off, the result would be a distribution of lifetimes for the trapped charge. It is interesting to note that a turbulence theory of $1/f$ noise, wherein similar instabilities are used to generate the turbulence, has been proposed.[19]

The experiment can be improved in a number of ways. An SP with a more favorable signal-to-noise ratio is essential. We also plan to digitize simultaneously the noise voltage from the two probes. In this way, the auto- and cross-correlation functions can be obtained directly for a wider range of frequencies with considerably reduced experimental running time. It is also of interest to measure the cross correlation between two SP's with separation distance as a variable.

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Interaction between the Electron-Cyclotron Emissions at $(n + \frac{1}{2})\Omega_e$ and the Ring-Current Protons in Space

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The interaction between the electron-cyclotron emissions at $(n + \frac{1}{2})\Omega_e$ and the ring-current protons is studied by using the linear-response theory of a turbulent plasma. The effective turbulent collisions come from the decay interactions between the electron-cyclotron turbulence and electrostatic ion-cyclotron waves. The ion-cyclotron waves generated by the generalized parametric resonance are effective for the loss of the ring-current protons. The critical electron-cyclotron–turbulence amplitude is consistent with the observed value.

One of the most controversial subjects in space-plasma physics is the turbulent-loss mechanism of the ring-current protons. According to recent observations,[1-5] the ring-current protons suffer turbulent losses beyond the plasmapause. There are three candidates for the turbulent-loss mechanism of the ring-current protons. The first is electromagnetic ion-cyclotron turbulence just inside the plasmapause.[6] The second is electrostatic ion-cyclotron turbulence[7,8] beyond the plasmapause. The third is the mixing-mode mechanism.[9] In this Letter, I propose a new mechanism likely to occur for the ring-current protons.

According to recent observations in the magnetosphere,[10-12] there usually are naturally occurring electron-cyclotron emissions at $(n+\frac{1}{2})\Omega_e$ near the geomagnetic equator outside the plasmapause. I propose that the generalized parametric interactions between the electrostatic electron-cyclotron emissions and the electrostatic ion-cyclotron waves play an essential role in the loss of ring-current protons.

According to the linear-response theory of a turbulent plasma,[13] the response of the plasma to the finite- (but small-) amplitude electron-cyclotron–turbulence random pump fields ($\vec{E}_H$) is composed of two parts. One is the high-frequency fluctuating part (δf_H) and the other is the low-frequency fluctuating part (δf_L). The coupled kinetic equations for them are

$$\frac{\partial \delta f_H}{\partial t} + \vec{v}\cdot\nabla\delta f_H + \frac{e}{m}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial \delta f_H}{\partial \vec{v}} = \frac{e}{m}\vec{E}_H\cdot\frac{\partial \delta f_L}{\partial \vec{v}} + \frac{e}{m}\delta\vec{E}_L\cdot\frac{\partial f_{1e}}{\partial \vec{v}} + \frac{e}{m}\delta\vec{E}_H\cdot\frac{\partial f_{0e}}{\partial \vec{v}}, \tag{1}$$

and

$$\frac{\partial \delta f_L}{\partial t} + \vec{v}\cdot\nabla\delta f_L + \frac{e}{m}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial \delta f_L}{\partial \vec{v}} = \frac{e}{m}\left\langle\vec{E}_H\cdot\frac{\partial \delta f_H}{\partial \vec{v}}\right\rangle + \frac{e}{m}\delta\vec{E}_L\cdot\frac{\partial f_{0e}}{\partial \vec{v}} + \frac{e}{m}\left\langle\delta\vec{E}_H\cdot\frac{\partial f_{1e}}{\partial \vec{v}}\right\rangle, \tag{2}$$

where the angular brackets indicate a time average. f_{0e} represents the space-time–averaged electron distribution function and f_{1e} the high-frequency fluctuating part which carries the linear electron-cyclotron waves. δE_H and δE_L are the small electric fields associated with the electron-cyclotron and electrostatic ion-cyclotron waves.

