2π de-excitation of paracharmonium II.

[14]The value of $\alpha_s = 0.26$ at 3.1 GeV was obtained in Ref. 1 from the leptonic branching ratio of orthocharmonium I. Asymptotic freedom reduces this value to 0.22 at 3.7 GeV.

[15]E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975) (this issue). As pointed out by these authors in the transition orthocharmonium II → paracharmonium I $+\gamma$, the orthogonality of the wave functions may make our upper limit a gross overestimate.

Spectrum of Charmed Quark-Antiquark Bound States*

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The discovery of narrow resonances at 3.1 and 3.7 GeV and their interpretation as charmed quark-antiquark bound states suggest additional narrow states between 3.0 and 4.3 GeV. A model which incorporates quark confinement is used to determine the quantum numbers and estimate masses and decay widths of these states. Their existence should be revealed by γ-ray transitions among them.

Recently two astonishingly narrow resonances have been discovered[1,2] at 3.105 and 3.695 GeV. In our view the most plausible explanation of this phenomenon is that of Appelquist and Politzer,[3] to wit, that they are $c\bar{c}$-bound states of charmed quarks c which lie below[4] the threshold M_c for the production of a pair of charmed hadrons.[5] Because of its similarity to positronium this system has been called charmonium.[3] This note is devoted to the spectrum of charmonium.[6] Many of the phenomena that we shall discuss are accessible to existing experimental techniques.

If the strong interactions are described by an asymptotically free theory, one may hope[3] that the short-distance structure of charmonium (in particular, its decay into leptons, and probably also hadrons) is adequately described by perturbation theory in terms of a small "running" coupling constant. In this regime the $c\bar{c}$ interaction would be Coulombic, with a small strong "fine-structure" constant α_s. At larger $c\bar{c}$ separation, on the other hand, there are rather compelling arguments that gauge theories provide for quark confinement.[7]

If α_s is small and the observed levels do not lie far below the threshold M_c, nonrelativistic quantum mechanics should provide a sound zeroth-order guide. Given[8] the sizable electronic widths Γ_e of $\psi(3695)$ and $\psi(3105)$, it is natural[9] to assign them to the states $2\,^3S_1$ and $1\,^3S_1$, respectively. This being said, it is at once clear that there should be other levels below M_c, for any confining potential will raise[10] the $2S$ Coulomb level above its previously degenerate partner $2P$. One must therefore expect a multiplet of narrow P states below $\psi(3695)$, fed from the latter by $E1$ γ transitions, and decaying in turn into $\psi(3105)$. If 3.7 GeV is not too close to M_c, bound D states could also exist.

It goes without saying that many qualitative features of the spectrum can be surmized without resorting to a detailed model. Nevertheless, we have found it informative to simulate the intricate $c\bar{c}$ interaction by a simple potential that incorporates both the Coulomb and confinement forces:

$$V(r) = -(\alpha_s/r)[1-(r/a)^2]. \tag{1}$$

That the interaction is far from Coulombic follows from the large $2S$-$1S$ mass difference, and the fact that[8]

$$\eta \equiv \left|\frac{\psi(1\,^3S; r=0)}{\psi(2\,^3S; r=0)}\right|^2 \approx \left(\frac{3.1}{3.7}\right)^2 \frac{\Gamma_e(3105)}{\Gamma_e(3695)} \approx 1.4, \tag{2}$$

in contrast to Ref. 8 for a Coulomb field.[11] In analogy with electrodynamics there must also be spin-spin, spin-orbit, and tensor forces, but hopefully they play a secondary role. Near M_c a treatment that accounts for coupling to decay channels is necessary.

