^{3}He was always significantly greater than that of the CMN. To ensure that the CMN was coupled to the ^{3}He rather than directly to the dilution refrigerator, experiments were also carried out with no ^{3}He in the chamber, with only ^{3}He vapor, and with liquid ^{3}He. The temperature was measured by means of a resistor calibrated against the CMN susceptibility at zero magnetic field. As an added check, it was determined that at the temperatures of our measurement the relation between $\chi(0)$, the susceptibility of CMN in zero magnetic field, and $\chi(H)$, the susceptibility in a dc magnetic field H parallel to the susceptibility measuring field, was given by[6]

$$\chi(0)/\chi(H) - 1 \propto H^2 \quad (1)$$

to a high precision.

By applying a magnetic field and then reducing it to a desired field value we quickly demagnetized the CMN powder, thereby lowering its temperature ($\Delta T/T \sim 3\%$), and then observed the subsequent equilibrium time constant as it warmed back to the ambient temperature of the ^{3}He. These time constants are shown in Fig. 1 at dif-

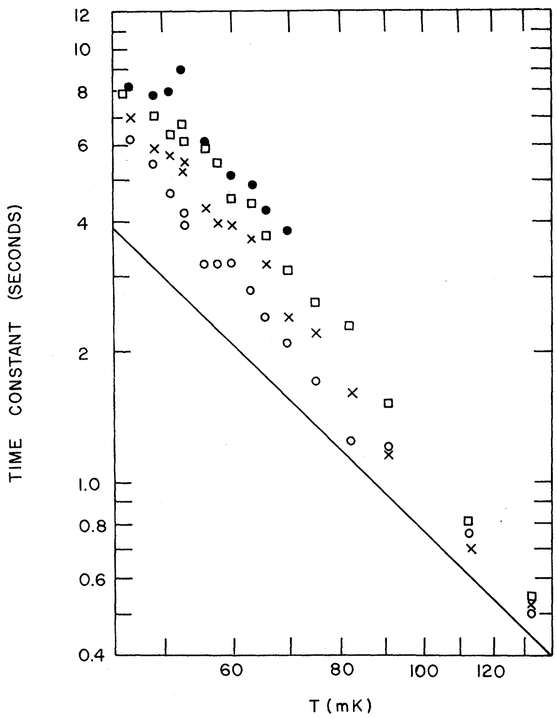

FIG. 1. Thermal time constants between CMN and solid ^{3}He in an applied magnetic field. Open circles, 55 G; crosses, 96 G; squares, 137 G; solid circles, 178 G; solid line, 0 G.

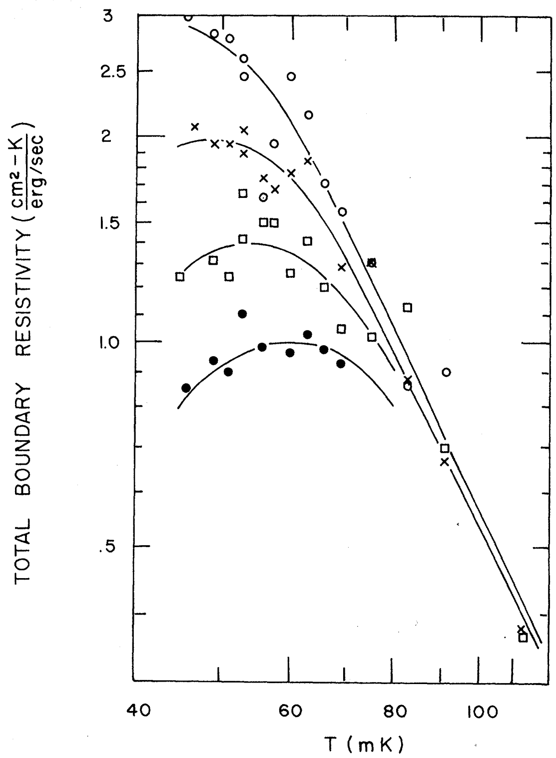

FIG. 2. Total boundary resistivity between powdered CMN and solid ^{3}He (23.9 cm^3/mole) in an applied magnetic field. Open circles, 55 G; crosses, 96 G; squares, 137 G; solid circles, 178 G. The solid lines are plots of the equation

$$\rho_B(H) = \rho_m(H)\rho_P/[\rho_m(H) + \rho_P],$$

where the values of $\rho_m(H)$ and ρ_P are derived experimentally.

ferent final magnetic fields.

