gut and L. Susskind, "Hamiltonian Formulation of Wilson's Lattice Gauge Theories" (to be published); L. Susskind, "Quark Confinement in Gauge Theories of Strong Interactions," in Lectures given at the Bonn Summer School, 1974 (to be published).

[12]J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[13]*Handbook of Mathematical Functions*, edited by M. Abramowitz and J. A. Stegun (National Bureau of Standards, Washington, D. C., 1964).

[14]J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, Mass., 1973), Vol. II.

[15]H. T. Nieh, T. T. Wu, and C. N. Yang, Phys. Rev. Lett. 34, 49 (1975).

COMMENTS

ϵ-Expansion Solution of Wilson's Exact Renormalization-Group Equation*

Geoffrey R. Golner and Eberhard K. Riedel

Department of Physics, Duke University, Durham, North Carolina 27706

(Received 18 November 1974)

The ϵ-expansion formula for the critical exponent η derived by Shukla and Green from Wilson's exact renormalization-group equation is found to yield the standard result $\eta = \frac{1}{54}\epsilon^2 + O(\epsilon^3)$, *independent* of the choice of the incomplete-integration function. The result is obtained analytically in the limit $a \to \infty$ and numerically for arbitrary a, where a denotes the normalization constant of the Gaussian fixed-point functional.

Shukla and Green[1] have studied Wilson's exact renormalization-group equation[2] by using the ϵ-expansion technique about $d=4$ dimensions. The authors found an expression for the critical exponent η which, they conclude, may violate the universality hypothesis, since it apparently depends on redundant parameters[3] of the calculation. Here we show that their result for η in fact is a universal quantity independent of redundant parameters and equal to the standard result for the continuous-spin Ising model, $\eta = \frac{1}{54}\epsilon^2 + O(\epsilon^3)$.[4,5]

Shukla and Green's ϵ expansion starts from Eq. (11.17) of Ref. 2. If we choose their incomplete-integration cutoff function $\beta(k)$ to have the general form

$$\beta(k) = ck^n, \tag{1}$$

then, at the Gaussian fixed point, u_{20}^* becomes

$$u_{20}^*(k) = \frac{ak^2}{ak^2 + \exp[-(2c/n)k^n]}, \tag{2}$$

where a is the arbitrary kinetic energy normalization. The quantities $\Psi(k)$, $g(k)$, and A defined in Shukla and Green's Letter[1] then become

$$\Psi(k) = (1+ck^n)\frac{\exp[-(2c/n)k^n]}{\{ak^2 + \exp[-(2c/n)k^n]\}^2}, \tag{3}$$

$$g(k) = \tfrac{1}{2}\{ak^2 + \exp[-(2c/n)k^n]\}^{-1}, \tag{4}$$

$$A = a^2/3\pi^2. \tag{5}$$

Introducing the functions

$$G(k) = 2\int d^4q\, \psi(q) g(|\vec{k}+\vec{q}|) \tag{6}$$

and

$$H(k) = 2\int d^4q\, \psi(q)\int_0^{|\vec{k}+\vec{q}|} \frac{dq'}{q'}[G(q') - G(0)], \tag{7}$$

we obtain for the coefficient of the ϵ^2 term in the expression of the critical exponent η, $\eta = \eta_2\epsilon^2 + O(\epsilon^3)$,

$$\eta_2 = -(A^2/a)H''(0), \tag{8}$$

where $H''(0) = [d^2H(k)/dk^2]_{k=0}$. The question is whether the apparent dependence of this result on the normalization parameter a and the cutoff function $\beta(k)$ is a spurious one.

The discussion is based on the expression for $H''(0)$ below, obtained from Eq. (7) by performing the differentiation with respect to k and the integration over the angular variables:

$$H''(0) = -\pi^2\int_0^\infty dq\, q^2\psi'(q)[G(q) - G(0)]. \tag{9}$$

With transformation to the variable $x = aq^2$, the substitutions

$$\psi'(x) = \hat{\psi}'(x)\, dx/dq, \quad G(q) = (\pi^2/2a^2)\hat{G}(x), \tag{10}$$

with $\hat{G}(0) = 1$ [by definition $G(0) = \frac{1}{6}A^{-1}$], and the partial integration of the $\hat{G}(0)$ term, Eqs. (5), (8), and (9) yield

$$\eta_2 = \frac{1}{18}\left[1 + \int_0^\infty dx\, x\hat{\psi}'(x)\hat{G}(x)\right]. \tag{11}$$

It is not hard to evaluate the expression for $\hat{G}(x)$ and the integral in Eq. (11) analytically in the limit where a tends to infinity. Then all dependence on the cutoff function $\beta((x/a)^{1/2})$ is trivially eliminated. To leading order in $1/a$ one obtains

$$\hat{G}(x) = [x(4+x)]^{-1/2} \times \ln\left|\frac{x(3+x)+(1+x)[x(4+x)]^{1/2}}{x-[x(4+x)]^{1/2}}\right|, \tag{12}$$

$$\hat{\psi}'(x) = -2/[1+x]^3. \tag{13}$$

The value of the integral in Eq. (11) is $\frac{2}{3}$, which implies

$$\eta = \frac{1}{54}\epsilon^2 + O(\epsilon^3), \tag{14}$$

in agreement with the result obtained previously by other techniques.[4,5]

For general values of the parameter a Eqs. (6) and (11) can be evaluated numerically. We have done this for two choices of the cutoff function: $\beta(k) = 2k^2$ and $\beta(k) = k^4$. In each case the result of Eq. (14) is obtained. Since there is nothing special about these choices of $\beta(k)$ we conclude that, as expected, η is independent of the redundant parameters present in Wilson's incomplete-integration renormalization-group approach.

*Work supported in part by the National Science Foundation through Grant No. GH-36882.

[1]P. Shukla and M. S. Green, Phys. Rev. Lett. 33, 1263 (1974). We thank the authors for sending us a report of this work prior to publication.

[2]K. G. Wilson and J. Kogut, Phys. Rep. 12C, 75 (1974).

[3]F. J. Wegner, J. Phys. C: Proc. Phys. Soc., London 7, 2098 (1974). See also G. Jona-Lasinio, in *Collective Properties of Physical Systems,* edited by B. Lundqvist and S. Lundqvist (Academic, New York, 1973), p. 38.

[4]K. G. Wilson, Phys. Rev. Lett. 28, 548 (1972).

[5]F. J. Wegner and A. Houghton, Phys. Rev. A 8, 401 (1973).

ERRATUM

NONLINEAR OPTICAL PROCESSES BY VAN DER WAALS INTERACTION DURING COLLISION. S. E. Harris and D. B. Lidow [Phys. Rev. Lett. 33, 674 (1974)].

The following papers whose content partially anticipates and overlaps that of our recent Letter have been brought to our attention: L. I. Gudzenko and S. I. Yakovlenko, Zh. Eksp. Teor. Fiz. 62, 1686 (1972) [Sov. Phys. JETP 35, 877 (1972)]; S. I. Yakovlenko, Zh. Eksp. Teor. Fiz. 64, 2020 (1973) [Sov. Phys. JETP 37, 1019 (1973)].