We have determined α_s, a, and the charmed-quark mass m_c by solving the wave equation numerically,[12] and by imposing the constraints (a) $M(2\,^3S) - M(1\,^3S) = 0.59$ GeV; (b) $\Gamma_e(1\,^3S) = 5.5$ keV; (c) 1.5 GeV $\lesssim m_c \lesssim$ 2.0 GeV; and (d) $0.1 \lesssim \alpha_s \lesssim 0.4$. Constraint (c) is the requirement that the system be nonrelativistic, and that $\psi(3695)$ lie below M_c; naive quark phenomenology would set

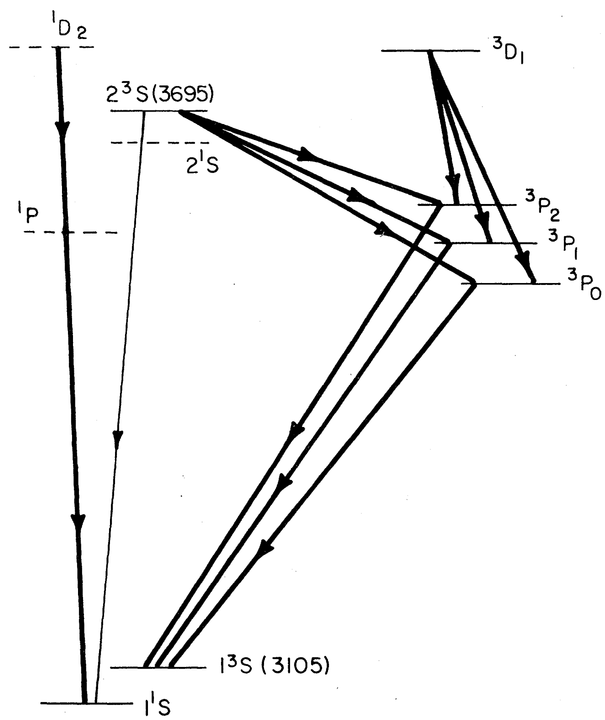

FIG. 1. The spectrum of charmonium. The vertical scale is schematic; our predictions of masses for the P and D levels are given in the text. The $^3D_3$ and $^3D_2$ levels are not shown as their position relative to $^3D_1$ is sensitive to $2\,^3S_1$-$^3D_1$ mixing. Heavy lines are allowed $E1$ γ transitions; the $2\,^3S \to 1\,^1S$ decay is a highly suppressed $M1$ transition. Dashed levels are unlikely to be produced or fed from above at an e^+e^- storage ring. Transitions among levels of an LS multiplet are probably unobservable, while γ transitions between states having the same value of $C = (-1)^{L+S}$ are rigorously forbidden.

$M_c \simeq 2m_c + 0.7$ GeV. Condition (d) is imposed so that the Coulomb term can be understood as a short-distance effect.[13]

A good fit was obtained with $m_c = 1.6$ GeV, $\alpha_s = 0.2$, and $a = 0.2$ fm. This is not an empty exercise. It is not a foregone conclusion that our naive model could simultaneously fit the data, conform to the qualitative features of asymptotic freedom, and satisfy the requirement that $(v/c)^2 \ll 1$. But we do find[13] a small value of α_s, and because $(m_c a)^{-1} \simeq 0.6$, we can safely assume that $c\bar{c}$ annihilation occurs in the short-distance regime. Finally, $(v/c)^2 \approx (m_c r_t)^{-2} \approx 1/25$, where $r_t \simeq 3.4a$ is the classical turning point of the $1S$ state.

We shall now describe the salient features of the level scheme (see Fig. 1).

(a) The center of gravity (c.o.g.) of the lowest

TABLE I. γ ray widths.[a]

Transition	Γ_γ	Γ_γ (keV)
$2\,^3S \to {}^3P_2$	$5I_1\alpha k^3$	120
$\to {}^3P_1$	$3I_1\alpha k^3$	70
$\to {}^3P_0$	$1I_1\alpha k^3$	25
$^3P_2 \to 1\,^3S$	$I_2\alpha k^3$	240
$^3P_1 \to 1\,^3S$	$I_2\alpha k^3$	240
$^3P_0 \to 1\,^3S$	$I_2\alpha k^3$	240
$^1P_1 \to 1\,^1S$	$I_2\alpha k^3$	240
$^3D_1 \to {}^3P_2$	$1I_3\alpha k^3$	7
$\to {}^3P_1$	$15I_3\alpha k^3$	110
$\to {}^3P_0$	$20I_3\alpha k^3$	150
$2\,^3S \to 1\,^1S$	$I_4\alpha k^7$	~1

[a]In the second column $1/\alpha = 137$, k is the energy of the transition, and I_n is a radial integral. The last column is based on our wave functions and energy differences, with fine-structure splittings and S-D mixing ignored.