Equations (1) and (2) can be solved by integrating with respect to time along the unperturbed orbit[14]:

$$\begin{aligned}\delta f_H(\vec{K},\Omega) = {} & \frac{e}{m}\int_0^\infty \vec{E}_H(\vec{K}-\vec{k}',\Omega-\omega')\cdot\frac{\partial}{\partial\vec{v}}\delta f_L(\vec{k}',\omega')\exp[-i\varphi(\tau')-\eta\tau']\,d\tau' \\ & + \frac{e}{m}\int_0^\infty \delta\vec{E}_L(\vec{k}',\omega')\cdot\frac{\partial}{\partial\vec{v}}f_{1e}(\vec{K}-\vec{k}',\Omega-\omega')\exp[-i\varphi(\tau')-\eta\tau']\,d\tau' \\ & + \frac{e}{m}\int_0^\infty \delta\vec{E}_H(\vec{K},\Omega)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}\exp[-i\varphi(\tau')-\eta\tau']\,d\tau',\end{aligned} \tag{3}$$

where $\varphi(\tau') = \vec{K}\cdot(\vec{r}-\vec{r}\,')-\Omega\tau'$; η is an infinitesimally small parameter to assure the adiabatic turning on of the disturbance; and $\vec{K}$, Ω and $\vec{k}'$, ω' are the wave numbers and frequencies for electron-cyclotron and ion-cyclotron waves, respectively. Substituting Eq. (3) into Eq. (2), we obtain

$$\begin{aligned}\delta f_L(\vec{k},\omega) = {} & \left[1+\left(\frac{e}{m}\right)^2\sum_{\vec{K}'}\int_0^\infty \vec{E}_H(\vec{K}',\Omega')\cdot\frac{\partial}{\partial\vec{v}}\int_0^\infty \vec{E}_H(-\vec{K}',-\Omega')\cdot\frac{\partial}{\partial\vec{v}}\exp[-i\varphi_1(\tau')-\eta\tau']\,d\tau'\exp[-i\varphi_0(\tau)-\eta\tau]\,d\tau\right] \\ & \times\left[\frac{e}{m}\int_0^\infty \delta\vec{E}_L(\vec{k},\omega)\cdot\frac{\partial}{\partial\vec{v}}f_{0e}\exp[-i\varphi_0(\tau)-\eta\tau]\,d\tau\right. \\ & + \left(\frac{e}{m}\right)^3\int_0^\infty\sum_{\vec{K}'}\vec{E}_H(\vec{K}',\Omega')\circ\frac{\partial}{\partial\vec{v}}\int_0^\infty\delta\vec{E}_L(\vec{k},\omega)\cdot\frac{\partial}{\partial\vec{v}}\int_0^\infty\vec{E}_H(-\vec{K}',-\Omega')\circ\frac{\partial}{\partial\vec{v}}f_{0e} \\ & \left.\times\exp[-i\varphi_1(\tau'')-\eta\tau'']\,d\tau''\exp[-i\varphi_2(\tau')-\eta\tau']\,d\tau'\exp[-i\varphi_0(\tau)-\eta\tau]\,d\tau\right],\end{aligned} \tag{4}$$

where

$$\varphi_0 = \vec{k}\cdot(\vec{r}-\vec{r}\,')-\omega(t-t'),\quad \varphi_1 = (\vec{k}-\vec{K}')\circ(\vec{r}-\vec{r}\,')-(\omega-\Omega')(t-t'),\ \text{and}\ \varphi_2 = -\vec{K}'\circ(\vec{r}-\vec{r}\,')+\Omega'(t-t');$$

