spacelike and timelike photons gives cutoffs $\Lambda_+ > 35$ GeV and $\Lambda_- > 47$ GeV, which are considerably larger than the previous highest limits ($\Lambda_+ > 14.5$ GeV, $\Lambda_- > 23.6$ GeV) set by Beron *et al.*[1] No deviation of either the electron or muon form factor from unity has been observed, the cutoff parameters being always larger than 16 GeV. Our limit $\Lambda_{\mu e}$ on μ-e universality, defined by $1/\Lambda_{\mu e}^2 \equiv 1/\Lambda_\mu^2 - 1/\Lambda_e^2$, is $\Lambda_{\mu e^+} > 13$ GeV and $\Lambda_{\mu e^-} > 15$ GeV determined from the (correlated) difference between $1/\Lambda_\mu^2$ and $1/\Lambda_e^2$ from Table I.

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SU(4) Symmetry and the Possible Existence of New Hadrons*

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We derive SU(4) mass formulas taking mixing, whenever it arises, fully into account. With the masses of the usual hadrons and that of the newly discovered resonance ψ as input, we predict the masses of the new 1^- and 0^- mesons as well as the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons.

Recently a new resonance (hereafter called ψ) has been reported[1] in $e\bar{e} \to$ hadrons and $p + \mathrm{Be} \to e\bar{e} + \ldots$ at a mass value of 3.105 ± 0.003 GeV and a very narrow width $\Gamma < 1.3$ MeV. Assuming that it is a vector particle, we have recently proposed[2] the possibility that this resonance belongs to a $15 \oplus 1$ dimensional representation of the SU(4) group. This hypothesis is presently consistent with the various experimental features such as the total production cross section and the width that have been reported.

If our suggestion is correct, it opens up a whole world of new particles carrying a new quantum number, usually referred to as charm. In this note, we discuss the classification of these new particles under SU(4), and derive the SU(4) mass formulas for 1^- and 0^- mesons as well as for $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. With the mass of the ψ as input, we obtain numerical predictions for the masses of the new mesons and baryons. We assume that SU(4) symmetry is broken by the interaction[3]

$$H' = T_8 + \alpha T_{15}, \tag{1}$$

where T_8 and T_{15} belong to the same 15-dimensional representation of SU(4).

As suggested we assume that the 1^- mesons belong to a $15 \oplus 1$ dimensional representation of SU(4), which we may denote by V^α ($\alpha = 0, 1, \ldots 15$). Note that if we consider as usual that mesons are quark-antiquark structures, this is the SU(4) representation we obtain from $4 \otimes \bar{4} = 15 \oplus 1$. The SU(3) decomposition of the 15-plet is

$$15 \supset 8 \oplus 3 \oplus \bar{3} \oplus 1. \tag{2}$$

The components $V^{1,\cdots 8}$ and V^{15}, belonging to the $8 \oplus 1$ representation, describe the nonet of vector mesons. The representation 3 contains an SU(2) doublet $C_u(V), C_d(V)$ and an SU(2) singlet $C_s(V)$ with the quantum numbers displayed in Table I. The SU(3) representation $\bar{3}$ contains the corre-

TABLE I. Predicted masses of the new hadrons based on the SU(4) mass formulas. In using the masses of the usual hadrons as input, we have, wherever required, averaged over the masses of particles in an isospin multiplet.

