[1]J. E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[2]S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975).

[3]Since we anticipate that SU(4) symmetry would be violated more strongly than SU(3), we expect $\alpha \gg 1$, as indeed our numerical value (4) shows.

[4]For the assignment of quantum numbers of the four quarks, see S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1972).

[5]The widths for the decays $\psi \to K\bar{K}$, $\rho\pi$, etc., depend on the small departures of the state ψ from a pure $\bar{\mathcal{P}}'\mathcal{P}'$ state, which in turn depend rather sensitively on the input parameters, particularly the ρ mass. Of all the input masses, the quoted value of the ρ mass has the largest uncertainty ($M_\rho = 770 \pm 10$ MeV). We have also found that if we diagonalize the linear mass matrix, the resulting solution gives rather narrow width for ψ, consistent with the experimental observation. The value of α in this case changes to 9.81, and leads to masses for $C_u(V)$ and $C_s(V)$ to be 1.94 and 2.06 GeV, respectively. The possibility of using a linear mass formula for 1^- mesons and its consequences on the other baryon and 0^- meson mass spectra will be discussed elsewhere.

[6]We expect this mixing angle to be small since it results from SU(3) symmetry breaking.

[7]The New York Times, 23 November 1974, p. 52M.

[8]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).

[9]M. K. Gaillard, B. W. Lee, and J. L. Rosner, Fermilab Report No. Pub. 74/86-THY 1974 (unpublished).

COMMENTS

Comment on Radiative Corrections to $e^+e^- \to \psi(3105)$†

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The salient features of the radiative corrections to the line shape of the reaction $e^+e^- \to \psi(3105)$ are described, emphasizing the importance of treating the infrared divergence to all orders. A tentative value of 5.2 keV is obtained for the partial width for the ψ to decay to e^+e^-. An interesting but possibly fortuitous relation among the leptonic decay widths of the ρ^0, ω, φ, and ψ is noted.

Recently, new resonances have been discovered at Brookhaven National Laboratory[1] and Stanford Linear Accelerator Center.[2] They are presumably vector states which couple to e^+e^- pairs through a virtual-photon intermediate state, as do the ρ^0, ω, and φ. As a consequence of the fact that these new resonances are very narrow, certainly less than a few MeV, the radiative corrections to the line shape in e^+e^- annihilation become very significant, as is apparent in Fig. 1. The main feature is that the line becomes very skewed because the incoming electrons can emit soft photons before producing the resonance. In the first presentation of the data, the radiative line shapes were calculated to lowest order in α using an analysis of Bonneau and Martin.[3] However, for such narrow lines, this lowest-order estimate is rather poor and it becomes necessary to include the infrared contributions to all orders. The procedure is essentially no different than the one which has been known for many years in the case of elastic or inelastic electron scattering. However, it has apparently not been presented in a convenient form for the analysis of resonances in e^+e^- annihilation. This will be done here, and a preliminary analysis will be made of the published data in order to estimate the partial width for the decay of the vector mesons into e^+e^- pairs. A final analysis will require a careful least-squares fit to the data, and is best made by the experimentalists when the final data become available.

The observed line is formed from the convolution of three factors: a Breit-Wigner function to describe the line shape of the resonance; a radiative tail $\lambda(\epsilon)$ giving the probability of emitting a certain amount of radiation; and a resolution function for the beam energy, presumed to be a Gaussian function. Let $2E$ be the total beam energy available in the center-of-mass frame, ϵ

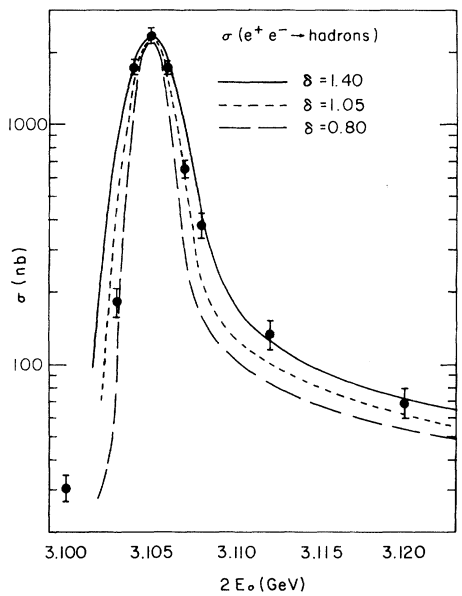

FIG. 1. Comparison of the data with three calculations of the radiative line shape depending on the energy resolution δ.