P multiplet lies about 230 MeV below that of the $2S$ levels. This energy difference is not very sensitive to our choice of parameters: It decreases to 160 MeV if α_s and m_c assume the unreasonable values of 0.8 and 0.9 GeV, respectively.

(b) The c.o.g. of the lowest D multiplet is 70 MeV above that of the $2S$ levels. These D levels may therefore lie below the threshold M_c.

(c) The $3S$ level lies at ~4.2 GeV. As no sharp resonance has been found in this region,[2] this implies that $M_c < 4.2$ GeV.

(d) The almost inevitable presence of tensor forces has an intriguing consequence, for it guarantees some mixing between the rather nearby $2\,^3S_1$ and $^3D_1$ levels.[14] Let $\psi(2\text{“}S_1\text{”})$ and $\psi(\text{“}D_1\text{”})$ be the true eigenstates. Because of the $2S$ component of $\psi(\text{“}D_1\text{”})$, this function does not vanish at $r = 0$, and therefore it can be produced in e^+e^- annihilation, albeit with a width that may be quite small. It is therefore essential to seek such a level with greater resolution than in previous searches.[1,2] At the other extreme,[15] it is even conceivable that $\psi(3695)$ is $\psi(\text{“}D_1\text{”})$! This requires very strong S-D mixing, and implies another spectacular e^+e^- resonance somewhat below $\psi(3695)$. According to Ref. 2, this mass region has not yet been examined systematically.

(e) If spin-orbit and tensor forces can be treated in first order,[14] ^1P lies at the c.o.g. of the ^3P_J levels. Furthermore, if these forces have the same ratio as in positronium, $5[M(^3P_2) - M(^3P_1)] = 4[M(^3P_1) - M(^3P_0)]$.

TABLE II. Angular distributions and correlations.[a]

Transition	W
$(J=1)\to(J=2)$	$1+(1/13)\cos^2\theta$
$(J=1)\to(J=1)$	$1-(1/3)\cos^2\theta$
$(J=1)\to(J=0)$	$1+\cos^2\theta$
$(J=1)\to(J=0)\to(J=1)$	1
$(J=1)\to(J=1)\to(J=1)$	$1+(1/13)\cos^2\theta_{12}$
$(J=1)\to(J=2)\to(J=1)$	$1+(69/377)\cos^2\theta_{12}$

[a]For levels fed directly by e^+e^-, W is the distribution in the angle θ with respect to the beam direction. In the case of sequential transitions, W is the correlation in the angle θ_{12} between the two γ rays, irrespective of the direction of the first γ. With the exception of the $(J=1)\to(J=0)$ transition, these W's do not incorporate recoil effects.

Finally we turn to the decay and production of these levels (see Fig. 1 and Tables I and II).

(i) There are $E1$ transitions between $\psi(3695)$ and the ^3P multiplet, as well as between the latter and $\psi(3105)$. Taking our $2S$-^3P energy difference (and wave functions) at face value, we find the remarkably large result $\Gamma_\gamma(3695)\simeq 210$ keV. The presently inferred[16] value of $\Gamma_{\rm tot}(3695)$ is well accounted for by Γ_γ, $\Gamma_{\rm lept}$, $\Gamma_{2\pi}(3695\to 3105)$, and[17] $\Gamma_{\rm had}$.