τ, τ', and τ'' are differences between pairs of times. The partial derivatives in the first integrals operate on both f_{0e} and the subsequent exponentials. In deriving Eq. (4), I have used the following identity:

$$\frac{1}{A+B} = \frac{1}{A} - \frac{1}{A}B\frac{1}{A} + \dots. \tag{5}$$

The second term of the first bracket of Eq. (4) shows the decay interaction between the electron-cyclotron turbulence and the ion-cyclotron waves. In other words, this shows the effective collision process between electrons. The imaginary part of it shows the generalized parametric destabilizing effect of the electron-cyclotron turbulence on the ion-cyclotron waves. This term causes a drastic modification of the resonant interactions between the electrons and the ion-cyclotron waves. I must stress the difference between the turbulent collisions proposed by Dupree[12,15] and those of this paper. The conventional turbulent collisions always have a *damping* effect on waves. On the other hand, the effective turbulent collisions considered here come from the coupling process between the high-frequency turbulence and the low-frequency waves. As is shown in the following, this effect always gives a *destabilizing* effect on the ion-cyclotron waves. My formulation is the generalization of the previous parametric theories[16,17] for many random pump fields.

The dielectric constant (ϵ) of the ion-cyclotron waves in the presence of the electron-cyclotron–turbulence fields ($\vec{E}_H$) is

$$\epsilon = \epsilon_0(\vec{k}, \omega) + \epsilon_N(\vec{k}, \omega). \tag{6}$$

$\epsilon_0(\vec{k}, \omega)$ is the linear response from the quiescent plasma,

$$\epsilon_0(\vec{k}, \omega) = 1 - \left(\frac{\omega_{pe}}{k}\right)^2 \int \frac{k_\parallel \, \partial f_{0e}/\partial v_\parallel}{\omega - k_\parallel v_\parallel} d^3v - \left(\frac{\omega_{pi}}{k}\right)^2 \int \sum_n \frac{J_n^2(k_\perp v_\perp/\Omega_i)[k_\parallel \, \partial f_{0i}/\partial v_\parallel + (n\Omega_i/v_\perp)\, \partial f_{0i}/\partial v_\perp]}{\omega - k_\parallel v_\parallel - n\Omega_i} d^3v. \tag{7}$$

$\epsilon_N(\vec{k}, \omega)$ is the linear response from the turbulent plasma and the lowest-order contribution is

$$\begin{aligned}
\epsilon_N(\vec{k}, \omega) = {} & \left(\frac{\omega_{pe}}{k}\right)^2 \left(\frac{e}{m}\right)^2 \int \sum_{\vec{K}} |\Phi_H(\vec{K}, \Omega)|^2 \frac{1}{\omega - k_\parallel v_\parallel} K_\parallel \frac{\partial}{\partial v_\parallel} \frac{J_1^2(K_\perp v_\perp/\Omega_e)}{-\Omega + \Omega_e + K_\parallel v_\parallel} \left(k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{k_\perp}{K_\perp}\frac{\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\right) \\
& \times \frac{J_1^2(K_\perp v_\perp/\Omega_e)}{\omega - \Omega + \Omega_e - (k_\parallel - K_\parallel)v_\parallel} \left(K_\parallel \frac{\partial}{\partial v_\parallel} + \frac{\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\right) f_{0e}\, d^3v \\
& + \left(\frac{\omega_{pe}}{k}\right)^2 \left(\frac{e}{m}\right)^2 \int \sum_{\vec{K}} |\Phi_H(\vec{K}, \Omega)|^2 \frac{1}{\omega - k_\parallel v_\parallel} K_\parallel \frac{\partial}{\partial v_\parallel} \frac{J_1^2(K_\perp v_\perp/\Omega_e)}{\omega - \Omega + \Omega_e - (k_\parallel - K_\parallel)v_\parallel} \\
& \times \left(K_\parallel \frac{\partial}{\partial v_\parallel} + \frac{\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}\right) \frac{1}{\omega - k_\parallel v_\parallel} k_\parallel \frac{\partial f_{0e}}{\partial v_\parallel} d^3v.
\end{aligned} \tag{8}$$

In deriving Eq. (8), we retain only $n=1$ contributions in summations of Bessel functions because the electron-cyclotron–turbulence frequency is about $\frac{3}{2}\Omega_e$. The $(n+\frac{1}{2})\Omega_e$ instability occurs in the magnetosphere only when a small quantity of cold plasma electrons is present in addition to the hot, anisotropic, unstable, electron component. This is because the cold-electron component gives a strong cyclotron damping at $\Omega \sim \Omega_e, 2\Omega_e$. Therefore, the electron-cyclotron emissions occur only at $\Omega \sim (n+\frac{1}{2})\Omega_e$. The hot-electron component is retained in Eq. (8).

