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Description of Nuclear Resonances with Mixed Isospin*

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It is pointed out that a formulation commonly used to describe mixed-isospin nuclear resonance is incorrect. An alternative approach is suggested and applied to some examples.

Overlapping resonances that mix by virtue of a symmetry-breaking interaction are present in many branches of physics.[1] For example, such mixing phenomena occur in elementary-particle physics in K-meson decays[2] as a result of CP-invariance violation, as well as in the ρ-ω system[3] as a result of electromagnetic G-parity nonconservation. An extensive theoretical formulation has been developed and applied to the above problems.[4]

In nuclear physics a familiar example of mixing occurs if two nuclear resonance levels with the same spin and parity are sufficiently overlapping that the electromagnetic interaction leads to a mixing of the isospin T of the levels.[5] The most striking example of this phenomena is the occurrence of a pair of 2^+ levels in ^{8}Be at 16.6 and 16.9 MeV that are thought[6] to be nearly equal mixtures of $T=0$ and $T=1$. It has been customary[7] to express the mixed states as linear combinations of the pure isospin states:

$$|16.6\rangle = |A\rangle = \alpha|T=0\rangle + \beta|T=1\rangle,$$
$$|16.9\rangle = |B\rangle = \beta|T=0\rangle - \alpha|T=1\rangle, \qquad (1)$$

with α and β taken to be real and the mixed states constructed so as to be orthogonal. It is apparently not widely realized[8] that the expansion (1) is not sufficiently general to treat the mixing problem and that except for unusual cases, it violates the unitarity of the S matrix in the channels coupled to the resonances.

To modify (1), consider the matrix elements of the full Hamiltonian for the nucleus in question in an orthonormal isospin basis $|i\rangle$. In this representation the off-diagonal elements will be nonzero because of the symmetry-breaking electromagnetic interaction. We require a linear transformation C to a set of physical states $|a\rangle$ such that the Hamiltonian matrix will be brought to diagonal form in the new representation and the diagonal elements will be the complex physical energies of the resonances. We then write

$$|a\rangle = \sum_i C_{ai}|i\rangle, \quad H|a\rangle = \tilde{E}_a|a\rangle, \qquad (2)$$

where $\tilde{E}_a = E_a - \frac{1}{2}i\Gamma_a$ are the physical energies, and since H has complex eigenvalues we expect C to be nonunitary and the states $|a\rangle$ to be not necessarily orthogonal. For a two-level problem, C could most generally contain four complex elements, but the arbitrary nature of certain phases, two normalization conditions, and time-reversal invariance leave only one complex mixing parameter ϵ unspecified.[9] Thus we replace (1) by

$$|A\rangle = (1+|\epsilon|^2)^{-1/2}(|1\rangle - \epsilon|0\rangle)$$
$$|B\rangle = (1+|\epsilon|^2)^{-1/2}(|0\rangle + \epsilon|1\rangle). \qquad (3)$$

The physical states $|A\rangle$ and $|B\rangle$ have been normalized to unity but their overlap is purely imaginary:

$$\langle A|B\rangle = 2i(1+|\epsilon|^2)^{-1}\,\mathrm{Im}\epsilon. \quad (4)$$

If we take the resonance positions to be $\tilde{E}_A = E_A - \frac{1}{2}i\Gamma_A$ and $\tilde{E}_B = E_B - \frac{1}{2}i\Gamma_B$ and permit decay into a set of channels $|f\rangle$, then enforcing the unitarity of the S matrix leads to the results[4] for the total widths

$$\Gamma_A = \sum_f |\langle f|t|A\rangle|^2 = \sum_f \gamma_{A\to f},$$
$$\Gamma_B = \sum_f |\langle f|t|B\rangle|^2 = \sum_f \gamma_{B\to f}, \quad (5)$$

where t is the amplitude responsible for the decay and the γ's are partial widths. Unitarity also leads to the Bell-Steinberger sum rule[10] which relates the overlap $\langle A|B\rangle$ to the amplitudes for decay of the resonances:

$$i(\tilde{E}_A{}^* - \tilde{E}_B)\langle A|B\rangle = \sum_f \langle f|t|A\rangle^* \langle f|t|B\rangle. \quad (6)$$

From this relation it is evident that if the final states $|f\rangle$ are such that none is populated from both $|A\rangle$ and $|B\rangle$, then $\langle A|B\rangle = 0$. Such a situation would prevail if isospin conservation were an exact symmetry. A second possibility is that the phases of the decay amplitudes conspire so as to lead to a vanishing sum in (6). This would not be expected to happen in general.

