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Space Correlation in Ion-Beam—Plasma Turbulence

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We investigate the turbulence excited by an ion beam in an unmagnetized plasma in connection with the linear theory of the ion-beam—plasma instability. Depending on beam velocity, one- or three-dimensional turbulence can be expected. These two types are experimentally observed by looking at space correlation functions.

The turbulence generated by beam-plasma interaction is known to diffuse particles in velocity space. Among the processes by which such diffusion occurs, theory[1] has pointed to the effect of unstable electrostatic waves propagating off angle from the beam velocity. Thus, whereas the E field can only be axial in one-dimensional (1D) systems, it can be oblique in 3D plasmas. The particle velocities are not only decelerated or accelerated, but also deviated.[2] This 3D effect has an important bearing on the final state of the interaction.[3] The object of this paper is to present a correlation method for measuring the structure of this random E-field turbulence. We chose the ion-beam—plasma system since its most unstable modes are off axis, unlike with the electron beam. Moreover, the occurrence of oblique propagation in this type of turbulence was seen in numerical results,[4] and observed in an experiment in which discrete frequency components[5] were correlated.

We first establish the 3D predictions of the linear theory of the ion-beam instability.[6] Experimental results obtained by correlation techniques are reported, showing two types of turbulence: plane 1D turbulence and 3D turbulence with oblique E fields.

The 3D linear instability.—The 3D dispersion relation for electrostatic waves in an unmagnetized plasma is written[7]

$$1+\sum_s \frac{\omega_s^2}{n_s k^2}\int d^3v(\omega-\vec{k}\cdot\vec{v})^{-1}\vec{k}\cdot\frac{\partial f_s}{\partial\vec{v}}=0, \qquad (1)$$

where the sum is over species s (electrons, plasma ions, and beam ions) of distribution function f_s and plasma frequency $\omega_s=(n_s e^2/\epsilon_0 m_s)^{1/2}$.

The distributions are assumed Maxwellian. A 1D solution[6] to Eq. (1) depends on four parameters: beam velocity V_b, relative density, relative temperature, and plasma-ion-to-electron temperature ratio. The 3D analysis introduces two new parameters: the perpendicular temperatures of the beam and of the plasma ions. Fortunately, the double-plasma-machine[8] method of ion-beam formation provides a useful simplification. The ions of the source plasma are assumed to have the same density and temperature as the plasma ions. Accelerating the source ions between two plane grids changes their parallel distribution into a positive tail of a truncated Maxwellian,[6,9] with uniquely defined density, velocity, and temperature, while the perpendicular temperature is unchanged. The electron-to-plasma-ion temperature ratio and the beam velocity are the only parameters left. Once these are known, Eq. (1) can be solved for each orientation (and modulus) of $\vec{k}$ to give the frequency and growth rate of unstable modes.