SU(3) Representation	Quantum Numbers	Predicted Mass (GeV)	
VECTOR MESONS			
3 $(C_u^+(V), C_d^0(V))$	$I=\frac{1}{2}$, $Y=-1$, $C=1$	2.190	
$C_s^+(V)$	$I=0$, $Y=0$, $C=1$	2.236	
PSEUDOSCALAR MESONS			
3 $(C_u^+(P), C_d^0(P))$	$I=\frac{1}{2}$, $Y=-1$, $C=1$	2.171	
$C_s^+(P)$	$I=0$, $Y=0$, $C=1$	2.222	
1 $\psi(P)$	$I=0$, $Y=0$, $C=0$	2.755	
$1/2^+$ BARYONS		Linear Formula	Quadratic Formula
6 (B_c^{++}, B_c^+, B_c^0)	$I=1$, $Y=0$, $C=1$	6.202	3.479
(B_c^+, B_c^0)	$I=\frac{1}{2}$, $Y=-1$, $C=1$	6.398	3.537
B_c^0	$I=0$, $Y=-2$, $C=1$	6.581	3.600
$\bar{3}$ (B_c^+, B_c^0)	$I=\frac{1}{2}$, $Y=-1$, $C=1$	4.831	2.982
B_c^+	$I=0$, $Y=0$, $C=1$	4.597	2.898
3 (B_c^{++}, B_c^+)	$I=\frac{1}{2}$, $Y=-1$, $C=2$	8.790	4.313
B_c^+	$I=0$, $Y=-2$, $C=2$	9.044	4.375
$3/2^+$ BARYONS			
6 $(\Delta_c^{++}, \Delta_c^+, \Delta_c^0)$	$I=1$, $Y=0$, $C=1$	4.261	3.215
(Δ_c^+, Δ_c^0)	$I=\frac{1}{2}$, $Y=-1$, $C=1$	4.414	3.277
Δ_c^0	$I=0$, $Y=-2$, $C=1$	4.562	3.342
3 $(\Delta_c^{++}, \Delta_c^+)$	$I=\frac{1}{2}$, $Y=-1$, $C=2$	7.291	4.377
Δ_c^+	$I=0$, $Y=-2$, $C=2$	7.444	4.422
1 Δ_c^{++}	$I=0$, $Y=-2$, $C=3$	10.320	5.289

sponding antiparticles $(\bar{C}_d, \bar{C}_u)$ and $\bar{C}_s$. The generalization of the Gell-Mann–Nishijima relation which serves to define the charm quantum number C, is given by[4]

$$Q = I_3 + \tfrac{1}{2}Y + C. \tag{3}$$

The quantum numbers for the SU(4) singlet V^0, and the components V^8 and V^{15} of the 15-plet are identical, so that these states would mix both by SU(4) as well as by SU(3) symmetry-breaking terms in Eq. (1). Diagonalization of the 3×3 quadratic mass matrix[5] then generates the physical states ω, φ, and ψ. This mixing problem has been solved in Ref. 2, and using the masses of ρ, K^*, ω, φ, and ψ as input, we obtain a complete solution, which, in particular, yields

$$\alpha = 21.61. \tag{4}$$

The masses of the charmed vector mesons can now be calculated from the following formulas

$$\frac{C_u(V)-\rho}{K^*-\rho} = \frac{C_s(V)-K^*}{K^*-\rho} = x, \tag{5}$$

where

$$x = \frac{2\sqrt{2}\alpha + 1}{3} = 20.71, \tag{6}$$

and the symbols in Eq. (5) represent the squared masses of the corresponding particles. The computed masses are given in Table I.

We classify the 0^- mesons also in a $15\oplus 1$ dimensional representation, whose SU(3) and SU(2) decomposition we have already discussed. If we denote the charmed pseudoscalar mesons by $C_u(P)$, $C_d(P)$, and $C_s(P)$ in analogy to the corresponding vector particles, we obtain the mass formulas analogous to (5):

$$\frac{C_u(P)-\pi}{K-\pi} = \frac{C_s(P)-K}{K-\pi} = x. \tag{7}$$

As for vector mesons, we employ squared masses in Eq. (7) to obtain the results displayed in Table I.

Knowing the parameter α in (1), we can now solve the P_0, P_8, P_{15} mixing problem, to predict the mass of $\psi(P)$, the pseudoscalar analog of ψ. Our calculations give

$$M_{\psi(P)} = 2.755\ \text{GeV}. \tag{8}$$

For the usual three-quark structure of a baryon, we have

$$4\otimes 4\otimes 4 = \bar{4}\oplus 20\oplus 20'\oplus 20''. \tag{9}$$

The SU(4) representation 20 is completely symmetric and, in analogy to the SU(3) decouplet, is suitable for the classification of the $\frac{3}{2}^+$ baryons. The other two twenty-dimensional representations possess mixed-symmetry property, and either one can be used for assigning the $\frac{1}{2}^+$ baryons.