Further restrictions are available to determine ϵ. Letting $\epsilon = |\epsilon|\exp(i\varphi_\epsilon)$, we have the following general result[11] and a weak-coupling ($|\epsilon|^2 \ll 1$) version:

$$\tan\varphi_\epsilon = \frac{(1+|\epsilon|^2)(\Gamma_B - \Gamma_A)}{(1-|\epsilon|^2)2(E_B - E_A)} \xrightarrow[|\epsilon|\sim 0]{} \frac{\Gamma_B - \Gamma_A}{2(E_B - E_A)}. \quad (7)$$

Thus a specification of the mixing reduces to the determination of $|\epsilon|^2$ from experiment, with φ_ϵ then given (modπ) by (7).

In the special case that only $T = 0$ particle channels are open, we may simplify (7) further. The ratio of the total widths is then

$$\frac{\Gamma_A}{\Gamma_B} = \frac{\sum_f |\langle f|t|A\rangle|^2}{\sum_f |\langle f|t|B\rangle|^2} = |\epsilon|^2, \quad (8)$$

where I have used (3) and assumed that t itself is isospin conserving. Using (8), (7) reduces to[12]

$$\tan\varphi_\epsilon = (\Gamma_A + \Gamma_B)/2(E_B - E_A) \quad (9)$$

so that, in this case, the phase angle φ_ϵ is determined only by the positions and widths.

I have applied the above expressions to the determination of the mixing parameters for several nuclear level pairs that are thought to be mixed. The experimental positions and widths are taken from published compilations[13] and I have applied no Coulomb or penetrability corrections to the widths. The results are given in Table I.

The 2^+ levels in ^{6}Li have been studied and an upper bound established[14] on the branching ratio for the deuteron decay of the nominally $T = 1$ level at 5.366 MeV. From the total width of this level and the elasticity of the $T = 0$ resonance in d-α elastic scattering,[15] we find that $\gamma_{A\to d}/\gamma_{B\to d} = |\epsilon|^2 \le 0.007$. The weak-coupling version of (7) then gives φ_ϵ.

The ^{8}Be 2^+ levels decay only to the α-α channel (neglecting narrow γ widths), and $|\epsilon|^2$ and φ_ϵ follow from (8) and (9).

The ^{8}Be 1^+ states are studied via the ^{10}B(d, α)^{8}Be result of Callender and Browne.[16] Assuming that the mechanism of the reaction is isospin conserving and averaging their yield ratios at 10, 11, and 12 MeV, we obtain $\sigma_A/\sigma_B \sim 0.09 = |\epsilon|^2$, and φ_ϵ follows from (7).

In the ^{12}C case, the partial widths for α decay of the two levels are known[17] and we take $\gamma_{A\to 3\alpha}/\gamma_{B\to 3\alpha} = |\epsilon|^2$; then φ_ϵ follows from (7). Balamuth, Zurmühle, and Tabor[17] have estimated penetrability corrections for these levels and find that a reduction of $|\epsilon|^2$ by a factor of 6 is possible. Note that these states are so far apart relative to their widths that they are close to orthogonality, even though $|\epsilon|^2$ is not negligible. It could be speculated that perhaps some contribution to the impurities in these levels, or to those in oth-

TABLE I. Values of mixing parameters.

Levels	E_A (MeV)	Γ_A (keV)	E_B (MeV)	Γ_B (keV)	$\lvert\epsilon\rvert^2$	φ_ϵ (deg)	$\lvert\langle A\vert B\rangle\rvert$
^{6}Li, 2^+	5.366	540	4.31	1500	$\le$0.007	−24.5	$\le$0.070
^{8}Be, 2^+	16.627	107	16.911	77	1.39	17.9	0.30
^{8}Be, 1^+	17.642	10.7	18.154	138	0.09	8.5	0.081
^{12}C, 1^+	15.109	0.0394	12.713	0.0146	0.11	3.7×10^{-4}	3.9×10^{-6}
^{18}F, 1^-	5.606	0.047	5.674	0.156	0.30	0.085	1.2×10^{-3}

er nuclei, may be related to an underlying configuration built on the impure ^{8}Be states discussed above.

Finally, the α widths of the 1^- levels in ^{18}F have been measured[18] and we take $\gamma_{A\to\alpha}/\gamma_{B\to\alpha} = |\epsilon|^2$. The phase φ_ϵ may be obtained from (9) since experimentally[19] $\gamma_\alpha \sim \Gamma$. It has been argued that in order to understand the γ-decay data from the ^{18}F levels, at least one additional 1^- level must be involved in the mixing.[20] The formalism for such three-state mixing has been developed[21] and, though quite complex, it could be applied to this problem if necessary. The three-state formalism could also find application in the description of the highly excited $T=2$ states in nuclei,[22] since these states could have both $T=0$ and $T=1$ admixtures.

