60 (1931), and Phys. Rev. 74, 817 (1948).
[4]W. Israel and G. Wilson, J. Math. Phys. (N.Y.) 13, 865 (1972).
[5]Z. Perjes, Phys. Rev. Lett. 27, 390 (1971).
[6]There appears to be a misprint in Eq. (26) of Ref. 4: namely, ∇ should read $-\nabla$. This sign was propagated by R. A. Kobiske and L. Parker, Phys. Rev. D 10, 2321 (1974), so that ω_φ should read $-\omega_\varphi$ throughout and J should read $-J$ in that reference.
[7]Kobiske and Parker, Ref. 6.
[8]One can argue that the stresses which the line singularities or "struts" exert (holding the charges in place) should be properly represented, within the Einstein, Maxwell, etc., system of equations, as exerted by other sources (gravitational, electromagnetic, etc.) without the above line singularities. It is possible that the present solutions without "struts" may give us insight as to what occurs in those more complicated "strutless" situations.
[9]J. B. Hartle and S. W. Hawking, J. Math. Phys. (N.Y.) 26, 87 (1972). For the case of vanishing spins, Hartle and Hawking also obtained Eqs. (23) and (24) (with $a_1 = a_2 = 0$).
[10]That result also holds when particle 2 is replaced by a subsystem of such particles, with line singularities permitted between pairs in the subsystem (R. A. Kobiske and L. Parker, unpublished).
[11]W. Israel and J. T. Spanos, Lett. Nuovo Cimento 7, 245 (1973).
[12]The sign of the square root in the expression for N is taken so that N vanishes in the limit of zero intrinsic spins, in agreement with the result for $a_1 = a_2 = 0$. The other root appears spurious, yielding an N which diverges as $a_2 - a_1 \to 0$ or $b_1 + b_2 \to \infty$, and which satisfies $N \gtrsim 10^{10}$ esu (a factor of 10^{20} greater than the electron charge).

K^+p Interactions at 100 GeV Using a Hybrid Bubble-Chamber–Spark-Chamber System and a Tagged Beam*

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We studied K^+p interactions at 100 GeV with the Fermi National Accelerator Laboratory 30-in. hydrogen bubble chamber and associated spark-chamber system. We find $\sigma_{tot}(K^+p) = 18.7 \pm 1.8$ mb and $\sigma_{el}(K^+p) = 2.0 \pm 0.4$ mb. We present the charged-multiplicity distribution and its moments, and the charge-transfer distribution. The average inelastic charged multiplicity is $\langle n_c \rangle = 6.65 \pm 0.31$ and the two-charged-particle correlation functions are $f_2^{cc} = 4.52 \pm 1.32$ and $f_2^{--} = 0.47 \pm 0.35$.

We present first results on K^+p multiparticle final states at Fermi National Accelerator Laboratory (FNAL) energies. We have analyzed approximately 130 000 pictures from the FNAL 30-in. hydrogen bubble chamber (BC) with a supplemental downstream wide-gap optical spark-chamber system[1] (WGOSC) exposed to an unseparated positive 100-GeV beam produced from the extracted 300-GeV proton beam. The beam contains 50% protons, 46% π^+, 2.7% μ^+, and 1.5% K^+ at the bubble chamber. Particle identities in the beam are established by (a) a 34-m-long differential Cherenkov counter[2] located 400 m upstream from the BC, and (b) muon tagging in a downstream scintillation-counter telescope using 1 m of Pb and 4 m of concrete shielding blocks. Three triples of multiwire proportional chambers (PWC)[3] with 2-mm wire spacing, located 175, 16, and 2.5 m upstream from the BC, determine beam position in the BC to better than 1 mm. We require that the BC measurement matches the projected PWC coordinates of one and only one beam track within an ellipse ± 1.5 mm in X and ± 3.0 mm in Z (Z is the BC stereoscopic depth direction). There is a 20% *unbiased* loss of data because of the 10 cm width of the PWC's, dead time in the system, upstream beam interactions, and match failures.

The differential Cherenkov counter uses a secondary mirror with a central hole to separate the K^+ mesons, which give Cherenkov light at angles of less than 5 mrad on the first phototube

($C1$), from π^+ mesons which give light at angles greater than 5 mrad onto a second phototube ($C2$). A detailed study of the Cherenkov-counter pressure curve (not shown) shows that the only measurable contamination to the ($C1\cdot\overline{C2}$) K^+ tag comes from protons, which are below threshold but can be accompanied by a δ ray giving light only in $C1$. We find that 1.7% of all particles entering the BC are tagged as K^+, and of these (9 ± 3)% are proton contamination and another 2% or less are π^+ from decays of K^+ near the BC.