The time constants at $H=0$ (H is the applied magnetic field) were fitted by a straight line described by

$$\tau_{PB} = 7.4\times10^{-3}T^{-2}\ \mathrm{sec} \qquad (2)$$

which is the expected "phonon bottleneck" observed in CMN at these temperatures.[7] It has been shown[8] that the phonon-bottleneck time constant in CMN, τ_{PB}, is field independent in the high-temperature approximation. Thus, if one assumes that the phonon bottleneck is in series with the total thermal boundary resistivity ρ_B, which is associated with the time constant τ_B through the relationship

$$\tau_B = (V/A)\, c_V(H)\rho_B, \qquad (3)$$

where $c_V(H)$ is the heat capacity of CMN per unit volume[9] and equals $c_V(0) + CH^2/T^2$ with C the Curie constant, V is the CMN volume, and A is its surface area,[10] then $\tau_B(H, T)$ can be determined by measuring the total relaxation time $\tau(H, T)$ in a magnetic field at a temperature T, and subtracting the value of the field-independent $\tau_{PB}(T)$, given by Eq. (2). These results are shown in Fig. 2.

The value of V/A for the powdered CMN sample was estimated according to the method used by Bishop *et al.*[3] However, the reliability of this estimate is in question and might possibly be smaller by an order of magnitude because the particles are in contact with each other and have anisotropic heat conductivity. Thus the absolute value of ρ_B is only approximate and the significant result is the dependence of ρ_B on T.

The total thermal boundary resistivity consists of a parallel combination of phonon resistivity, ρ_P, and magnetic dipolar resistivity, $\rho_m(H)$.[3] We can write

$$\rho_B(H) = \rho_P\rho_m(H)/[\rho_P + \rho_m(H)]. \qquad (4)$$

By assuming that the field-independent ρ_P takes the form $\rho_P \propto T^{-3}$ we calculated from the data in Fig. 2 that

$$\rho_P = [5.7\times10^{-4}\ \mathrm{K^4\ cm^2/(erg/sec)}]T^{-3}. \qquad (5)$$

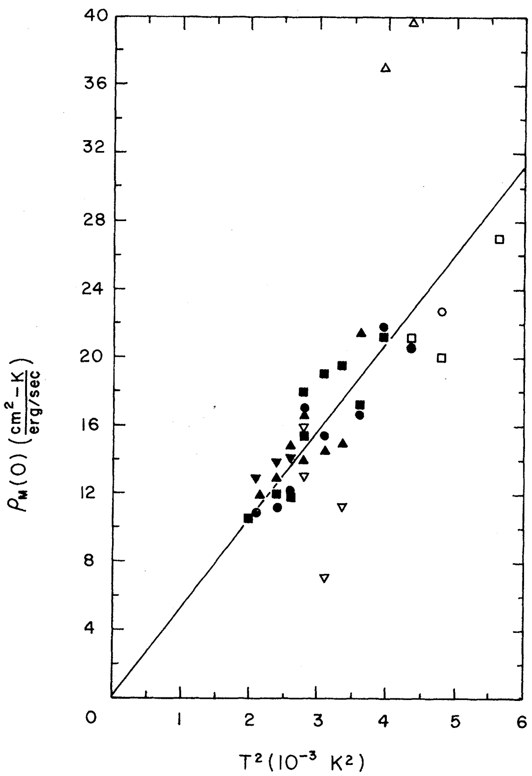

FIG. 3. Zero-field magnetic boundary resistivity $\rho_m(0)$, derived from measured values of $\rho_m(H)$ in the applied fields of 55 G, inverted triangles; 96 G, erect triangles; 137 G, squares; and 178 G, circles. Open symbols indicate data points with an estimated systematic error greater than ±25%. The straight line is a weighted "least-squares" fit of the data by a line of the form $\rho_m(0) = \text{const} \times T^2$.

From this, the values of $\rho_B(H, T)$, and Eq. (4) we obtain $\rho_m(H, T)$. In order to compare our data with theoretical predictions, it was useful to relate $\rho_m(H, T)$ to $\rho_m(0, T)$. Using a Redfield-like theory[11] Bishop derives the relation

$$\rho_m(H, T) = [H_d{}^2/(H_d{}^2 + \tfrac{1}{2}H^2)]\,\rho_m(0, T), \qquad (6)$$

where H_d is the CMN rms local dipolar field and H is the applied field. Equation (6) is valid for the condition of $H, H_d \ll k_B T/\gamma h$ (γ is the CMN gyromagnetic ratio), which holds in our measurements, and implies that one improves thermal contact by increasing the applied field.

