nucleon, $\sigma(\psi N)$, by using the relation

$$\sigma^2(VN) = 16\pi(g^2/e^2)[d\sigma(\gamma N \to VN)/dt]_{t=0}.$$

In this model the V couples to the photon with the coupling constant e/g_V, where e is the electric charge and $g_V^2/4\pi$, obtained from the e^+e^- colliding-beam measurements,[1,5] is given in Table I. We have also used

$$\left[\frac{d\sigma(\gamma N \to VN)}{dt}\right]_{t=0} = \exp(b\,|t_{\min}|)\left[\frac{d\sigma(\gamma N \to VN)}{dt}\right]_{t=t_{\min}}. \qquad (1)$$

The upper limits on $\sigma(\psi N)$ are given in Table I. These limits are quite dependent upon the value assumed for b; nevertheless we observe that $\sigma(\psi N)$ is less than even the smallest of the total cross sections, namely $\sigma(\varphi N)$.

On the other hand the e^+e^- colliding-beam production of the $\psi(3105)$ may have nothing to do with vector-meson–dominance ideas. For example, one might speculate on a direct electron-electron coupling. In that case one might expect some direct electroproduction of the $\psi(3105)$. We find an upper limit with 90% confidence of 0.46 nb/nucleon for the direct electroproduction of the $\psi(3105)$ by 20.5-GeV electrons. In this calculation we have used steps (2), (3), and (4), although there is no reason to assume step (2) in this case.

The branching ratio of the $\psi(3695)$ into e^+e^- is not yet known. However, based on the data in Fig. 2, the 90% confidence upper limit on the diffractive photoproduction of the $\psi(3695)$ is 4.5 nb divided by the branching ratio into e^+e^-, for $b = 4$ $(\text{GeV}/c)^{-2}$.

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Direct Evidence for Independent Emission of Clusters

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In the distribution of rapidity gaps between charged particles produced at 100–400 GeV/c, we find direct evidence for the independent production of clusters with density approximately one per unit of rapidity. This cluster-density measurement and the observed charged-particle density imply that the mean number of charged particles per cluster is about two. Implications of these numbers and techniques for refining them are discussed.

Although considerable circumstantial evidence suggests that the short-range correlations among produced particles observed at Fermilab and the CERN intersecting storage rings may be ascribed to the independent emission of clusters,[1] no direct confirmation of this picture has previously been presented. Moreover, because the observable effects cited as evidence for clustering are insensitive to the detailed properties of clusters, the characteristics of clusters—other than range—have been established only deviously, with heavy reliance on fits to specific models.[2] Recently it was shown[3] that the distribution $P(r)$ of rapidity gap lengths between charged particles adjacent in rapidity contains the same sort of information as the two-particle correlation function in (it was hoped) more accessible form. In this Letter we establish the usefulness of the rapidity-gap distribution by proving that (i) the form $P(r) \propto \exp(-\rho r)$ assumed by the rapidity-gap distribution for large gap lengths is a direct consequence of independent cluster emission; (ii) the exponential slope ρ is the density in rapidity of independently emitted clusters which decay into at least one charged particle; (iii) a deviation from exponential behavior at small separations, in the form of upward curvature, establishes that clusters decay in the mean into more than one charged particle. In addition to these new qualitative results, we shall present and interpret numerical information based on 100–400 GeV/c pp collisions.

In the independent-cluster-emission model it is assumed that clusters of hadrons are produced independently (therefore according to a Poisson multiplicity distribution) in the available rapidity interval. Each cluster decays independently of its fellows, the probability g_M for decay into M particles being the same for all clusters. The M decay products of a cluster emitted at rapidity $\hat{y}$ are distributed in an uncorrelated manner according to

$$D(\hat{y}-y_1)D(\hat{y}-y_2)\ldots D(\hat{y}-y_M),$$

where $D(y)\approx(2\pi\delta^2)^{-1/2}\exp(-y^2/2\delta^2)$ and the assumption of isotropic decay implies $\delta\approx 0.85$.

The distribution of gaps between independently emitted objects is[4]

$$P(r)=\rho\exp(-\rho r), \qquad (1)$$

where ρ is the density in rapidity of the emitted objects and r is the separation between adjacent objects. The experimental distribution of gaps between charged particles is shown in Fig. 1. It is not described by a single exponential, which is unsurprising because the charged-particle multiplicity distribution is known to be broader than Poissonian ($f_2>0$). We now argue that if clusters are produced, Eq. (1) is appropriate for the (unobservable) gaps between clusters but is inappropriate for gaps between particles unless the particles are so widely separated as to be products of different clusters. If clusters are localized, short-range objects, the last-mentioned situation is attainable and we should observe the onset of the exponential behavior of (1) when the gap length exceeds the typical range of a cluster (approximately 1 unit of rapidity). An exponential dependence can indeed be ascribed to the large-gap data; the slope implies a cluster density $\rho\approx 1$.

