Stabilization of the Charged Vacuum Created by Very Strong Electrical Fields in Nuclear Matter*

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The expectation value of electrical charge in charged vacuum is calculated utilizing the Thomas-Fermi model. We find almost complete screening of the nuclear charge. For any given nuclear density there is an upper bound for the electrical potential. For normal nuclear densities this value is -250 MeV. This suggests that the vacuum is stable against spontaneous formation of heavy, charged particles.

The behavior of electrons in strong electromagnetic fields gives rise to phenomena outside the range of the usual perturbation-theory treatment of quantum electrodynamics (QED). One is forced to give up the usual concept of a neutral ground state (the vacuum) of QED as the strength of the potential passes through the critical point ($Z_{\rm cr} \approx 172$ for ordinary nuclei). It was shown by Rafelski, Müller, and Greiner[1] and, independently, by Fulcher and Klein,[2] that the supercritical vacuum is charged.

In this note we explore the consequences of the idea of a changing vacuum[1] when applied to the description of extended bulk nuclear matter, e.g., charged domains in neutron stars or abnormal nuclear states.

It has been shown[1] that the proper value of the Fermi energy is $E_{\rm F}=-m$, even for supercritical fields. Even if no bound states between $E=-m$ and $E=+m$ are occupied by real electrons, the ground state thus defined has charge $-2e$ ($172 \leqslant Z \leqslant 185$), $-e$ ($185 \leqslant Z \leqslant 215$), etc. From this it is obvious that with increasing Z the vacuum becomes more and more charged. Ultimately it can be expected that the situation is adequately described by a relativistic extension of the Thomas-Fermi statistical model.

The density of electrons is as usual related to the Fermi momentum $k_{\rm F}$ by

$$e\rho = -e^2/3\pi^2 k_{\rm F}^3, \tag{1}$$

where e is the electron's charge. The relativistic relation between the Fermi energy $E_{\rm F}$ and Fermi momentum is

$$k_{\rm F}^2 = [(E_{\rm F}-V)^2 - m^2]\,\theta(E_{\rm F}-V-m), \tag{2}$$

where $V=eA_0$ is the electrostatic potential. The step function θ guarantees that $k_{\rm F}^2$ is a positive quantity (we have used $E_{\rm F}-V>0$).

From Eq. (1) we now obtain[3]

$$\langle 0|\rho(x)|0\rangle = -(e/3\pi^2)[(E_{\rm F}-V)^2-m^2]^{3/2} \times\theta(E_{\rm F}-V-m). \tag{3}$$

Introducing the total charge density ρ_T which is composed of the external "nuclear" charge ρ_N and the electron charge,

$$\rho_T = \rho_N + \rho, \tag{4}$$

and using the Coulomb law

$$\Delta V(\vec{r}) = e\rho_T(\vec{r}), \tag{5}$$

we find

$$\Delta V(\vec{r}) = e\rho_N(\vec{r}) - (e^2/3\pi^2)[(E_{\rm F}-V)^2-m^2]^{3/2} \times\theta(E_{\rm F}-V-m). \tag{6}$$

For neutral atomic systems one must take $E_{\rm F}=m$ which gives (for $-2mV \gtrsim V^2$) the usual Thomas-Fermi model. Our new choice is $E_{\rm F}=-m$, since we are only interested in the electrons that dived into the continuum of the negative energy states. This gives

$$\Delta V(r) = e\rho_N(r) - (e^2/3\pi^2)(2mV+V^2)^{3/2} \times\theta(-V-2m). \tag{6$'$}$$

For the "nuclear" charge distribution we take

$$\rho_N(r) = e\rho_0\theta(R_0-r) \tag{7}$$

with $4\pi\int \rho_0 r^2\,dr = Z$, the external "nuclear" charge. We assume $R_0 = r_0 A^{1/3}$, r_0 being determined by the density of the nuclear matter. Our numerical results were obtained with $r_0 = 1.2$ fm, $A=2Z$. The latter assumption is justified by the result:

We find almost total compensation of the nuclear Coulomb energy.

