Analysis of the Proton-Helium Scattering at 1 GeV*

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The proton-helium elastic scattering data at 1 GeV are analyzed using the effective-channel approach. The calculated spin-averaged cross section is in good agreement with the experiment from Saclay up to 35 deg, effectively with no free adjustable parameters. An estimate for the average excitation energy and for the ratio Ref/Imf of the nucleon amplitude is obtained.

The proton-helium elastic scattering at medium energies has been studied extensively in the past, both experimentally[1] and theoretically.[2,3] The analysis of the data has usually been carried out using either the multiple-diffraction theory[2] of Glauber[4] or the coupled-channel approach[3] of Feshbach derived from the multiple-scattering theory[5] and optical potentials.[6] Although both approaches were able to fit the earlier data[1] reasonably well with acceptable sets of values for the parameters which appear in the formalisms, the study was unable to provide improved information on the target correlations and on the off-shell behavior of the nucleon interactions. More recent data from Saclay[7] show a quite different angular distribution from the previous result.[1]

It is the purpose of this note to report the result of a calculation of the p-α cross section using the effective-channel theory.[3,8] In this approach, all the inelastic channel effects are consistently parametrized by a single effective inelastic channel function φ defined simply by

$$\varphi = N_0^{-1} Q\hat{V}P\psi_0, \quad \int \varphi^*(\vec{r}, \vec{R})\varphi(\vec{r}, \vec{R})d^3r = 1, \qquad (1a)$$

where $\psi_0(\vec{r})$ is the helium ground-state function generated by the target Hamiltonian $H_T(\vec{r})$, with eigenenergy E_0, and P and Q are the projection operators defined as $P = \psi_0\rangle\langle\psi_0$ and $Q = 1 - P$. $N_0(\vec{R})$ is the normalization factor which depends on the projectile-target center-of-mass coordinate $\vec{R}$. The form (1a) follows from a perturbation theoretic consideration of the total inelastic scattering function $Q\Psi_i$, which may be written, in the closure approximation,[8] as

$$Q\Psi_i \sim \varphi_0(\vec{r}, \vec{R})w(\vec{R}), \qquad (1b)$$

with $\varphi_0 \approx QVP\psi_0(\vec{r})$, where V is the projectile-target interaction. Thus, φ_0 is to simulate the inelastic channels. However, since φ_0 contains, in general, excessive high-momentum components, we correct it by replacing V with $\hat{V}$, thus obtaining (1a). $\hat{V}$ is here chosen to be similar in form to V, but with one adjustable parameter β'. This parameter β' will be determined in terms of the total cross section, as will be discussed later. Thus, within this approximation, the total scattering function Ψ_i can be represented by

$$\Psi_i \equiv P\Psi_i + Q\Psi_i \approx \psi_0(\vec{r})u_0(\vec{R}) + \varphi(\vec{r}, \vec{R})w(\vec{R}). \qquad (1c)$$

Using φ and ψ_0, we obtain[8] a set of coupled equations given by

$$(T_{\vec{R}} + V_{00} - E_0')u_0(\vec{R}) = -V_c w(\vec{R}), \qquad (2a)$$

$$(T_{\vec{R}} + \bar{V} + J - \bar{E}')w(\vec{R}) = -V_c^\dagger u_0(\vec{R}), \qquad (2b)$$

in which u_0 and w are the scattering functions to be solved. In (2a) and (2b), $T_{\vec{R}}$ is the projectile kinetic energy operator, and

$$\bar{V}(\vec{R}) = \langle\varphi|H_T + V - E_0|\varphi\rangle_{\vec{r}} - \bar{E},$$
$$V_{00}(\vec{R}) = \langle\psi_0|V|\psi_0\rangle_{\vec{r}}, \quad V_c = \langle\psi_0|V|\varphi\rangle_{\vec{r}}, \qquad (3a)$$
$$J(\vec{R}) = \langle\varphi|(T_{\vec{R}}\varphi)\rangle_{\vec{r}} \equiv \int d^3r\, \varphi^*(\vec{r}, \vec{R})T_{\vec{R}}\varphi(\vec{r}, \vec{R}),$$