With the WGOSC plus the BC we can measure a 100-GeV/c beam track momentum to ±4% as compared with ±25% accuracy for a *full length* beam track measured in the BC alone. The spark chambers were triggered whenever a deflection of the beam particle was detected and/or whenever a dE/dx counter near the BC exit window detected more than one outgoing particle. The overall trigger efficiency was about 60% for two-prong events and about 75% for four-prong and higher-multiplicity events. For BC events with a spark-chamber trigger, 74% of the tracks above 15 GeV/c hook up in the downstream WGOSC system. *All* tracks were measured in the BC and over 99% of them reconstructed successfully in TVGP and were used in our inclusive-particle studies.

We have a total of 234 K^+p interactions in a fiducial volume 42 cm long. The single-scanning efficiencies were determined to be (89 ± 6)% for the two-prong topology and (95 ± 2)% for the higher multiplicities. In addition, there is an intrinsic loss of small four-momentum transfer (t) two-prong events, which were corrected by fitting a large measured sample of our 100-GeV π^+, K^+, and proton two-prong interactions to a simple exponential in t. The resulting average correction of +(15 ± 5)% was applied to all of our K^+p two-prong events, and the results are quite insensitive to the expected variations in t slope between elastic and inelastic events. We obtain a K^+p total cross section of 18.7 ± 1.8 mb corrected for 9% proton contamination and for scanning, PWC matching, and small t losses. This compares excellently with the FNAL counter measurement of $\sigma_{tot}(K^+p) = 18.85 \pm 0.08$ mb.[4]

To obtain an elastic cross section two methods are used: (a) Elastic events are selected by kinematic fitting using the program in SQUAW and by a combination of cuts on coplanarity and the square of the missing mass recoiling against the slow proton. This yields 18 ± 5 elastic events. After a correction for losses, the resulting elastic cross section is $\sigma_{el}(K^+p) = 1.7 \pm 0.5$ mb. (b) The observed elastic t slope, assuming a simple exponential form, $d\sigma_{el}/dt = ae^{bt}$, is $b = 8.2 \pm 1.8$ GeV^{-2}. Using the optical theorem and the counter measurement of σ_{tot}, and assuming that the real part of the forward scattering amplitude is small, we obtain an elastic cross section $\sigma_{el} = 2.2 \pm 0.5$ mb. The weighted average of these two estimates is 2.0 ± 0.4 mb. The above value of b agrees well with the FNAL counter measurement of $b = 6.9 \pm 0.38$ GeV^{-2}.[5] Using the above assumptions, the counter value of b implies $\sigma_{el} = 2.64 \pm 0.15$ mb, which is used in the rest of this paper.

TABLE I. Topological cross sections and moments.

Prong number	Number observed	Corrected number [a]	$\sigma_n(K^+p)$ [b] (mb)
2	54	69.8	4.97 ± 0.80
Inel.			2.33 ± 0.85
4	46	48.4	3.50 ± 0.49
6	42	44.2	3.11 ± 0.47
8	47	49.5	3.79 ± 0.51
10	24	25.3	1.83 ± 0.38
12	13	13.7	1.00 ± 0.29
14	6	6.3	0.48 ± 0.20
16	0	0	0
18	2	2.1	0.18 ± 0.13

Charged	Negative
$\langle n_c\rangle = 6.65 \pm 0.31$	$\langle n_-\rangle = 2.33 \pm 0.16$
$f_2^{cc} = 4.52 \pm 1.32$	$f_2^{--} = 0.47 \pm 0.35$
$f_3^{ccc} = 3.34 \pm 6.57$	$f_3^{---} = -0.78 \pm 0.76$
Dispersion = 3.34 ± 0.19	
Skewness = 0.63 ± 0.27	

[a] Corrected for scanning and small t losses.

[b] Corrected for $\sigma_n(pp)$ and normalized to $\sigma_{tot}(K^+p)$ of Ref. 4.

Our inelastic topological cross sections are given in Table I and shown in Fig. 1, along with published values obtained at lower beam momenta. For K^+p interactions the four-prong cross section appears to be decreasing, while the two-prong inelastic cross section appears to be slightly decreasing or leveling off. The average K^+p charged-prong multiplicity of $\langle n_c\rangle = 6.65 \pm 0.31$ lies between the 100-GeV π^+p and pp values of 6.80 ± 0.14 and 6.49 ± 0.10.[6] The increase of $\langle n_c\rangle$ from 5.11 ± 0.05 at 32 GeV[7] is consistent with a ln(s) dependence, with a coefficient similar to the ln(s) dependence for other beam particle species.