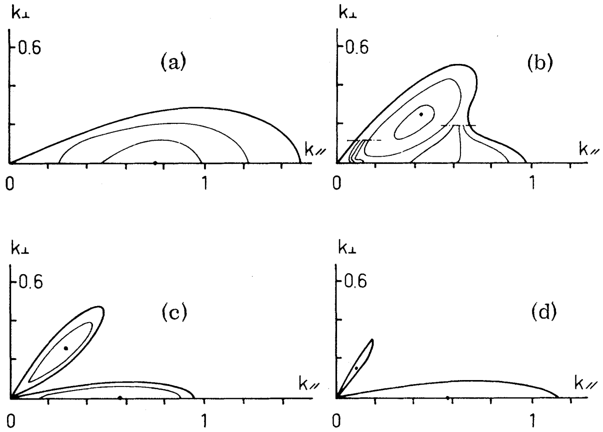

FIG. 1. Growth-rate contours of the ion-beam–plasma instability in a two-dimensional wave-vector space. k vectors are normalized to the Debye wave number. Electron–to–plasma-ion temperature ratio is 5, ion mass is 40 (argon). The most unstable wave is indicated by a dot; outer contours correspond to zero growth. (a) $V_b = 1.15$ (times the ion-acoustic speed C_s); growth rates γ (normalized to the ion-plasma frequency) on maximum 9.3×10^{-2}; contours for $\gamma = 8 \times 10^{-2}$ and 5×10^{-2}. (b) $V_b = 2.1$; $\gamma_{max} = 4.5 \times 10^{-2}$; contours for $\gamma = (4, 2, 1, 0.5, 0.1) \times 10^{-2}$. (c) $V_b = 2.5$; off-axis lobe, $\gamma_{max} = 2.2 \times 10^{-2}$, contour for $\gamma = 10^{-2}$; on-axis lobe, $\gamma_{max} = 6 \times 10^{-4}$, contour for $\gamma = 4 \times 10^{-4}$. (d) $V_b = 3.5$; off-axis lobe, $\gamma_{max} = 3.7 \times 10^{-3}$; on-axis lobe, $\gamma_{max} = 9.2 \times 10^{-4}$.

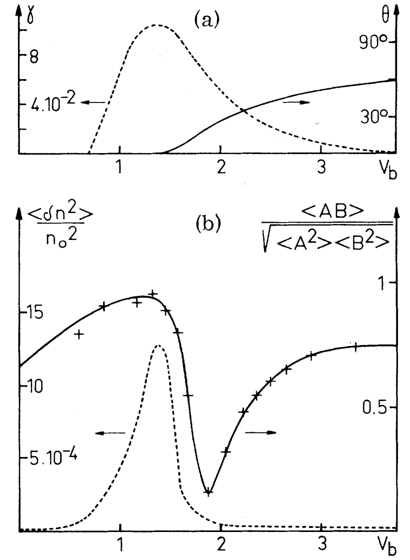

FIG. 2. Unstable modes of linear theory compared with turbulence characteristics, as a function of V_b. (a) Theoretical results on the most unstable wave: full line is the emission angle θ (right scale); dashed line, the growth rate γ (left scale). (b) Experimental results on the correlation coefficient at radial distance $x = 6$ mm (full line, right scale); and the mean square density fluctuations at maximum (normalized to the square of the mean density) (dashed line, left scale). Correlation coefficient falls from 0.96 to 0.16 when θ rises from 0° to 20° (and V_b from 1.35 to 1.9).

For $T_e/T_i = 5$, in argon, four such solutions are shown in Fig. 1 as growth-rate contours in the $(k_\parallel, k_\perp)$ plane. Wave numbers $\vec{k}$, growth rates γ, and V_b are normalized to the Debye wave number $(ne^2/\epsilon_0 T_e)^{1/2}$, ion-plasma frequency $(ne^2/\epsilon_0 m_i)^{1/2}$, and ion-acoustic velocity $C_s = (T_e/m_i)^{1/2}$, respectively. The points indicate the $\vec{k}$ vector extremities of the most unstable waves (largest growth rate γ). The outer contour is the limit for marginal stability ($\gamma = 0$).

As V_b increases, the instability starts when $V_b = 0.7$, and is concentrated on the axis. When $V_b = 1.15$ [Fig. 1(a)], the maximum growth rate is 9.3×10^{-2} on axis [contours shown are for (8 and 5)$\times 10^{-2}$], and the unstable k's have an half-angular spread of 22°. This is the ion-ion hydrodynamic instability, whose phase velocity is close to 1. This type of instability behaves more or less like Cherenkov emission of ion-acoustic waves by a supersonic beam.[10] As V_b rises above 1.4, the most unstable wave number becomes oblique, and the emission angle θ increases to 30° at $V_b = 2.1$ [Fig. 1(b)], to 42° at $V_b = 2.5$ [Fig. 1(c)], and to 56° at $V_b = 3.5$ [Fig. 1(d)]. At relatively large beam velocities [Figs. 1(c) and 1(d)], an extra, axial lobe develops. This is the electron-ion kinetic instability. Its growth rate is smaller than for the corresponding ion-ion instability, and the phase velocity identifies it as the slow-beam mode.[11] The growth rates γ of the most unstable modes (dashed line), and their emission angles (full line), are plotted in Fig. 2(a), as a function of V_b. γ is positive for $V_b > 0.7$, and reaches a maximum value of 0.104 when $V_b \simeq 1.4$. The angle θ is an increasing function of V_b starting from zero at $V_b = 1.4$.