with $\langle\,\rangle_{\vec{r}}$ denoting the integration over the target variables $\vec{r}$, and where

$$\bar{E} = \lim_{R\to\infty} \langle\varphi|H_T - E_0|\varphi\rangle_{\vec{r}},$$
$$E_0' = E - E_0, \quad \bar{E}' = E - \bar{E}. \qquad (3b)$$

In (3a), V is the projectile-target potential chosen to be of the form

$$V = \sum_{i=1}^{4} v(\vec{r}_i - \vec{R}), \qquad (4)$$

with the effective two-nucleon interaction defined by

$$v(\vec{s}) = g(\rho - i)e^{-s^2/2\beta} \equiv v(\vec{s}; E). \qquad (5)$$

Thus, the potential $\hat{V}$ in the function φ is the same form as V of (4) with β replaced by β'. In (2), both the relativistic kinematic correction[9] and the transformation from the p-nucleon to p-α center-of-mass system[6] are omitted for simplicity, but they are explicitly included in the actual calculation. In terms of the solution of (2), the cross section is given by

$$\frac{d\sigma}{d\Omega}=\frac{m^2}{4\pi^2\hbar^4}|(\Phi_f, V\Psi_i)|^2, \tag{6a}$$

where

$$\Phi_f=\psi_0(\vec{r})\exp(i\vec{K}_f\cdot\vec{R}), \tag{6b}$$

and Ψ_i is given by (1c). We neglect the Coulomb effect in our calculation.

We now discuss briefly the parametrization of various quantities which appear in the coupled equations (2). Firstly, the *effective* nucleon-nucleon potential $v(\vec{r},\vec{R};E)$ contains three parameters g, ρ, and β. Their values have been fixed by fitting the cross section, calculated by solving a relativistic two-particle equation[9] with this potential v, to the spin-averaged p-p and p-n cross sections.[10,1] The potential used, then, represents an energy-dependent effective interaction. Without additional terms in v, which may need an exchange term, we have obtained a reasonable fit to the nucleon-nucleon elastic cross section for angles $\theta_{\text{c.m.}}\lesssim 35°$ with the parameter values

$$g=690\pm70\text{ MeV},\quad \rho=0.2\pm0.3,$$

$$\beta=0.2\pm0.05\text{ fm}^2. \tag{7}$$

We note especially the large uncertainty in the value for ρ given in (7), which is consistent with the present uncertainty[1-3,10] in the ratio γ, where $\gamma\equiv(\text{Re}f/\text{Im}f)_{\theta=0}$, and where f is the spin-averaged nucleon-nucleon amplitude. The target wave function $\psi_0(\vec{r})$ was chosen to be of the simple form[3]

$$\psi_0(\vec{r})=8^{-1/2}(\nu/\pi)^{9/2}\exp(-\nu\sum_{i=1}^{4}r_i^2/2), \tag{8}$$

with one adjustable parameter ν, which was chosen to give roughly the correct binding energy[2,3]:

$$\nu=0.57\pm0.05\text{ fm}^{-2}. \tag{9}$$

Both for v and ψ_0, the Gaussian form was chosen in order to facilitate integrations involved in the quantities defined by (3), and not because it simulates the physical situation better. The center-of-mass correlation is included in the calculation by imposing the constraint on the internal variables, $\vec{r}=(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$, that

$$\sum_{i=1}^{4}\vec{r}_i=0.$$

The scattering equations (2) are solved for u_0 and w by first analyzing them into partial waves. For the lab energy of 1 GeV, approximately 30–40 partial waves were needed to obtain the cross section out to angles $\theta_{\text{c.m.}}\approx 35°$.