To proceed further, we observe that if the cluster density is approximately 1 and the directly observed[6] charged-pion density $\rho_\pi\approx 2$, the mean number of charged pions per cluster, $\langle M\rangle$, must be roughly 2. If clusters were neutral, we would

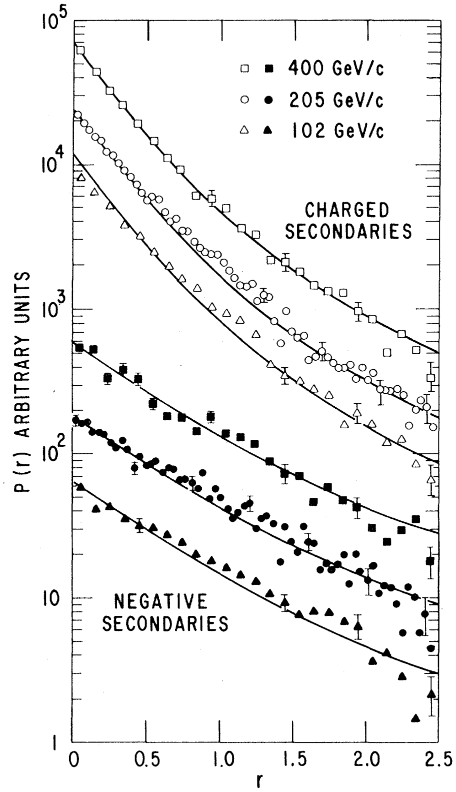

FIG. 1. Distribution of rapidity gaps between produced particles in 102-, 205-, and 405-GeV/c pp collisions (from Ref. 5). End gaps have been excluded from the data to minimize distortions due to diffractive events and kinematical constraints. The curves, which are predictions of a simple cluster model, are explained in the text.

expect (1) to hold down to small separations for gaps between negative particles, since the probability for two negatives to emerge from a single cluster would be exceedingly small. The remaining data in Fig. 1 show this to be the case: The slope for gaps between negatives coincides with the large-gap slope for the all-charged-particles distribution.

To this point we have confirmed directly, on a qualitative level, that clusters of limited extent in rapidity are produced independently. This conclusion does not rely upon any dynamical assumption concerning the underlying mechanism for cluster production, nor do we require any assumption about the intracluster multiplicity distribution g_M to obtain the result $\langle M\rangle\simeq 2$. To manifest the consistency of our interpretation, we next explain the gap-length distribution over its full observed range. Beyond providing an important independent test of our ideas, this exercise brings to light a means of determining more sensitively the parameters of a cluster model.

Once the cluster density and the intracluster multiplicity distribution are specified, the rapidity-gap distribution is completely determined. Indeed, it will turn out that in first approximation the inclusive rapidity-gap distribution is sensitive only to $\langle M\rangle$, and not to higher moments of the intracluster multiplicity distribution.

Each secondary emerging from a cluster at $\hat{y}$ has the independent probability

$$q_{12}(\hat{y}) = \int_{y_1}^{y_2} dy\, D(y-\hat{y}), \qquad (2)$$

to occupy the interval (y_1, y_2). Therefore, the probability that an M-particle cluster at $\hat{y}$ deposits no particles in the interval of interest is

$$G_M(y_1, y_2; \hat{y}) = [1 - q_{12}(\hat{y})]^M. \qquad (3)$$

If several clusters are produced, we combine these probabilities, weighting by the probability $\exp(-\langle N\rangle)\langle N\rangle^N/N!$ that N clusters be made and by the probability g_M that a cluster emits M observed particles. Thus, the probability that a single cluster is produced anywhere in the allowed rapidity interval $-Y/2, Y/2$ and deposits no particles in (y_1, y_2) is

$$\exp(-\langle N\rangle)\sum_M g_M(\langle N\rangle^1/1!)Y^{-1}\int_{-Y/2}^{Y/2} d\hat{y}[1-q_{12}(\hat{y})]^M, \qquad (4)$$

where we have weighted equally all cluster positions. At sufficiently large energies, this approximation introduces only a small error. Next, the probability that two independent clusters deposit no particles in (y_1, y_2) is

$$\exp(-\langle N\rangle)\sum_{M_1, M_2} g_{M_1}g_{M_2}(\langle N\rangle^2/2!)Y^{-2}\int_{-Y/2}^{Y/2} d\hat{y}[1-q_{12}(\hat{y}_1)]^{M_1}\int_{-Y/2}^{Y/2} d\hat{y}_2[1-q_{12}(\hat{y}_2)]^{M_2}. \qquad (5)$$

Summing over all cluster multiplicities, we obtain for the probability that no particles are produced in (y_1, y_2)

$$G(y_1, y_2; Z) = \exp\Big((\langle N\rangle/Y)\sum_{M=0}^{\infty} g_M Z^M \int_{-Y/2}^{Y/2} d\hat{y}\,\{[1-q_{12}(\hat{y})]^M - 1\}\Big), \qquad (6)$$

where we have weighted each coefficient g_M by Z^M to keep track of the number of produced particles. If the right-hand side of (6) is expanded in a power series in Z, the coefficient of Z^n is the probability that in an n-particle event, no particles appear in (y_1, y_2). The inclusive average is given by $Z=1$.