Spatial structure of the turbulence.—The experiment is performed in the multipole[12,13] double-plasma[8] device [diameter of homogeneous plasma 10 cm; argon pressure 4×10^{-4} Torr, $T_i = 0.1$ eV, $T_e = 0.5$–1 eV, $n = (5$–$10) \times 10^8$ cm^{-3}]. The noise characteristics are measured with two spherical probes of 0.7 mm diam. One of these probes (A) can be moved along the z axis (direction of beam velocity) while the other (B) can move radially (x axis). They are positively biased with low-impedance circuits; the collected electron current in the saturation branch serves as a measure of the density. The signals are read and ac amplified in identical, wide-band circuits, and fed into a multiplier. A low-pass filter then provides the mean value of the product: $\langle AB\rangle$. Also, connecting the same signal (A or B) to both multiplier inputs provides $\langle A^2\rangle$ and $\langle B^2\rangle$. The correlation coefficient can thus be calculated:

$$C=\langle AB\rangle/(\langle A^2\rangle\langle B^2\rangle)^{1/2}. \tag{2}$$

In Fig. 2(b), the two probes were set at the same distance from the beam source, $z_0=36$ mm, but $x=6$ mm apart. The dashed line is the mean square density fluctuation $\langle\delta n^2\rangle$, related to the square of the mean density n_0. It behaves much like the predicted maximum growth rate [Fig. 2(a)]. From the background noise level (rms density fluctuation of 5×10^{-3}), it starts to rise at $V_b \simeq 0.5$, reaches a maximum (rms fluctuation of 3.6×10^{-2}) when $V_b=1.4$, and then decays down to its initial value.

The most instructive result is the correlation coefficient C (full line) between two points in the same z plane. When $\theta=0°$ [Fig. 2(a)], C is very close to 1, as expected for a one-dimensional spectrum of waves, all propagating along the z axis. But when θ is predicted to increase from 0° to 20°, C falls down sharply to 0.16: This destroyed coherence characterizes a three-dimensional turbulent spectrum. The further increase of C merely connects it to the background noise value.

In the conditions of 3D turbulence, C is a rapidly decreasing function of the radial distance x between the two probes: It decays from 1 to 0.2 within a distance of the order of the wavelength.[14]

For 1D turbulence, a typical structure is shown in Fig. 3, where the beam velocity was 1.35. The upper plot is the square of the relative-density fluctuation, as a function of z: Starting from the source, it undergoes spatial growth by a factor of 10^2 before reaching saturation and subsequent decay. The correlation function $\langle AB\rangle$ is plotted as a function of the relative probe position (a vector of components z along the beam velocity direction, and x across). $\langle AB\rangle$ is independent of x, except for $x>7$ cm (the plasma edge). The correlation coefficient (2) has a value close to unity on the $z=0$ line, i.e., when the two probes lie in the same plane parallel to the grid. Then, by Schwartz's theorem, the random density (or potential) functions of time, at any two points in such a plane, are linearly dependent. The fluctuations there have the same phase, and propagation is thus one dimensional, along the z axis. Furthermore, the space-time correlation function identifies the dispersion relation of these fluctuations as being the ion-ion hydrodynamic unstable mode.[14] Space-Fourier transforming Fig. 3 results in the spectral density function. This spectral density peaks on the $k_\parallel$ axis ($k_\perp=0$), with a maximum at $k_\parallel=0.25k_D$ and a relative half-width of 0.3.

