ticle density in two dimensions using the complete dispersion relation for surface waves [Eq. (2)]. There are two temperature regimes for a given thickness, one corresponding to each term in the dispersion relation, and in these two limits our calculation agrees with those of Kuper and Padmore. For all the film thicknesses and temperatures realized in our experiments, this calculation produces Kuper's $T^{5/3}$ dependence as opposed to the observed T^3 dependence.

We have considered only gapless excitations in our calculation. Padmore[13] has done the Feynman-Cohen roton calculation for rotons in two-dimensional films and finds that the energy gap is reduced 2 to 3 deg below that for bulk rotons. However, we see no exponential contribution to ρ_n between 0.2 and 0.7 K for any of our films.

We are faced with two problems. One, what sort of excitation produces the observed T^3 temperature dependence in the areal excitation density? Two, why do we not see Kuper's surface-wave excitations? The T^3 behavior clearly indicates two-dimensional, phononlike excitations. In our case the natural velocity for these excitations in the long-wavelength limit is the measured third-sound velocity. However, this velocity depends strongly on film thickness [(Eq. (3)] while the excitation velocity we obtain from our measured values of α and Eq. (9) (76 ± 2 m/sec) is nearly independent of film thickness.

The films seem to behave as though their surface is rigid for short-wavelength (100 Å) thermal excitations while remaining mobile for the much longer-wavelength (1 cm) third-sound waves. If so, we must consider a two-dimensional phononlike excitation, perhaps a compressional wave between the surface and the substrate, with a velocity approximately $\frac{1}{3}$ the first-sound velocity for bulk liquid helium. This velocity is also above the maximum third-sound velocity which can be attained on argon. A partial answer to these puzzles is that the excitation we are seeing might be nonhydrodynamic.

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Frequency and Temperature Dependence of Sound Velocity in Liquid Helium-4: A Test of the Phonon Dispersion Relation*

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Changes $\Delta C/C$ in sound velocity C in liquid helium-4 over the frequency range 1 to 15 MHz were studied as a function of temperature for $0.45 \geq T \geq 0.1$ K. The experimental results are in qualitative agreement with predictions of positive phonon dispersion for small momentum values.

Earlier experiments by Whitney and Chase,[1] who measured the changes ΔC in the velocity C of sound in liquid He^4, indicated that at frequencies lower than 12 MHz, the velocity change $\Delta C/C$ as a function of temperature increases with increasing frequency over the temperature range

0.1–0.6 K. At higher frequencies, however, it has been subsequently demonstrated that $\Delta C/C$ was a decreasing function of frequency in the same temperature range.[2]

Recent results of Roach *et al.*[3] suggest an even more complicated dependence of the velocity on frequency and temperature. They have reported that below approximately 0.2 K $\Delta C/C$ increases with increasing frequency over the frequency range 12 to 105 MHz, while above 0.2 K $\Delta C/C$ decreases with increasing frequency. Within the framework of then existing theories, a reversal in the frequency dependence of the change in sound velocity was inexplicable.

Maris and Massey[4] proposed that the discrepancy between the results of ultrasonic attenuation measurements and theory could be resolved by a reevaluation of the shape of the phonon dispersion curve. Previously the dispersion curve had been generally thought to have the form

$$E = C_0 p(1 + \gamma p^2 + \ldots), \quad (1)$$

where γ is a negative quantity. Maris and Massey explored the consequences of assuming that γ is a positive quantity. Under this assumption Maris calculated the attenuation and velocity change as a function of frequency at 0.35 K.[5] His results suggest that indeed $\Delta C/C$ should have a complicated behavior exhibiting a maximum and minimum as a function of frequency.

Since previous measurements in the low-frequency region were not sufficient to determine the validity of this calculation, we carried out experiments with improved accuracy in the temperature range below 0.45 K. In particular, we wish to report measurements of $\Delta C/C$ made at the frequencies 1, 3, 5, 9, and 15 MHz under saturated vapor pressure in the range $0.45 \geq T \geq 0.1$ K. Maris has since completed a more extensive computation of the velocity change,[6] covering several additional temperatures, which allows a comparison of calculated and experimental results on the temperature dependence of the sound velocity.

The sample cell that was used for our experiments consisted of a cylindrical chamber sealed at one end with a soldered cap. The remaining end was sealed by means of a demountable plate utilizing an indium O-ring seal, facilitating changes of transducers. The measurements were performed using a pair of quartz transducers separated by a 5-cm-long copper-plated Pyrex spacer, whose ends were ground flat and parallel to within better than 2×10^{-4} rad. An annular space surrounding the cell served as the mixing chamber of a dilution refrigerator which cooled the sample below 80 mK. A germanium thermometer was mounted on the outside of the chamber where the temperature was monitored. The procedure followed in making a measurement was to cool the sample to below 80 mK and then, as the temperature was raised, to record the changes in the velocity with respect to the value obtained at the lowest temperature. At a temperature for which the velocity was to be measured, the temperature was stabilized for the duration of the velocity measurement (10–15 min). After reaching the highest temperature for which the measurement of velocity was desired (sometimes limited by attenuation), the refrigerator was allowed to cool the sample to its initial temperature in order to verify whether there were any significant drifts in the velocity measuring system.