If we assume that $H_d = 40$ G and apply Eq. (6) to the obtained $\rho_m(H, T)$ values we obtain the data shown in Fig. 3. It is seen here that our data are in agreement with the predicted form

$$\rho_m(0, T) = \text{const} \times T^2 \qquad (7)$$

with a rms deviation of ~15%. The constant (which, as stated before, is not reliable) is equal to 5×10^3 cm^2 sec/erg K. This is about 10 times the value of the predicted constant.[1]

In a similar experiment and extrapolation of the data we were able to observe the magnetic Kapitza resistance between liquid ^{3}He and CMN. The data thus obtained show agreement with the equation

$$\rho_m(0, T) = [760\ \text{cm}^2/(\text{erg/sec})]\,T\ \text{(liquid)} \qquad (8)$$

with a 20% rms deviation in the temperature region between 50 and 70 mK. The precision of the liquid measurement was smaller than that of the solid measurement because at the lowest temperatures, about 50 mK, $\rho_m(0, T)$ of the liquid is a factor of 5 greater than that of the solid. The observation of $\rho_m(0, T)$ in the liquid at these temperatures seems to contradict the interpretation by Harrison and Pendrys[12] of the data of Bishop *et al.* in terms of only phonon Kapitza boundary resistance and the phonon bottleneck.

To our knowledge, this is the first experimental observation of the T^2 dependence of ρ_m between CMN and solid ^{3}He. Although the precision of the data is not sufficient to establish the T^2 dependence precisely, the rms deviation of the data from this law is 15% and thus it definitely rules out the T dependence obeyed between CMN and liquid ^{3}He. This result is also significant because of its implication on paramagnetic cooling of ^{3}He solid by CMN where the limiting resistance will now become the phonon bottleneck. τ_{PB}, however, depends on the particle size[12] and could thus be made small. This is important in light of the recently reported nuclear magnetic ordering in solid ^{3}He.[13]

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Intense Focusing of Relativistic Electrons by Collapsing Hollow Beams

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Low-impedance diodes with hollow tapered cathodes produce strong self-pinching in intense relativistic electron beams. Early in the pulse a thin hollow beam is formed, based on evidence of electrons striking the anode. This hollow beam collapses, accelerating toward the diode axis with velocities (1 to 5 mm/nsec) which depend locally on the anode material. An efficient and stable pinch, less than 3 mm in diameter, is formed at the anode. In the center 0.1 cm^2, the power rises to 10^{11} W in less than 3 nsec. About 50% of the total diode energy (~ 9 kJ) is dissipated within the pinch region.

There has been considerable interest recently in intensely focused electron beams, primarily in connection with studies aimed at the achievement of pulsed fusion[1] in a manner analogous to that proposed using high-power lasers.[2] Typical electron-beam parameters considered necessary for pellet fusion are more than 10^{14} W of electrons of energies less than 3 MeV, efficiently delivered to a pellet approximately 3 mm in diameter in times less than 10 nsec.[1] In this Letter, we report on experiments with the U. S. Naval Research Laboratory's generator[3] Gamble-I [750 keV, 500 kA, 70 nsec] which demonstrate efficient focusing of high-current relativistic electron beams to areas of about 0.1 cm^2. Simultaneously, we have been able to overcome the limitations on current rise time posed by the Gamble-I generator (30 nsec), obtaining power rise times an order of magnitude shorter. Such rise-time limitations are quite general for high-power electron-beam accelerators.

Observations show that tapered hollow cathodes[4] initially produce a thin hollow beam which collapses into a tight pinch at the anode center. In contrast to the pinch-formation process in plasma-channel diodes,[5] negligible electron current strikes the center of the anode before the pinch is formed. The lack of early current near the anode center is responsible for the observed power rise time on axis of less than 3 nsec. More than 50% of the total diode energy is deposited in the pinch region at a rate corresponding to 70% of the total diode power. Pinch currents and power densities in excess of 1.6×10^6 A/cm^2 and 10^{12} W/cm^2 have been measured over areas of 0.1 cm^2. Although similar current and power densities had been previously reported, the irradiated areas in these experiments are an order of magnitude larger.

The experiments described here were all done on the Gamble-I accelerator at total electron-beam energies of 8 to 9 kJ. A hollow cathode with an outer diameter of 84 mm and an inner diameter of 39 mm was used. The front face of the cathode was slightly tapered (at a 6° angle) with the anode-cathode spacing smallest at the inside edge. The majority of the shots were fired with a diode gap of 3.7 mm, giving a 3-Ω-impedance beam. In some shots, the impedance was reduced to about 2 Ω by narrowing the gap. Figure 1(a) shows typical pulse forms of the diode current, voltage, and impedance. Note the flat plateau in the impedance curve which is characteristic of hollow cathodes operating in the self-pinch mode.[6] The shot-to-shot reproducibility of these signals was about ± 3%. This diode never failed to produce a tight and reasonably well-centered pinch. Pinch locations all fell within a 3-mm radius around the diode axis as witnessed by damage patterns on the anode plates.

The spatial distribution of the current reaching the anode and its time behavior were investigated