The cross section for the case in which the inelastic excitation effect is neglected (with $V_c=0$) is given in Fig. 1, and denoted by σ_P. Also given is the full cross section σ which contains the distortion effect. The experimental value σ_{exp} from Saclay[7] is also plotted for comparison. Although the overall inelastic effect represented by $Q\Psi_i$ seems small, the contributions of the individual terms contained in $\bar{V}$ of (3a) are quite large, when taken separately. Up to $\theta_{\text{c.m.}}\approx 35°$, the fit is reasonable, when the parameter values defined above in (7)–(9) are used. Because of a poor fit of the nucleon-nucleon cross section, however, we do not expect our calculation to be as meaningful for angles larger than 35°, although the effect of ν seems to be more dominant at these

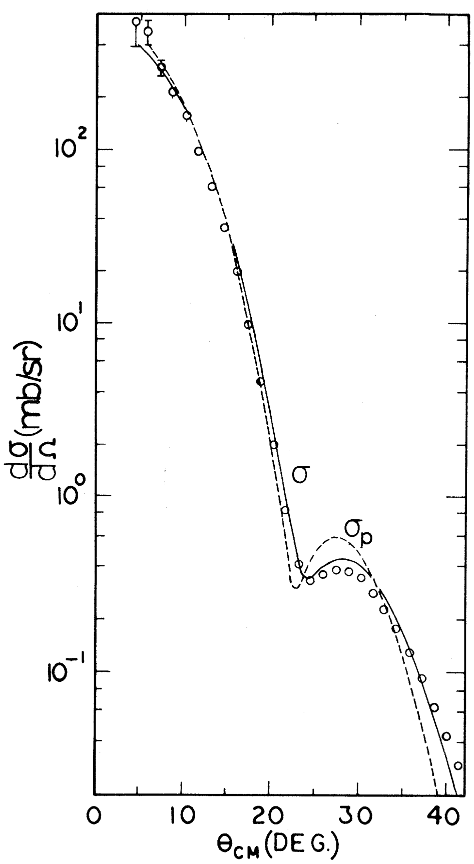

FIG. 1. The elastic proton-helium cross section at 1 GeV lab energy (in mb/sr) as a function of the center-of-mass angle θ. The circles represent the experimental points obtained by the Saclay group (Ref. 7). The dashed line neglects the distortions and thus includes only the V_{00} part, while the solid curve is the full cross section obtained by solving the coupled equations (2).

angles. The parameter β' for the curve σ in Fig. 1 is chosen to be

$$\beta' = 1.5 \pm 0.5 \text{ fm}^2, \tag{10}$$

such that the resulting total cross section, via the optical theorem, is made slightly lower than the experimental value.[1] This was done to make the calculation consistent with the nucleon-nucleon input, where a similar situation was studied earlier in terms of the spin-averaging effect.[10] Thus, β' was effectively determined *a posteriori* by the total cross section. Consequently, within the uncertainty in the input parameters as given by (7), (9), and (10), there are effectively no free parameters in the entire calculation. The average excitation energy $\overline{E}$, which comes in the energy $\overline{E}'$ of (2b), is a sensitive function of this parameter β' and we have calculated it to be

$$\overline{E} = 48 \pm 15 \text{ MeV}. \tag{11}$$

In the evaluation of $\overline{E}$ by (3b), a proper adjustment has been made for the fact that the (collective) center-of-mass variable has to be extracted[11,12] from the target function $\psi_0(\vec{r})$.