The completely symmetric 20, has the following SU(3) decomposition

$$20 \supset 10\oplus 6\oplus 3\oplus 1. \tag{10}$$

Further decomposition to isospin reveals the content of the charmed SU(3) multiplets as shown in Table I, where we have used the generic symbol Δ_c for all the $\frac{3}{2}^+$-charmed particles. We obtain the following mass formulas:

$$\Delta_c(I=1, Y=0, C=1) - \Delta = \Delta_c(I+\tfrac{1}{2}, Y=-1, C=1) - Y_1{}^* = \Delta_c(I=0, Y=-2, C=1) - \Xi^*$$
$$= \tfrac{1}{2}[\Delta_c(I=\tfrac{1}{2}, Y=-1, C=2) - \Delta] = \tfrac{1}{2}[\Delta_c(I=0, \ Y=-2, C=2) - Y_1{}^*]$$
$$= \tfrac{1}{3}[\Delta_c(I=0, Y=-2, C=3) - \Delta] = x\delta, \tag{11}$$

where

$$\delta = Y_1{}^* - \Delta = \Xi^* - Y_1{}^* = \Omega^- - \Xi^*. \tag{12}$$

In computing the unknown masses from Eq. (11) we use an averaged value for δ. Furthermore, we list in Table I the calculated masses using both the linear and the quadratic masses in Eqs. (11) and (12). According to the usual folklore, one usually regards baryon mass formulas to be linear in mass, but in practice the SU(3) mass formulas do not clearly prefer the linear formula over the quadratic one. For the charmed baryons, the tabulated mass values are, in fact, quite different in the two cases. We should remark that the formulas (11) and (12) can also be obtained from a naive quark model if we take the mass of the $\frac{3}{2}^+$ baryon consisting of three quarks, q_μ, q_ν, q_λ, to be $M_{\mu\nu\lambda} = m_0 + a(m_\mu + m_\nu + m_\lambda)$.

The SU(4) representation 20′ in Eq. (9) can be decomposed in terms of the representation of SU(3) as follows

$$20' \supset 8 \oplus 6 \oplus 3 \oplus \bar{3}. \tag{13}$$

The classification of the charmed baryons is again shown in Table I. The SU(3) representations 6 and 3 for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons have similar content. In the present case, note that SU(3) symmetry breaking in Eq. (1) mixes the states $B_c{}^{+,0}$ in the SU(3) representations 6 and $\bar{3}$. We have taken the mixing into account, and obtain the mass formulas

$$[B_c(I=1, Y=0, C=1) - N]/(\Sigma - N) = [B_c(I=0, Y=-2, C=1) - \Xi]/(\Sigma - N)$$
$$= [B_c(I=0, Y=0, C=1) - N]/(\Lambda - N) = [B_c(I=\tfrac{1}{2}, Y=-1, C=2) - N]/(\Xi - N)$$
$$= [B_c(I=0, Y=-2, C=2) - \Sigma]/(\Xi - N) = x, \tag{14}$$

and

$$B_c{}^{1,2}(I=\tfrac{1}{2}, Y=-1, C=1) = \frac{\Lambda+\Sigma}{2} + \left(\frac{\Lambda+\Sigma}{2} - N\right)x \pm \left(\frac{\Sigma-\Lambda}{2}\right)\frac{(x-\frac{1}{2})}{\cos 2\theta},$$

where[6]

$$\tan(2\theta) = -\sqrt{3}/(2x-1). \tag{15}$$

As for Δ_c, we have tabulated the masses of charmed baryons B_c using both linear and quadratic masses.

It is interesting to note from Table I that the new 0^- and 1^- mesons are almost equally heavy, and that the linear mass formula gives the lowest mass of the new $\frac{3}{2}^+$ baryons to be slightly smaller than the corresponding mass of the new $\frac{1}{2}^+$ baryons. The low-lying charmed mesons as well as baryons (from the quadratic mass formula) are expected to have narrow widths, since they would presumably decay by weak interactions into ordinary hadrons and leptons. Needless to say, the discovery of these new particles would provide a direct confirmation of our hypothesis regarding the ψ resonance.