The measuring system used was based on a pulsed ultrasonic interferometric technique developed by Blume.[7] Some modifications of the basic Blume design have been made; however, the system still utilizes changes in the frequency of a cw oscillator to measure changes in the ultrasonic velocity. The measurement of changes in velocity has the advantage of being easily recorded and accurate, but there is one minor disadvantage. As the frequency is changed, any narrow-band component of the system will cause an apparent (spurious) change of velocity. With the transducer configuration used, the transducers are essentially unloaded, which results in a Q (quality factor) between 100 and 200, making them the narrowest band element of the system. By means of a known delay introduced into the system, through the use of a delay line, the effect of all the tuned components can be determined. We have found, using the first through-transmission echo, that the measurements of apparent velocity changes were systematically low by approximately 10%, the exact value of the correction depending on the particular set of transducers used. The measurements that we show on our graphs have therefore been corrected upward by this amount. It is important to note that since the transducers are the bandwidth-determining element, this correction is a multiplicative one, and is constant for all harmonics of a particular set of transducers. Thus this correction cannot affect the qualitative frequency dependence of the measurements.

We have utilized two sets of transducers with

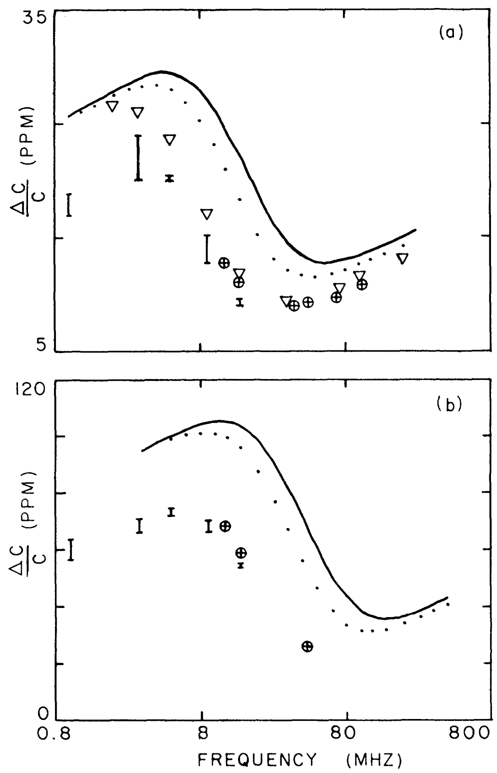

FIG. 1. Change in velocity of sound $\Delta C/C$ in helium-4 as a function of frequency. (a) $\Delta C/C$ at 0.25 K; (b) $\Delta C/C$ at 0.35 K. (Note difference in vertical scales). Vertical bars show experimental results of present work; crosses in circles show experimental results of Roach *et al.* (Ref. 3). Continuous line and dotted line show calculated results of Maris (Ref. 6) with values for γ of 8×10^{37} and 10×10^{37} cgs, respectively [see Eq. (1) in the text]. The inverted triangles are the results of previously unpublished calculations by Maris with $\gamma = 15\times 10^{37}$.

the fundamental frequencies of 1 and 3 MHz. Thus, two sets of overlapping data covering (a) 1, 3, and 5 MHz and (b) 3, 9, and 15 MHz were obtained. At each frequency several scans of velocity versus temperature were performed with each set of transducers being cycled from room temperature at least twice. The scatter in the results within a scan, due to the uncertainty in the measurement of the frequency shift, was less than the scatter between scans, so that the uncertainty in the measurements can be taken to be the scatter between scans. In Fig. 1 the data are plotted as a function of frequency at two different temperatures, so that the comparison with the theory of Maris, especially with regard to the occurrence of a maximum in $\Delta C/C$ in the frequency range studied, will be straightforward.

In addition to the present results, Fig. 1 also shows the results of Roach *et al.*[3] It should be noted that the calculations of Maris, as well as the measurements of Roach *et al.*,[3] are at constant density. The present data have, therefore, been corrected to constant density through the Grüneisen relation, using 2.84 as the value of the Grüneisen constant. Furthermore, since neither the calculated results nor the experiments of other authors are at precisely the same temperatures as those obtained in this experiment, the comparisons are made with points determined by interpolation through the use of a T^4 dependence of $\Delta C/C$.

Examination of the plotted data shows that there is indeed a maximum in the velocity change as a function of frequency. For all temperatures studied, the position of the maximum shifts to higher frequency as the temperature is increased. The frequency intervals at which measurements were made are too large, however, to specify precise frequencies at which the maxima occur. The above observations regarding the shape of the curves of velocity change versus frequency and the temperature dependence of the maxima are in qualitative agreement with the calculations of Maris.

Quantitative agreement between these results and those of other authors is fair. Small differences can be explained by an uncertainty in the correction factor that was applied to the data, and/or to differences between temperature scales used. Since the velocity change has an approximate fourth power dependence on temperature, a few percent error in temperature can result in a larger error in velocity.

The fact that the measured values of the velocity change lie below those calculated previously from theory suggests the possibility that γ in Eq. (1) may be larger in magnitude than the value used in the calculation. In order to check this possibility, $\Delta C/C$ was calculated using Maris's theory, with $\gamma = +15\times 10^{37}$ in cgs units, instead of the previously used values of $+8\times 10^{37}$ and $+10\times 10^{37}$ in cgs units. This calculation was done for $T = 0.25$ K, and for nine frequencies, including those covered in the present experiments. The results are in close agreement with the measured values of $\Delta C/C$ though still systematically somewhat higher, indicating that γ may be even larger. It is also noted that $\gamma = +15\times 10^{37}$ is not inconsistent with the interpretation of Whitworth's viscosity measurements.[8] Furthermore, although the γ found here is substantially larger than the value of $\approx 4\times 10^{37}$ cgs originally obtained from specific-heat measurements,[9] it is well within the range of the values deduced in a subsequent analysis of the same (specific-heat) data.[10] In conclusion, we note that the present results add further support to the resonancelike behavior of $\Delta C/C$ predicted by Maris,[11] and permit one to bracket more closely the value of γ to be used in the *positive* dispersion relation for small phonon momentum values in helium-4.

The authors had frequent discussions with H. J. Maris concerning his calculations and the use of his computer programs in relation to the present work.

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