(ii) The ^3P_J states decay mainly by γ transitions with appreciable rates. Because $\psi(^3P; r=0)=0$, their direct decay into hadrons should be negligible ($\Gamma_{\rm had}<30$ keV). Furthermore, $^3P_0$ and $^3P_2$ cannot decay into $1\,^1S+2\pi$, and $\Gamma_{2\pi}(^3P_1\to 1\,^1S)$ should be very small because of the lack of phase space and the angular momentum barrier.

(iii) The $M1$ transition between $\psi(3695)$ and the paracharmonium ground state $1\,^1S$ is strongly hindered because the radial wave functions are orthogonal. The only hope for seeing $1\,^1S$ at an e^+e^- storage ring is by the rare 2π decay of $^3P_1$.

(iv) The partial γ widths, γ angular distributions, and γ-γ angular correlations are listed in Tables I and II.

(v) Because of the tensor force, the value of η [Eq. (2)] computed from our wave functions ($\eta\simeq 1.15$) does not give $\Gamma_e(3695)$ directly; rather, it gives $\Gamma_e(2\,\text{"}S_1\text{"})+\Gamma_e(\text{"}D_1\text{"})$. This may account for the discrepancy between 1.15 and the value quoted in Eq. (2). (Naturally there are also experimental, and deeper theoretical uncertainties in this ratio.)

(vi) $p\bar{p}$ annihilation can only feed states having an appreciable total width,[18] and therefore provides access to paracharmonium S states. It is possible that 1^1S has already been seen.[19] But it may prove very difficult to observe the levels ^1D, ^1P, $^3D_3$, and $^3D_2$, even if they exist. This remark also applies to the Primakoff effect.

(vii) It is important to scrutinize the region just above charm threshold with some care. A vestige of the bound $3\,^3S$ could perhaps be seen.

If future experiments reveal a spectrum bearing a resemblance to these predictions, it could well be that charmonium is the "hydrogen atom" of strong interaction physics. For it may then be possible to subject the gauge theories of strong interactions to fairly stringent tests in a reasonably simple setting. Much of hadronic physics could then be related to charmonium spectroscopy as molecular spectra are related to that of hydrogen.

We are very grateful to S. D. Drell and J. D. Jackson, and to many colleagues at Stanford Linear Accelerator Center, for providing us with invaluable information. We have also benefited from stimulating conversations with S. Rudaz, K. G. Wilson, and D. R. Yennie.

Note added.—Some of our expectations appear to be borne out by the new results on $\sigma_{\rm tot}(e^+e^-)$ reported by Augustin *et al.*[20]: (i) $\sigma_{\rm tot}$ rises rapidly for $\sqrt{s}\gtrsim 3.8$ GeV; this is to be compared with the charm threshold of naive quark phenomenology, $M_c\simeq 2m_c+0.7$, which gives 3.9 GeV with our value of m_c. Observe that M_c would be ~ 4.7 GeV if the Coulombic interaction were to be deleted, for then[13] $m_c=2.0$ GeV. (ii) $\sigma_{\rm tot}$ has a resonancelike structure at $\sqrt{s}\simeq 4.15$ GeV, with an electronic width comparable to $\psi(3105)$ and $\psi(3695)$. As stated in the text our $3S$ level lies at ~ 4.2 GeV (to be precise, 4.18 with our parameters). The $2S$ and $3S$ states are insensitive to the Coulombic interaction, whence[11] $\Gamma_e(4.2)=(3.7/4.2)^2\Gamma_e(2S)=0.8\times\Gamma_e(3695)$, provided one ignores the influence of decay channels on $\psi(3S; r=0)$.

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[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); C. Bacci *et al.*, Phys. Rev. Lett. 33, 1408 (1974).

[2]G. S. Abrams *et al.*, Phys. Rev. Lett. 33, 1453 (1974).

[3]T. Appelquist and D. H. Politzer, Phys. Rev. Lett. 34, 43 (1975). These authors suggested the existence of charmonium before the discoveries announced in Refs. 1 and 2. [See also A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975).]