Since the charge density of the vacuum must be confined to the vicinity of the external charge, we require that

$$V(r) \underset{r\to\infty}{\rightarrow} -\gamma\alpha/r$$

(α is the fine structure constant). For every choice of Z, γ is determined by the boundary condition

$$\partial V/\partial r|_{r=0} = 0. \tag{8}$$

Equations (6′) and (8) are therefore eigenvalue equations for γ, which has a clear physical meaning—it is the unscreened part of the nuclear charge. $Z-\gamma$ gives the charge of the vacuum.

Although this model can only be expected to be true when $Z-\gamma \gg 1$, i.e., when the vacuum screens almost all of the nuclear charge, we find reasonably good agreement with single-particle calculations[1] performed in the vicinity of the critical potential. We also obtain the right value for the critical potential.

In Fig. 1 we show the behavior of the eigenvalues γ and $Z-\gamma$. We note two key features: (a) γ is monotonically rising as a function of Z and (b) for $Z\to\infty$ we find $\gamma/Z \to 0$. The single-particle calculations[1] differ only slightly from our statistical model. The fact that $\gamma/Z\to 0$ corresponds to the situation shown in Fig. 2(a), i.e., $|V|$ tends to an upper limit. This limit is, incidentally, obtained from Eq. (6′) by requiring $\rho_T = 0$ at r 0:

$$V_{\rm lim} = -m - [m^2 + (3\pi^2 e\rho_0/e^2)^{2/3}]^{1/2}. \tag{9}$$

Since $m \ll |V_{\rm lim}|$ for electrons, we have approximately

$$V_{\rm lim} = -(9\pi)^{1/3}/2r_0 = -(300\ {\rm MeV\ fm})/r_0. \tag{10}$$

In ordinary nuclear matter we obtain $V_{\rm lim} = -250$ MeV.

Figure 2(b) shows the behavior of the real vacuum polarization charge. Its radius follows the nuclear density. In terms of the single-particle description of Ref. 1 this implies that inclusion of electron-electron interaction prevents the resonances from diving below -250 MeV. Equivalently, the sum of single-particle energy (which

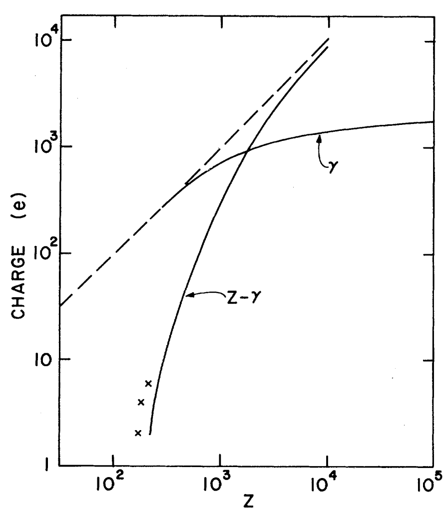

FIG. 1. The unscreened charge γ and the total charge of the vacuum $(Z-\gamma)$ as a function of Z. The crosses denote points from Hartree-Fock calculations. The dashed line denotes the nuclear charge Z.