and $\vec{K}$ be the energy and momentum carried off by radiation, and M be the mass of the hadronic system. It is easy to see that we may ignore the kinetic energy of the hadronic system ($\cong \vec{K}^2/2M$) and set

$$\epsilon = 2E - M, \tag{1}$$

The Breit-Wigner function for the resonance is characterized by a central mass M_0, a total width Γ, partial widths $\Gamma(V \to l^+ l^-)$ and $\Gamma(V \to \text{hadron})$, and a total area $\mathcal{A}$. This area is related to the other quantities through

$$\mathcal{A} = 6\pi^2 \frac{\Gamma(V \to e^+e^-)}{M_0^{\,2}} \frac{\Gamma(V \to \text{hadron})}{\Gamma}. \tag{2}$$

Determination of this area, together with the branching ratio into leptons, yields the two partial widths. The total beam energy is centered on $2E_0$ with a dispersion δ ($= \langle 4(E - E_0)^2 \rangle^{1/2}$). For $2E_0 = 3.1$ GeV, the value of δ predicted theoretically from quantum fluctuations of synchrotron radiation[1] is 0.8 MeV.

Finally, we need the probability for emitting total radiation energy ϵ. The important parameter characterizing this spectrum is αA and it is obtained by integrating the probability for emitting a single photon over all angles. For the present process this parameter turns out to be

$$\alpha A = \frac{2\alpha}{\pi}\left[\ln \frac{2p_+ \cdot p_-}{m_e^{\,2}} - 1\right] = \frac{2\alpha}{\pi}\left[2 \ln \frac{M_0}{m_e} - 1\right] \tag{3}$$

(emission of photons from hadrons has been neglected; this could affect the overall width by terms of order α, but not the shape of the observed line). To obtain a satisfactory expression for the shape of the radiative tail, real and virtual infrared terms must be summed to all orders.[4] Noninfrared terms may be calculated to lowest order in α. The result is

$$\lambda(\epsilon) = \frac{\alpha A}{\epsilon}\left(\frac{2\epsilon}{M_0}\right)^{\alpha A}\left[1 + \frac{13}{12}\alpha A + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right)\right]. \tag{4}$$

The predicted line shape is now obtained by multiplying these three factors together and integrating with respect to M and E. In the preliminary curves presented here, the Breit-Wigner function and Gaussian resolution are replaced by a single, "equivalent" Gaussian function. Three such curves with the values $(\delta, \mathcal{A}) = (0.8, 7600)$, $(1.05, 9700)$, and $(1.4, 12\,600)$ (where $\mathcal{A}$ has units nb MeV) are compared with the data in Fig. 1. The curve for $\delta = 0.8$ is clearly too narrow, indicating some source of beam-energy spread which is not yet understood.[5] (Note, the resonance line itself is too narrow to account for the additional spread.) There are some simple features of these curves which are worth noting since they may be useful for a quick appraisal of such data: (1) The peak height is given by

$$\sigma_{pk} \cong \frac{\mathcal{A}}{(2\pi)^{1/2}\delta}\left(\frac{2\sqrt{2}\delta}{M_0}\right)^{\alpha A}[1 + 0.79\alpha A]. \tag{5}$$

This measures essentially the quantity $\mathcal{A}/\delta$. For $\psi(3105)$, the radiative correction to the peak height is approximately a factor of 0.6. Also, the the peak is shifted about 10% of δ above the resonant mass; this is, of course, negligible compared to the uncertainty in the total energy. (2) The radiative tail above the peak energy (say for $2E_0 - M \gtrsim 5\delta$) is rather insensitive to the resolution and is given by

$$\sigma_{\text{tail}} \cong \mathcal{A}\frac{\alpha A}{2E_0 - M_0}\left(\frac{2E_0 - M_0}{\frac{1}{2}M_0}\right)^{\alpha A} \times \left[1 + \frac{13}{12}\alpha A + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right)\right]. \tag{6}$$