The moments for the inelastic interactions are shown in Table I. Our value of $f_2^{cc}(K^+p)$

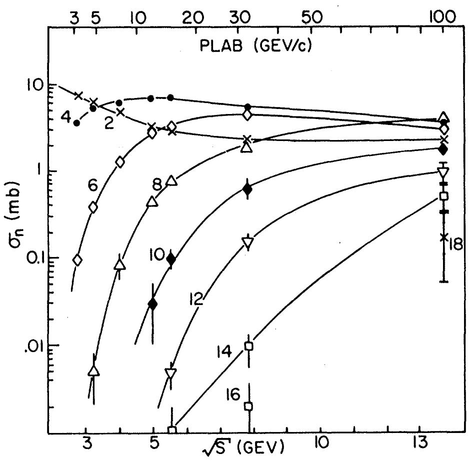

FIG. 1. Inelastic topological cross sections of K^+p interactions as a function of center-of-mass energy.

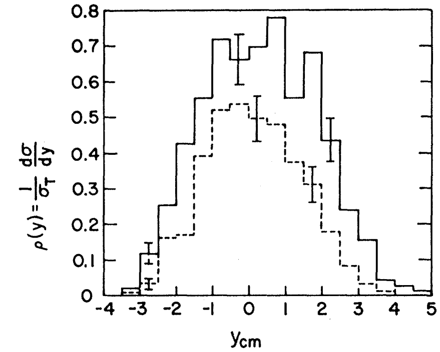

FIG. 2. Inclusive y distributions for π^- (dashed) and π^+ or K^+ with slow protons removed (solid).

$= 4.52 \pm 1.32$ shows that the integral over the two-charged-particle correlation function has risen considerably from the 32-GeV value $f_2^{cc}(K^+p) = -0.24 \pm 0.20$.[7] Removal of diffractive dissociation events[8] in the spirit of a two-component model[9] leaves a value of $f_2^{cc} = 3 \pm 2$ which is still positive. While our value of $f_2^{--} = 0.47 \pm 0.35$ is consistent with a Poisson distribution, it is probably crossing from negative toward large positive values at larger s, as is found for pp and π^-p interactions.

The single-particle inclusive interactions $K^+p \to \pi^- +$anything and $K^+p \to X^+ +$anything have been studied, where protons with ionization greater than 1.5 times minimum have been identified by scanning. All other charged particles will be treated as if they were pions. Elastic events have been removed as described above. Figure 2 shows the rapidity distributions $\rho^\pm = (1/\sigma_{tot})d\sigma^\pm/dy$, as functions of rapidity, $y = \ln(p_\parallel^* + E^*)/(m^2 + p_\perp^2)^{1/2}$. At $y = 0$ in the center of mass, there has been a considerable rise in ρ^- from that found at 12.7 GeV/c.[10] If our value of ρ^- at $y = 0$ is plotted against $p_{\text{beam}}^{-1/4}$ as suggested by Ferbel,[11] it is consistent with a straight line connecting the lower energy K^+p points and the presumed universal asymptotic value of 0.75 at infinite beam momenta. Our value of $\rho^+(0) \simeq 0.7$ is about 0.2 units above the value of $\rho^-(0)$ and is already close to the presumed asymptotic value, as is the case for $\pi^+p \to \pi^+ +$anything.[12]

Shown in Fig. 3 is the Feynman x distribution for the reaction $K^+p \to$ slow proton + anything. Elastic events are included. After scanning and small t corrections, we estimate 42.6 ± 8.0 two-prong events and 15.8 ± 4.1 four-prong and higher-multiplicity events due to beam diffraction and elastic scattering. (We include events with $x < -0.9$.) This gives a combined cross section of 4.3 ± 0.7 mb for these two processes, *not* including target diffractive dissociation.

Figure 4 shows the transverse-momentum distribution for negative particles, which is roughly exponential in $p_\perp^2$ but with an excess at large values of $p_\perp^2$. We have used PWC beam directions and WGOSC momentum information on outgoing tracks, where available, to improve substantially the accuracy of $p_\perp$. The average value of the square of the transverse momentum is $\langle p_\perp^2 \rangle^- = 0.181 \pm 0.025$ (GeV/c)^2 for $K^+p \to X^-$, for $p_\perp < 1.5$ GeV/c. The corresponding average transverse momentum magnitude is $\langle |p_\perp| \rangle^- = 0.34$ GeV/c. These values are compatible with either a logarithmic increase with s or a power-law dependence.