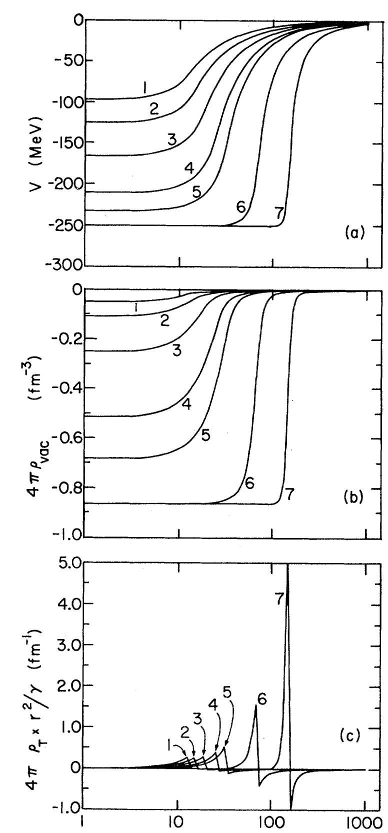

FIG. 2. The solutions of the relativistic statistical potential equation (6′) for selected values of the nuclear charge as a function of r. Curve 1, $Z = 600$; curve 2, 1000; curve 3, 2000; curve 4, 5000; curve 5, 10 000; curve 6, 10^5; curve 7, 10^6. (a) The self-consistent potential; (b) the corresponding charge distribution of the vacuum; (c) the total charge densities, scaled with γ (see Fig. 1).

now grows only with Z) becomes negligible with respect to the correlation energy

$$E_{\rm corr} = -(e^2/8\pi)\int\langle 0|\rho(x)|0\rangle|x-y|^{-1}\langle 0|\rho(y)|0\rangle\, d^3x\, d^3y, \tag{11}$$

which keeps on growing as Z^2 and cancels the Coulomb energy of the nuclear charge. Figure 2(c) confirms our assumptions about the charge distribution. For very large Z we obtain essentially a dipole layer at the edge of the external field source.

Incidentally, the value of $\rho(0) = -\rho_N(0)$ for large Z [curves 6 and 7, Fig. 2(b)] serves to confirm the numerical accuracy of our calculations. This behavior can be easily understood for infinite "nuclear" matter. At this point, we would like to mention that use of a charge distribution with a more "realistic" nuclear charge does not influence our results, if the mean charge radius is kept constant.

The result with the most important consequences is that there is a limit to the electrostatic potential. Depending on the density of the external charge distribution—which is for our purposes independent of electromagnetic interactions—only a finite number of species of charged elementary particles can create a charged vacuum or form a Bose condensate.[4] Whether the formation of a pion condensate for $Z \gtrsim 2000$, as discussed in Ref. 4, is possible, depends on the details of the pion-nucleus interaction.[5]

In the Lee and Wick[6] theory of abnormal nuclear states, there is a significant gain in the energy (ca. 150–300 MeV) of the vacuum excitation whenever a baryon is absorbed. Because of the compensation of the repulsive electric forces inside the nucleus by the polarization of the vacuum and the existence of the limiting potential, the Coulomb interaction will not, most likely, stop the growth of the abnormal nuclear state. If such strongly bound states could be created, they would continue to grow by absorption of normal nuclear matter. We find below that an abnormal state will not continue to grow only if the binding energy per baryon does not exceed $90(\rho_B/\rho_0)^{1/3}$ MeV [see Eq. (14)] (ρ_B is the density of the abnormal state, ρ_0 is the normal nuclear density).

We can calculate the energy per nucleon due to the remaining, Z-proportional, part of the Coulomb interaction and single-particle energies. A suitable Hamilton function is obtained, noting that Eq. (6′) can be obtained by minimizing

$$H[\rho] = \int d^3x\, 2\int[d^3k/(2\pi)^3]\,\theta(k_F^2[\rho(x)]-k^2)[(k^2+m^2)^{1/2}+m] + \tfrac{1}{2}\int d^3x\, d^3y[\rho_N(x)+\rho(x)](4\pi|x-y|)^{-1}[\rho_N(y)+\rho(y)]. \tag{12}$$

With respect to ρ, H is obviously the total electron and Coulomb energy of the system. k_F is given by

$$k_F = [-(3\pi^2/e^2)e\rho]^{1/3}. \tag{13}$$

Neglecting the second part of Eq. (12) we obtain the Coulomb + electron energy per nucleon to be

$$H/A = \tfrac{3}{8}|V_{\rm lim}| = (112\ {\rm MeV\ fm})/r_0. \tag{14}$$

Thus, addition of a proton and neutron to the abnormal nuclear state releases ca. 300–600 MeV of baryonic energy and costs only ~200 MeV in Coulomb + electron energy, if, by a spontaneous positron-production mechanism, the positive charge of the proton is neutralized.