[4]That the large value of $\sigma(e^+e^- \rightarrow \text{hadrons})$ for $s \gtrsim 16$ GeV^2 may be due to charmed hadron production was suggested by K. G. Wilson (discussion remarks at Proceedings of the Seventeenth Conference on High Energy Physics, London, 1–10 July 1974), and by R. Shrock and F. Wilczek, unpublished.

[5]J. D. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964).

[6]As this manuscript was being prepared for publication, we learned of a forthcoming paper on the same topic by T. Appelquist, A. De Rújula, D. H. Politzer, and S. L. Glashow, this issue [Phys. Rev. Lett. 34, 365 (1975)].

[7]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. 31, 792 (1973); K. G. Wilson, Phys. Rev. D 10, 2445 (1974).

[8]J. D. Jackson, private communication; D. R. Yennie, Cornell Univ. Report No. CLNS 291 (to be published).

[9]That $\psi(3695)$ was not observed by J. J. Aubert *et al.* [Phys. Rev. Lett. 33, 1624 (1974)] can be understood as a consequence of two small factors. The first is $\Gamma_e(3.7)/\Gamma_{\text{tot}}(3.7) \approx 0.14[\Gamma_e(3.1)/\Gamma_{\text{tot}}(3.1)]$. The second refers to the production which can be estimated with the Drell-Yan model. This gives a ratio of 0.10 for the production cross sections.

[10]The $2S$ state is raised to a higher energy than the P state since the former has a node.

[11]For S states $|\psi(0)|^2 = (m_c/4\pi)\langle dV/dr\rangle$; therefore $\eta = 1$ for a linear potential.

[12]We thank K. G. Wilson for providing us with this program.

[13]A purely linear potential gives virtually the same level scheme as our "fit." (The only significant change is that m_c shifts to 2.0 GeV.) This is only an acceptable model if one ignores (Ref. 11) the present indication that $\eta \neq 1$. Thus a precise measurement of η is important. From a theoretical standpoint the Coulomb force should dominate at distances $\lesssim m_c^{-1}$ if the model is to explain the small values of Γ_{tot}.

[14]P-F mixing is negligible.

[15]Should $\psi(3695)$ be $\psi(``D_1")$, our numerical predictions would require some change, but the general features shown in Fig. 1 and Tables I and II would survive.

[16]J. D. Jackson, private communication; S. Rudaz, Cornell Univ. Report No. CLNS-293 (to be published).

[17]Direct hadron decays are discussed in Ref. 3.

[18]E. L. Berger, private communication.

[19]H. Braun *et al.*, Phys. Rev. D 2, 488 (1970); G. P. Yost *et al.*, Phys. Rev. D 3, 642 (1971).

[20]J.-E. Augustin *et al.*, SLAC Report No. SLAC-PUB 1520-LBL-3621 (to be published).

COMMENTS

Resonant X-Ray Raman Scattering

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A theory of the resonant x-ray Raman effect is presented and compared with recent experimental data of Sparks. Excellent agreement between theory and experiment is found for the integrated intensity of the scattering, the spectral density, and the output polarization. The potential importance of this newly discovered spectroscopic probe is discussed.

Although the x-ray Raman effect has been known for many years,[1] the resonant enhancement in the scattering cross section that occurs when the input frequency ω_1 is near an atomic absorption edge has been discovered only very recently by Sparks.[2] As Sparks correctly points out, a theoretical description of this phenomenon requires that the $\vec{p}\cdot\vec{A}$ term in the interaction Hamiltonian be taken to second order in perturbation theory. Sparks also notes correctly that the dispersion corrections to the atomic scattering factor[3] require a similar treatment, and he postulates a conservation law which he claims enables him to use the known form of these dispersion corrections to describe the resonance x-ray Raman effect. The justification for this postulated conservation law is claimed to be the existence of a similar law for the A^2 terms in the interaction Hamiltonian. Unfortunately, *neither* of these conservation laws exist,[4] and the theoretical expressions presented by Sparks[2] are inadequate.

We present here a theory of the resonant x-ray Raman effect which does not employ *ad hoc* postulates, but represents, instead, a direct evalua-