Now let us estimate the magnitude of Eq. (8). The frequency-shift effect is negligible unless $\sum_{\vec{K}} |E_H|^2 / N\kappa T_e \sim 1$, where κ is the Boltzman constant. The main imaginary-part contribution comes from the cyclotron-resonance interaction between the electron-cyclotron turbulence and the electrons. We must gather all the resonance contributions from the pole of $(\Omega - K_\parallel v_\parallel - \Omega_e)^{-1}$. By repeating the partial-integration method, and after a lengthy but straightforward calculation, we obtain the Fokker-Planck–type collision terms

$$\begin{aligned}
& \left(\frac{\omega_{pe}}{k}\right)^2 \left(\frac{e}{m}\right)^2 \int \frac{k_\parallel^2}{(\omega - k_\parallel v_\parallel)^4} \sum_{\vec{K}} \frac{K_\parallel^2 |\Phi_H(\vec{K}, \Omega)|^2 J_1^2(K_\perp v_\perp/\Omega_e)}{K_\parallel v_\parallel - \Omega + \Omega_e} k_\parallel \frac{\partial f_{0e}}{\partial v_\parallel} d^3v \\
& + \left(\frac{\omega_{pe}}{k}\right)^2 \left(\frac{e}{m}\right)^2 \int \frac{k_\parallel}{(\omega - k_\parallel v_\parallel)^3} \sum_{\vec{K}} \frac{K_\parallel^2 |\Phi_H(\vec{K}, \Omega)|^2 J_1^2(K_\perp v_\perp/\Omega_e)}{K_\parallel v_\parallel - \Omega + \Omega_e} k_\parallel \frac{\partial^2 f_{0e}}{\partial v_\parallel^2} d^3v \\
& + \left(\frac{\omega_{pe}}{k}\right)^2 \left(\frac{e}{m}\right)^2 \int \frac{k_\parallel^2}{(\omega - k_\parallel v_\parallel)^3} \sum_{\vec{K}} \frac{K_\parallel |\Phi_H(\vec{K}, \Omega)|^2 J_1^2(K_\perp v_\perp/\Omega_e)}{K_\parallel v_\parallel - \Omega + \Omega_e} \frac{\Omega_e}{v_\perp} \frac{\partial^2 f_{0e}}{\partial v_\perp \partial v_\parallel} d^3v.
\end{aligned} \tag{9}$$

In deriving Eq. (9), we omit all terms higher than the third derivative of f_{0e}. We must note that the contributions from f_{1e} [the first term of Eq. (8)] give a minor contribution, by a factor $k_\parallel/K_\parallel$, in Eq. (8). Accordingly, the results obtained in this Letter do not depend on the detailed form of f_{1e}.

If we take the electron and ion distribution functions as

$$f_{0e}=\left(\frac{m}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{mv^2}{2T_e}\right),\quad f_{0i}=\frac{M}{2\pi T_{i\perp}}\left(\frac{M}{2\pi T_{i\parallel}}\right)^{1/2}\exp\left(-\frac{Mv_\perp^2}{2T_{i\perp}}-\frac{Mv_\parallel^2}{2T_{i\parallel}}\right), \tag{10}$$

then the effective growth rate (γ_e) of the ion-cyclotron waves in the presence of the electron-cyclotron-turbulence fields is