The above considerations clearly show that in a macroscopic domain the concept of the charged vacuum[1] smoothly goes over into the regular assumption that effects of a constant potential are unobservable; we are left with the usual theory of the electron gas, i.e., with the first part of Eq. (12).

This is understandable if we note that our time-independent statistical approach has its limitations in macroscopic distances ($r \gg \hbar/mc$). We have assumed that the spontaneous production of positrons is essentially sudden, which is certainly true for potential wells of the size not much larger than $m^{-1} \approx 400$ fm. It would be wrong to conclude that the limiting potential described above is an upper bound for macroscopic devices. This is understood for potential barriers $\gtrsim 2m$; although the channel for spontaneous pair production is already open, the tunneling factor $\exp(-\int V dr)$ is too small by hundreds of orders of magnitude. Therefore a macroscopic experiment to determine the value of the limiting potential is not feasible.

Finally, let us say again that our Eq. (6′) is most suitable to calculate the effects of real vacuum polarization in very strong external fields. Such calculations performed before only in the vicinity of the critical potential[7] already required great numerical effort.

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[4]A. Klein and J. Rafelski, Phys. Rev. D (to be published).

[5]It has been suggested by N. Kroll [discussion at the American Physical Society conference on Particles and Fields, Williamsburg, Virginia, September 1974 (to be published)] that the electronic charge distribution of the vacuum might prevent the formation of Bose condensation as discussed in Ref. 4. Since $V(0)\sim 2m_\pi$ the nuclear forces play a decisive role.

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[7]B. Müller, J. Rafelski, and W. Greiner, Nuovo Cimento 18A, 551 (1973); M. Gyulassy, Phys. Rev. Lett. 33, 921 (1974).

Is the Photon g-Spin Blind?*

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I consider a six-quark model for hadrons. The three new ones possess charges +, 0, −, and an additional quantum number g (for "gentleness"). To explain the recently observed resonances, a selection rule is proposed: Electromagnetism violates g spin and the photon is blind to gentleness. Experimental tests for the idea are discussed.

Recently, in experiments on $p+p\to l^+ + l^- + x$ at Brookhaven National Laboratory (BNL) and on $e^+ + e^-\to$ hadrons at SPEAR, sharp peaks (width <1.3 MeV) have been observed at, respectively, the final and initial lepton-pair invariant mass of 3.1 GeV.[1] This has been confirmed by experiments at Frascati.[2] A second similarly sharp peak has also been discovered at SPEAR at 3.7 GeV.[3] In this note I shall propose a plausible and interesting explanation for the observed phenomena and suggest additional tests for the basic idea.

Let us preserve the gauge-theoretic scheme for unifying weak and electromagnetic interactions for *leptons* as suggested in the classical work of Weinberg[4] and Salam.[5] To recapitulate, very briefly, the theory has the leptons arranged in a SU(2) ⊗ U(1) doublet (in all that follows I shall adhere as far as possible to the notation of Abers and Lee in their definitive review[6])

$$L_l=\begin{pmatrix}\nu_l\\ l^-\end{pmatrix}_L \tag{1}$$

and a right-handed singlet $R_l=l_R^-$. In addition, the theory incorporates a Higgs doublet

$$\varphi=\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} \tag{2}$$

which undergoes a spontaneous symmetry breakdown such that

$$\langle\varphi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \theta\end{pmatrix}. \tag{3}$$

As a result, three of the four gauge fields pick up mass while the fourth remains massless and is identified with the photon.

In an extension to a unified gauge theory for weak and electromagnetic interactions for hadrons instead of the traditional Glashow, Iliopoulos, and Maiani (GIM)[7] scheme of four quarks, let us assume that hadrons are constructed out of six quarks; that is, in addition to ($\mathcal{P}^+$, $\mathfrak{N}^0$, λ^0), there are three new ones which are collectively denoted by G,

$$G=\begin{pmatrix}g^+\\ g^0\\ g^-\end{pmatrix}, \tag{4}$$