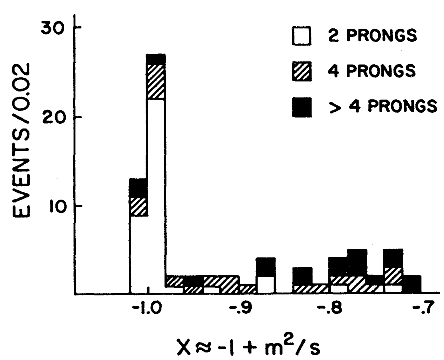

FIG. 3. Uncorrected Feynman x distribution for identified protons up to $x = -0.7$.

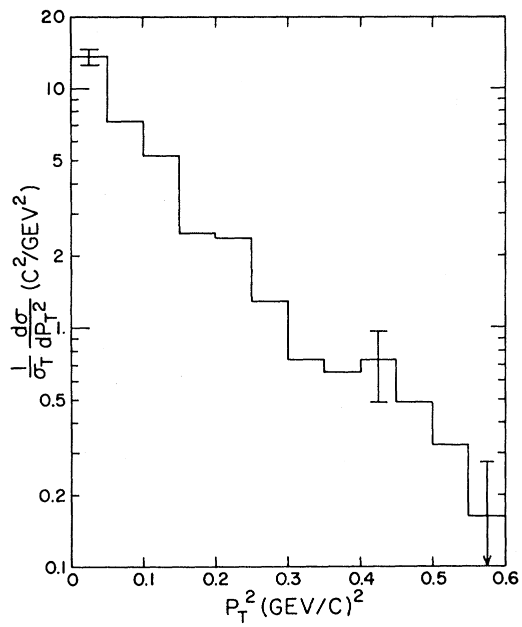

FIG. 4. The inclusive π^- transverse-momentum squared distribution.

The charge transferred in the center of mass between the forward and backward hemispheres is given by $u=\frac{1}{2}(Q_f - Q_b)$, where Q_f (Q_b) is the net charge of the final-state particles in the forward (backward) hemisphere. The charge-transfer cross sections are 7.8 mb for $u=0$, 3.5 mb for $u=\pm 1$, 0.61 mb for $u=-2$, 0.69 mb for $u=+2$, and 0.16 mb for $u \geqslant +3$. The u distribution is consistent with being symmetrical about $u=0$. The 100-GeV/c pp interactions also have a similar charge transfer distribution.[13] Our value of $\langle u^2 \rangle = 0.87 \pm 0.10$ is consistent with $\langle u^2 \rangle$ for those reactions, and is more compatible with multiperipheral production models than with the Yang fragmentation picture.[14]

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[1]G. A. Smith, in *Particles and Fields—1973*, AIP Conference Proceedings No. 14, edited by H. H. Bingham, M. Davier, and G. R. Lynch (American Institute of Physics, New York, 1973), p. 500.

[2]J. Lach and S. Pruss, FNAL Internal Report No. TM-298, 1971 (unpublished).

[3]Proportional Hybrid System Consortium, FNAL Proposal No. 154 (unpublished).

[4]A. S. Carroll *et al.*, Phys. Rev. Lett. 33, 932 (1974).

[5]C. W. Akerlof *et al.*, in Proceedings of the Seventeenth International Conference on High Energy Physics, London, 1973 (to be published), Session *A*1, and University of Michigan Report No. UMHE 7420 (to be published).

[6]J. Erwin *et al.*, Phys. Rev. Lett. 32, 254 (1974).

[7]G. A. Akopdjanov *et al.*, Nucl. Phys. B75, 401 (1974).

[8]Beam diffraction is defined as an event with an identified proton with Feynman x less than -0.90. We also remove target diffraction using the 102-GeV pp data, J. W. Chapman *et al.*, Phys. Rev. Lett. 32, 257 (1974), and assuming that factorization holds.

[9]J. Lach and E. Malamud, Phys. Lett. 44B, 474 (1973).

[10]S. Stone *et al.*, Phys. Rev. D 5, 1621 (1972).

[11]T. Ferbel, Phys. Rev. Lett. 29, 448 (1972).

[12]J. Erwin *et al.*, Phys. Rev. Lett. 33, 1352 (1974).

[13]C. Bromberg *et al.*, University of Rochester Report No. UR-460, 1973 (unpublished), and University of Michigan Report No. UMBC 73-19, 1973 (unpublished).

[14]See, for example, Figure 99, J. Whitmore, Phys. Rep. 10C, 275 (1974).