$$\gamma_e=\gamma_0-\gamma_N. \tag{11}$$

Here, γ_0 is the linear damping rate of the ion-cyclotron waves. We assume that the ring-current protons are stable to the linear cyclotron resonance. Then the damping rate is

$$\gamma_0=\left(\frac{k}{k_\parallel}\right)^2\frac{\omega_k^3}{\omega_{pi}^2}(2\pi)^{1/2}\left(\frac{k_i}{k}\right)^2\sum_n\left(\frac{\omega_k-n\Omega_i}{k_\parallel(T_{i\parallel}/M)^{1/2}}+\frac{T_{i\parallel}}{T_{i\perp}}\frac{n\Omega_i}{k_\parallel(T_{i\parallel}/M)^{1/2}}\right)\exp\left(-\frac{(\omega_k-n\Omega_i)^2}{2k_\parallel^2(T_{i\parallel}/M)}\right)\Lambda_n(\beta_i); \tag{12}$$

here $\Lambda_n(\beta_i)=I_n(\beta_i)\exp(-\beta_i)$, where I_n is the modified Bessel function of order n and $\beta_i=k_\perp^2T_i/M\Omega_i^2$. The leading term of the generalized parametric effect of the electron-cyclotron turbulence on the ion-cyclotron waves is

$$\gamma_N=\left(\frac{k}{k_\parallel}\right)^2\frac{|\omega_k|^3}{\omega_{pi}^2}\left(\frac{\omega_{pe}}{k}\right)^2\left(\frac{e}{m}\right)^2\frac{\Omega_e^2}{\pi}\left(\frac{m}{T_e}\right)^3\sum_{\vec{K}}\left(\frac{K_\parallel}{\Omega-\Omega_e}\right)^2\frac{|\Phi_H(\vec{K},\Omega)|^2}{|k_\parallel|K_\perp}\frac{\Omega}{\Omega_e}\exp\left[-\frac{m}{2T_e}\left(\frac{\Omega-\Omega_e}{K_\parallel}\right)^2\right], \tag{13}$$

where $\omega_k=\pm\omega_{pi}k_\parallel/(k^2+k_e^2)^{1/2}$, and k_i and k_e are the Debye wave numbers for ions and electrons. In deriving Eq. (13), we expand the Bessel function in the large-argument limit. We must note that the sign of Eq. (9) depends on the slope of the electron distribution function at the resonance velocity. But as is shown above, the collective modes include both forward ($\omega_k>0$) and backward ($\omega_k<0$) waves. Therefore, our generalized parametric effect always gives a destabilizing effect[16] on the ion-cyclotron waves. This is the basic difference between the orbit-modification theory[15,12] and this paper's theory.

We can construct a steady turbulent state with the ion-cyclotron damping effect and the destabilizing effect caused by the turbulent collision effect. Equating Eq. (11) to zero, and assuming $K_\parallel=\frac{1}{10}k_e$, $k_\parallel=\frac{1}{100}k_e$, $K_\perp/K_\parallel=1$, $\Omega=\frac{3}{2}\Omega_e$, $T_i=40$ keV, $T_e=10$ keV, $\omega_{pe}/\Omega_e=5$, and $N=10$, we obtain the critical threshold intensity of the electron-cyclotron turbulence,

$$\sum_{\vec{K}}|E_H(\vec{K},\Omega)|^2/4\pi N\kappa T_e=6.3\times10^{-6}. \tag{14}$$

Equation (14) gives the critical amplitude $|E_H|_{\rm cr}\sim30$ mV/m. According to the observations in the magnetosphere,[10] the fluctuating amplitudes are from several tens to hundreds of millivolts per meter which is consistent with Eq. (14). The generated ion-cyclotron waves are absorbed by the ring-current protons through the cyclotron-resonant interactions. Such ion-cyclotron waves are observed[2] just outside the plasmapause. One must admit that very few electrons can penetrate into the region where the ring current maximizes ($3<L<5$ in the dusk-midnight sector) either because they are all precipitated in the outer zone or because of the electro-Alfvên layer effect. Hence the $(n+\frac{1}{2})\Omega_e$ emissions should be weak in the ring-current peak region. According to the recent observations,[10] the $(n+\frac{1}{2})\Omega_e$ emission peak is in the region $5<L<7$. We can conclude that the ring-current protons suffer strong pitch-angle diffusion and energization through the generalized parametric interactions with the naturally occurring electron-cyclotron emissions beyond the plasmapause. The electron-cyclotron turbulences at $(n+\frac{1}{2})\Omega_e$ control the stability and heating of the ring-current protons in the magnetosphere.