In the course of the study of the effect of variations of the parameters involved on the cross section, we have found that the p-α cross section (spin averaged) at the first diffraction minimum ($\theta_{\text{c.m.}} \approx 24°$) is rather sensitive to the parameter ρ in the effective two-nucleon potential v. Recalling the fact that the value given in (7) has a large uncertainty, it was possible to improve the accuracy of ρ by fitting the p-α data near the diffraction minimum. The dependence on ρ of the p-α cross section is such that the value used in our calculation is

$$\rho \approx 0.20 \pm 0.05, \tag{12}$$

which in turn gives

$$\gamma \equiv [\text{Re}f/\text{Im}f]_{\theta=0} \approx -0.25 \pm 0.05, \tag{13}$$

which should be compared with the presently available value,[1,10] $-0.1 \lesssim \gamma \lesssim -0.5$. This value would be affected somewhat by the correlation and spin effects in the p-α system.[13,3] In addition to the variation in ρ, we have also studied[11] the effect of the parameters β, ν, g, and β'. As the input parameters are varied over their allowed range of values, the cross section changes sufficiently to mask any additional effects due to dynamical correlations and off-shell effects. Therefore, it seems premature to attribute any discrepancies between the theory and experiment to new physical phenomena at this stage of the development. However, we have effectively shown that the latest p-α data[7] are completely consistent with the available input data on the nucleon interaction and the ground-state target structure. On the other hand, for the purpose of obtaining new dynamical information, an analysis of the inelastic scattering data in conjunction with the elastic scattering seems to be more desirable.

The formalism being used here can be adapted, with minimal changes, to incorporate directly the two-nucleon amplitude in (2) and (3), in place of the effective potential v of (5). However, a preliminary calculation indicates that the cross section is reproduced less well at large angles. Reasons for this are not yet clear, but there are several major differences in the physical content of the two theoretical formulations which require further analysis.

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Relative Velocity Measurements of Electrons and Gamma Rays at 15 GeV

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Measurements were made to detect differences in the velocity of propagation of γ rays and electrons in the energy range 15–20 GeV, by using a time-of-flight technique with 1-psec sensitivity and a flight path of ~1 km. A relative velocity difference larger than $1-\beta_e$ ($\sim 5\times10^{-10}$) would imply a breakdown of special relativity. No significant difference in the velocities of light and electrons was observed to within ~2 parts in 10^7.

Previous efforts have been made to detect a frequency-dependent shift in the velocity of light from visible wavelengths up to GeV energies. All such experiments are in essence attempts to detect some departure from the predictions of special relativity.

The most recent contribution preceding the present work was made by Brown *et al.*[1] Using a time-of-flight (TOF) technique, they compared the velocities of short pulses of visible light and 7-GeV γ rays. The results gave a relative velocity difference of $(1.8\pm6)\times10^{-6}$. An additional measurement using 11-GeV electrons and visible light gave a relative velocity difference of $(-1.2\pm2.7)\times10^{-6}$. The precision of these measurements was limited in part by the time resolution of photomultiplier detectors.

In the present experiment at the Stanford Linear Accelerator Center the relative velocities of ~15-GeV γ rays and electrons with energies ranging from 15 to 20.5 GeV were measured by using an rf separator (RFS) synchronized with the accelerator's rf system as the timing element. Ultimate time resolution was limited, in part, by the characteristic bunch length of the Stanford Linear Accelerator Center beams, ~5 psec or 5 deg of rf phase. Results more than 1 order of magnitude smaller in $\Delta v/c$ at energies higher than the previous best measurements were achieved.

Electrons accelerated to 15 GeV in about $\frac{2}{3}$ of the full length of the accelerator strike a thin, annular target at the end of sector 22, scatter, and produce bremsstrahlung photons [Fig. 1(a)]. Measurements were made both with and without further acceleration. With acceleration, electron energy increased continuously to 20.5 GeV at the end of the accelerator. After a common flight path of 1015 m a small fraction of the photons and scattered electrons strike two thin targets which serve as positron sources for the beam transport system [Fig. 1(b)]. The upstream (downstream) target fills the upper (lower) half of the beam aperture. Their axial displacement within the field of vertical bending magnet B60, in conjunction with subsequent collimation and momentum analysis, is such that only 13.5-GeV positrons produced by electrons (photons) in the upper (lower) target are retained in the transport