Hereafter we identify the cluster density as

$$\rho = \langle N\rangle/Y, \qquad (7)$$

and work in the high-energy limit ($Y\to\infty$), in which G becomes a function only of $r\equiv y_2 - y_1$ and Z. The frequency distribution for at least one particle to occur at y_1 and no particles to occur in (y_1, y_2) is[7] $(\partial/\partial y_1)G(y_1, y_2; Z)$. The frequency with which at least one particle occurs at the other end of the gap $y_2 = y_1 + r$, independent of y_1, is precisely the rapidity-gap distribution. It is given by

$$P(r;Z) = \frac{-1}{\rho\langle M\rangle}\frac{\partial}{\partial y_2}\frac{\partial}{\partial y_1}G(y_1, y_2; Z) = \frac{1}{\rho\langle M\rangle}\frac{\partial^2}{\partial r^2}G(r;Z), \qquad (8)$$

which depends only on the gap length and Z.

To compare this result with the data, we compute $P(r; Z=1)$ using a Poisson intracluster multiplicity distribution with $\langle M\rangle = 2$. The results are shown in Fig. 1 to agree well with the data over the entire measured range. We have verified that other choices for g_M, constrained to give $\langle M\rangle \simeq 2$, are equally satisfactory.[8] Distinctions among various models can be drawn by confronting predictions with data for $Z\neq 1$. The relevant parameter for arbitrary Z is $\langle M\rangle_Z$. Given this average for all values of Z we could in principle extract the distribution g_M.

The exponential behavior of the present data establishes that multihadron clusters are emitted independently. Deviations from the exponential behavior at small rapidity gaps demonstrate yet again the existence of short-range correlations. We term this evidence direct since no calculation or curve fitting is needed to justify it. The cluster density is obtained as the slope of the observed exponential. Previous estimates of this number, and of the related mean number of charged particles per cluster $\langle M\rangle$, have been obtained less directly. Nevertheless, many of these estimates—which like our detailed numerical computation require additional assumptions—are in qualitative agreement with the direct measurement. For example, the one-channel Chew-Pignotti multiperipheral model, in which produced particles are strongly ordered in rapidity, relates the density of produced objects to the input trajectory α_0 by $\rho = 2 - 2\alpha_0$. Thus the usual meson exchanges with $\alpha_0 = \frac{1}{2}$ give unit density, consistent with our measurement.[9]

Two-particle rapidity correlations are consistent with the shape predicted by the isotropic-cluster-decay picture. Assuming independent cluster emission to be the source of short-range correlations, and under specific assumptions about the diffractive component of multiple production, one can estimate $\langle M(M-1)\rangle/\langle M\rangle$. If in addition a specific form is assumed for g_M, $\langle M\rangle$ can be extracted.[10] The uncertainties introduced by the indirectness of these estimates are indicated by the fact that estimates vary from[10] $\langle M\rangle \simeq 2$ to[11] $\langle M\rangle \simeq 4$.

Another estimate, based upon a direct measurement of charge mobility but a specific "ω" cluster model for change-transfer fluctuations, leads to $\rho \simeq 1$.[12] The consistency of these many estimates, together with the direct evidence we have presented for independent cluster emission, argues convincingly in favor of the cluster description.

Our measurements of ρ and $\langle M\rangle$ carry a number of additional important implications. That ρ is clearly not less than 0.5 nor more than 2 will severely constrain overlap-function calculations which have in the past been based upon widely varying guesses for ρ.[13] The small value we find for $\langle M\rangle$ indicates that *the properties of clusters closely resemble those of the prominent meson resonances.* In our opinion, this adds to the intuitive appeal of the cluster picture.

We intend to investigate several additional properties of clusters with the techniques outlined here. One of the most important questions concerns the energy-dependence of the cluster parameters. By extracting g_M from the semi-inclusive data at various energies, we can test the attractive assumption that the intracluster multiplicity distribution is only weakly energy dependent. A related issue is whether, at a given energy, high-multiplicity events are produced by more clusters or bigger clusters. Finally, it is possible to study gap distributions for the exchange across the gap of a definite charge. These are sensitive to the net charge carried by clusters, and can test whether the cluster-production mechanism is a multiperipheral exchange.

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