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Nonlinear Saturation of the Trapped-Ion Mode

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A nonlinear model of the collisional trapped-ion mode is presented, in which the energy in long-wavelength instabilities is transferred to short-wavelength modes which are then damped by ion-bounce resonances. Near marginal stability, the saturation of a single unstable Fourier mode is computed. Far from marginal stability, steady-state nonlinear solitary waves containing many Fourier modes are found. Particle transport is estimated in both cases.

It is well known[1-4] that plasma confinement in toroidal devices may be seriously impaired by the development of instabilities associated with the trapped particles, namely that class of particles which oscillate in magnetic wells created by the inherent magnetic field inhomogeneity. In the next generation of tokamaks, the ion temperature should be sufficiently large for the ions to enter the banana regime. In this parameter range (where the effective ion collision frequency ν_i^{eff} is less than the trapped-ion bounce frequency ω_{bi}^{T}) the dissipative trapped-ion mode, a drift wave driven unstable by electron collisions, is theoretically predicted to appear.

Several authors[1-4] have studied the linear development of this instability which appears in the limit where the mode frequency ω_0 is much less than both the trapped-ion bounce frequency and the effective electron collision frequency ($\nu_- \equiv \nu_e^{\text{eff}} = \nu_e/\epsilon$, where $\epsilon = r/R$ is the inverse aspect ratio). In this limit, the linear dispersion relation is

$$\omega = \omega_0 + i(\omega_0^2/\nu_- - \nu_+ - \gamma_{\text{LD}}), \tag{1}$$

where $\omega_0 \simeq \epsilon^{1/2}\omega_*/2$, ω_* being the electron diamagnetic drift frequency, $\nu_+ = \nu_i^{\text{eff}} = \nu_i/\epsilon$ is the effective ion collision frequency, and γ_{LD}, which represents the effect of Landau damping by ion-bounce resonances,[2-4] is given by

$$\gamma_{\text{LD}} = A'(1 - \tfrac{3}{2}\eta_i)\omega_0^4/(\omega_{bi}^{T})^3, \tag{2}$$

where A' is a constant of order unity and $\eta_i = d\ln T_i/d\ln n$ is required to be less than $\frac{2}{3}$ to ensure Landau damping rather than growth.[2-4]

In this Letter, we study the nonlinear evolution of this mode in order to determine the saturation level of the fluctuating electric fields. Knowing the saturation level we then compute the particle transport caused by this instability. The analysis is performed using a slab model, first proposed by Kadomtsev and Pogutse,[5] which includes the nonlinear motion $\vec{E}\times\vec{B}$ of the trapped particles. Other nonlinear effects, such as particle detrapping by the electrostatic potential,[6] are not included in this treatment. The basic mechanism for the saturation of the mode is the effective transfer of energy from long-wavelength to short-wavelength modes which are then damped by ion-bounce resonances for sufficiently weak temperature gradients ($\eta_i < \frac{2}{3}$).

The basic model consists of the two-dimensional continuity equations describing the field-line-averaged $\vec{E}\times\vec{B}$ convection of the trapped particles,

$$\partial n_{e,i}^{T}/\partial t + c(\hat{e}\times\nabla\varphi/B)\cdot\nabla n_{e,i}^{T} = -\nu_{-,+}[n_{e,i}^{T} - \epsilon^{1/2}n_0\exp(\pm e\varphi/T)], \tag{3}$$