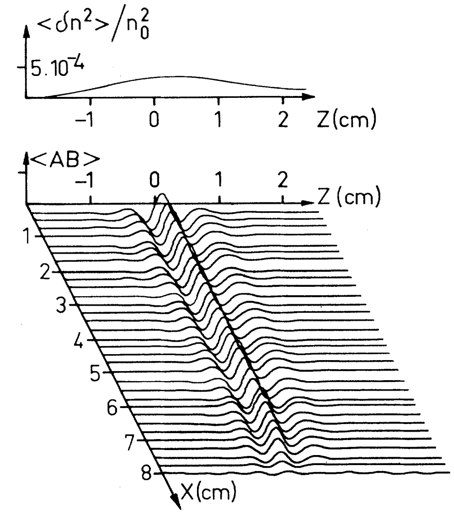

FIG. 3. Spatial correlation function in a plane turbulence. V_b is 1.35. Upper plot is the mean square relative density fluctuation along the axis. Correlation coefficient is conserved across the plasma radius, and close to 1 when $z=0$. This correlation function gives an accurate idea of a typical wave pattern.

The ion-beam–plasma turbulence thus follows rather closely the characteristics predicted from its linear unstable modes. Agreement is particularly good for the onset of instability and on- or off-axis emission of the most unstable modes. When the turbulence is three-dimensional, random E fields perpendicular to the axis can diffuse beam particles in velocity space by deflecting their trajectories. The space correlation method which we describe could be a very efficient way to look at this heating process.

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Dispersion Theory Approach for the Critical Correlation Function

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A one-parameter scaling function $s(x)$ of $x = k\xi$, where ξ is the correlation length and k the wave number of a density or concentration fluctuation in a fluid, is derived from a phenomenological field theoretic approach. On the basis of a positive-definite spectral function, we exploit the requirements of a three-particle threshold and the correct asymptotic approach as $x \to \infty$. The resulting $s(x)$ is compared with the Fisher-Burford approximant obtained from numerical studies of the three-dimensional Ising model.

The anomalous-dimension critical exponent η is a measure of the deviation of the critical correlation function of a fluid from Ornstein-Zernike[1] (O-Z) or mean-field behavior.[2,3] High-precision light-scattering measurements by Lunacek and Cannell[4] gave $\eta = 0.074 \pm 0.035$ for CO_2 and recent neutron-diffraction experiments by Warkulwiz, Mozer, and Green[5] show that $\eta = 0.11^{+0.03}_{-0.02}$ for neon. These experiments make it impossible to ignore any longer deviations from O-Z behavior. Here we obtain a new expression for the correlation function of the fluctuations in a fluid at a temperature just above its critical temperature T_c. Our procedure is to make use of the spectral representation of the correlation function and some of its general features developed in a previous paper[6] devoted to the screening approximation or so-called "n^{-1} expansion."[7,8] Bray[9] has recently reviewed the status of the n^{-1} expansion. It is clear that for $n = 1$, the case at hand, we cannot use the n^{-1} expansion explicitly, but instead only as a source of ideas. We construct a one-parameter phenomenological form for the spectral weight which is positive definite, has the correct threshold, and has an asymptotic form consistent with Griffiths's theorem.[10] Then, using this, we obtain an explicit one-parameter expression for the correlation function. We conclude by comparing our form with the Fisher-Burford approximant, obtained from numerical studies of the three-dimensional Ising model.[11]

For $T - T_c > 0$, the Fourier transform of the correlation function, g, is finite for all real values of the wave number, or "momentum." But continued analytically in the complex momentum plane, g has a pole at $\sqrt{-1}$ times the reciprocal correlation length, or "mass." (We will use the field-theory analogy[12] and particle language throughout this paper.) Farther out along the negative momentum-squared axis there is a cut which begins at the branch point corresponding to the three-particle production threshold. It is advantageous to work with $g^{-1}(z)$, where z is the momentum normalized such that the branch point occurs at $z^2 = -1$. This converts the pole at $z^2 = -\frac{1}{9}$ into a zero, which does not contribute to the