This determines the parameter $\mathcal{A}$. (3) The area under the peak up to $2E_0$ (where $2E_0 - M \gtrsim 5\delta$) is given by

$$\mathcal{A}' = \mathcal{A}\left(\frac{2E_0 - M_0}{\frac{1}{2}M_0}\right)^{\alpha A} \times \left[1 + \frac{13}{12}\alpha A + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right)\right]. \tag{7}$$

This yields another method for determining $\mathcal{A}$.

Undoubtedly, the proper way to determine $\mathcal{A}$ is to make a least-squares fit using $\mathcal{A}$, δ, and M_0 as parameters. Since the peak is being remeasured more carefully, there is no point to doing that with the original data. Instead we shall make a rough estimate, assuming that $\delta = 1.2$ MeV and that the peak height (less background) is 2300 nb. This gives for the area

$$\mathcal{A} = 11\,000 \text{ nb MeV}, \tag{8}$$

Taking the branching ratio of leptons to hadrons to be about 14%,[6] we then find

$$\Gamma(\psi \to e^+e^-) = 5.2 \text{ keV}. \tag{9}$$

The very tentative nature of this number should be obvious to the reader. When the final data are carefully analyzed by the experimentalists, it could easily change by 10 to 15%.

It has been suggested that the new vector meson at 3.105 GeV might be a state of "orthocharmonium",[7] that is, a bound state of charmed quarks. If that is so, its coupling to the photon ought to be related to those of the other vector mesons by[8]

$$e^2/f_\rho^2 : e^2/f_\omega^2 : e^2/f_\varphi^2 : e^2/f_\psi^2 = 9:1:2:8. \tag{10}$$

This turns out not to work terribly well (the ψ coupling is too small by a factor of 4 or so). However, if we instead compare widths to e^+e^- pairs [which are proportional to e^2M_v/f_v^2 or $|\psi(0)|^2/M_v^2$], we find the following interesting result[9]

$$\frac{\Gamma(\rho\to e^+e^-)}{9} : \frac{\Gamma(\omega\to e^+e^-)}{1} : \frac{\Gamma(\varphi\to e^+e^-)}{2} : \frac{\Gamma(\psi\to e^+e^-)}{8} = (0.72\pm 0.10):(0.76\pm 0.08):(0.67\pm 0.07):(0.65\pm 0.07). \tag{11}$$

Until one understands why the lepton-pair widths are the best quantities for comparison in the case of broken symmetry, this result must be regarded as fortuitous.

I wish to thank my colleagues at Cornell University for stimulating discussions about the new vector mesons. J. D. Jackson at Berkeley and G. Feldman at Stanford Linear Accelerator Center were generous and helpful in providing information by telephone and correspondence.

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[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[2]J. E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406, 1453 (1974); the discovery in e^+e^- annihilation was confirmed at Frascati [C. Bacci *et al.*, Phys. Rev. Lett. 33, 1408 (1974)] and DESY.

[3]G. Bonneau and F. Martin, Nucl. Phys. B27, 381 (1971).

[4]For example, see D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (Leipzig) 13, 379 (1961).

[5]G. Feldman, private communication.

[6]J. D. Jackson, private communication. Using the same formalism and the area method of analysis, Jackson has found $\mathcal{A} = 11\,500$ nb MeV and $\Gamma(\psi\to e^+e^-) = 5.5$ keV for the data of Ref. 2.

[7]T. W. Appelquist and H. D. Politzer, private communication.

[8]M. K. Gaillard, B. W. Lee, and J. L. Rosner, Fermilab Report No. Pub-74/86-Thy (unpublished).

[9]The values for ρ^0, ω, φ were obtained by multiplying total widths by the e^+e^- branching ratio, using the tables of particle properties [V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974)]. This may not be the best way to use the data. The error in the ψ width was taken to be 10%.