Very recently Stanford Linear Accelerator Center has reported another new resonance[7] (call it ψ') at 3.7 GeV. We believe this could possibly be a radial excitation of the $\bar{\mathcal{P}}'\mathcal{P}'$ quark system. If ρ' and $K^{*\prime}$ denote similar radial excitations of the quark-antiquark structures of ρ and K^*, we may assume that ψ' together with ρ' and $K^{*\prime}$ fall into another $15 \oplus 1$ representation of SU(4). It is interesting to note that using the quadratic mass formula, we obtain the mass of ψ' to be 3.7 GeV, if we use $\rho' = 1.600$ GeV and $K^{*\prime} = 1.680$ GeV. Particle states ρ' and $K^{*\prime}$ at or near such mass values have in fact been reported[8] in some experiments.

On completion of this work, our attention was drawn to a paper by Gaillard, Lee, and Rosner,[9] who have also derived some of the SU(4) mass formulas without the benefit of the new resonance necessary to treat the mixing problems in detail.

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[1]J. E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[2]S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975).

[3]Since we anticipate that SU(4) symmetry would be violated more strongly than SU(3), we expect $\alpha \gg 1$, as indeed our numerical value (4) shows.

[4]For the assignment of quantum numbers of the four quarks, see S. L. Glashow, J. Illiopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1972).

[5]The widths for the decays $\psi \rightarrow K\bar{K}$, $\rho\pi$, etc., depend on the small departures of the state ψ from a pure $\bar{\mathcal{P}}'\mathcal{P}'$ state, which in turn depend rather sensitively on the input parameters, particularly the ρ mass. Of all the input masses, the quoted value of the ρ mass has the largest uncertainty ($M_\rho = 770 \pm 10$ MeV). We have also found that if we diagonalize the linear mass matrix, the resulting solution gives rather narrow width for ψ, consistent with the experimental observation. The value of α in this case changes to 9.81, and leads to masses for $C_u(V)$ and $C_s(V)$ to be 1.94 and 2.06 GeV, respectively. The possibility of using a linear mass formula for 1^- mesons and its consequences on the other baryon and 0^- meson mass spectra will be discussed elsewhere.

[6]We expect this mixing angle to be small since it results from SU(3) symmetry breaking.

[7]The New York Times, 23 November 1974, p. 52M.

[8]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).

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COMMENTS

Comment on Radiative Corrections to $e^+e^- \rightarrow \psi(3105)$†

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The salient features of the radiative corrections to the line shape of the reaction $e^+e^- \rightarrow \psi(3105)$ are described, emphasizing the importance of treating the infrared divergence to all orders. A tentative value of 5.2 keV is obtained for the partial width for the ψ to decay to e^+e^-. An interesting but possibly fortuitous relation among the leptonic decay widths of the ρ^0, ω, φ, and ψ is noted.

Recently, new resonances have been discovered at Brookhaven National Laboratory[1] and Stanford Linear Accelerator Center.[2] They are presumably vector states which couple to e^+e^- pairs through a virtual-photon intermediate state, as do the ρ^0, ω, and φ. As a consequence of the fact that these new resonances are very narrow, certainly less than a few MeV, the radiative corrections to the line shape in e^+e^- annihilation become very significant, as is apparent in Fig. 1. The main feature is that the line becomes very skewed because the incoming electrons can emit soft photons before producing the resonance. In the first presentation of the data, the radiative line shapes were calculated to lowest order in α using an analysis of Bonneau and Martin.[3] However, for such narrow lines, this lowest-order estimate is rather poor and it becomes necessary to include the infrared contributions to all orders. The procedure is essentially no different than the one which has been known for many years in the case of elastic or inelastic electron scattering. However, it has apparently not been presented in a convenient form for the analysis of resonances in e^+e^- annihilation. This will be done here, and a preliminary analysis will be made of the published data in order to estimate the partial width for the decay of the vector mesons into e^+e^- pairs. A final analysis will require a careful least-squares fit to the data, and is best made by the experimentalists when the final data become available.

The observed line is formed from the convolution of three factors: a Breit-Wigner function to describe the line shape of the resonance; a radiative tail $\lambda(\epsilon)$ giving the probability of emitting a certain amount of radiation; and a resolution function for the beam energy, presumed to be a Gaussian function. Let $2E$ be the total beam energy available in the center-of-mass frame, ϵ