The two-state mixing problem discussed above involves the three complex numbers $\tilde{E}_A$, $\tilde{E}_B$, and ϵ. Alternatively we may view the system in the original isospin basis in terms of the three independent elements of the Hamiltonian matrix H_{00}, H_{11}, and H_{01} ($=H_{10}$). The element H_{01} is of considerable interest since it is the matrix element of the symmetry-breaking interaction in the isospin basis. Its value is readily obtained by reversing the diagonalization carried out in (2) and we obtain

$$H_{01} = \epsilon(\tilde{E}_B - \tilde{E}_A)/(1+\epsilon^2). \tag{10}$$

For example, in the case of the 2^+ levels in ^{8}Be, where penetrability corrections are small, we obtain a value of $H_{01} = 147.2 + i0.0$ keV, a result somewhat larger than recent shell-model estimates.[23]

It was long ago suggested by Barker and Mann[24] that isospin mixing in the nominally $T=1$ giant dipole resonance in self-conjugate nuclei would manifest itself as a deviation from unity in the decay ratio Γ_p/Γ_n. Since their arguments are based on the expansion given in (1) with real coefficients, there are modifications due to the additional phase. On expanding $|0\rangle$ and $|1\rangle$ into neutron and proton states, the modified result is easily obtained:

$$\Gamma_p/\Gamma_n = |1+\epsilon|^2/|1-\epsilon|^2. \tag{11}$$

This result implies that a single measurement of $\Gamma_p/\Gamma_n = R$ does not lead to a unique specification of $|\epsilon|$ and φ_ϵ since (11) defines only a circle in the ϵ plane centered at $\mathrm{Re}\epsilon = A = R + 1/R - 1$ and of radius $(A^2-1)^{1/2}$. In the absence of other information on the positions and widths of the components of the giant dipole state, (11) leads only to the inequalities

$$A-(A^2-1)^{1/2} \le |\epsilon| \le A+(A^2-1)^{1/2}, \quad |\tan\varphi_\epsilon| \le (A^2-1)^{1/2}. \tag{12}$$

It could be concluded that apart from the information of a purely nuclear character that is obtained from a study of these mixed levels, the nuclear cases provide the most varied examples available of the phenomenon of level mixing itself.

Many helpful conversations with W. D. McGlinn are gratefully acknowledged.

*Research supported in part by the National Science Foundation under Grant No. GP-32167X.

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Isolation of the Giant Quadrupole Resonance in ^{58}Ni via Deuteron Inelastic Scattering

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An investigation of the reaction ^{58}Ni(d, d') at $E_d = 46$ and 70 MeV has demonstrated significant advantages in studying isoscalar giant resonances with deuterons compared with other projectiles. The differential cross section at 70 MeV for the resonance at $E_x \approx 63A^{-1/3}$ MeV provides strong evidence for an $E2$ interpretation. A comparison of measurements in ^{58}Ni(d,d') and (p,p') provides evidence for excitation of the giant dipole resonance by protons.

The giant resonance region of the nuclear continuum has been extensively studied via inelastic scattering of electrons, protons, ^{3}He particles, and α particles.[1] The primary purpose of previous measurements has been to characterize the pronounced resonance structure at $E_x \approx 63A^{-1/3}$ MeV, which is 2–3 MeV below the well-known $E1$ giant dipole resonance.

The spin of this resonance is usually deduced by comparing measured differential cross sections with predictions of the distorted-wave Born approximation (DWBA) normalized to transition strengths based on depletion of the linear energy-weighted sum rule (EWSR).[1] Electron scattering and initial proton scattering results could not distinguish between excitation of an $E2$ giant quadrupole resonance and an $E0$ giant monopole resonance.[1] Later proton cross-section measurements[2,3] showed a preference for the $E2$ interpretation.[4] The angular distributions for 71-MeV ^{3}He ions could not distinguish between $E2$ and $E0$ excitations, but the $E2$ interpretation was preferred from the predicted EWSR strengths.[5,6] An angular distribution for 115-MeV α particles showed some preference for an $E2$ excitation, but an $E0$ excitation cannot be ruled out by the EWSR strengths.[7]

In this Letter, we report a study of the giant resonance region in ^{58}Ni using inelastic scattering of deuterons, a projectile not previously employed in such measurements. We find that three significant advantages are obtained with deuterons. (1) The cross section for the resonance structure relative to the cross section for the underlying, unstructured nuclear continuum is significantly increased compared with results for other projectiles. (2) Since, to a good approximation, the isovector $E1$ resonance is not excited by isoscalar projectiles, the resonance at $E_x \approx 63A^{-1/3}$ MeV can be isolated from the $E1$ resonance. (3) DWBA predictions suggest that the resonance differential cross section in deuteron inelastic scattering is sensitive to the transition multipolarity, particularly in distinguishing between $E2$ and $E0$ excitations.

Most of the measurements were made using a