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Nonlinear Evolution of Stimulated Raman Backscatter in Cold Homogeneous Plasma

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We present an analytic solution for Raman backscatter in a cold homogeneous plasma, valid until saturation by breaking of the longitudinal waves, or by self-consistent depletion of the pump. We give quantitative values for the pump strength necessary for the former to occur first, and the transmission at saturation. Nonlinear particle bunching causes growth of electromagnetic sidebands. In quarter-critical plasma, a sideband at $\omega=\frac{3}{2}\omega_0$ reaches 8% of the pump intensity at wave breaking.

Two phenomena of the radiation-plasma interaction, beat heating[1-3] and the stimulated-Raman-scattering (SRS)[4-9] instability, are based on resonant excitation of a longitudinal electron mode by the beat between two electromagnetic waves. When the cold-plasma approximation[6,10,11] is valid, i.e., when the thermal velocity of an electron is smaller than its directed velocity, the excited longitudinal modes are resonant plasma oscillations.[11] Using Lagrangian methods,[10-13] we can then follow the course of these phenomena analytically until wave breaking[10] occurs, thus exposing the physics directly. The analysis fails at that point, but cold-plasma computer simulations[6] show that growth of SRS stops when the longitudinal waves break. Thus, the analysis is valid until saturation begins.

Here we report some results of a study of Raman backscatter in a cold, infinite homogeneous plasma. The physical system is shown in Fig. 1: Two counterpropagating electromagnetic waves, one a high-intensity pump at frequency ω_0 [Fig. 1(a)], the other a reflected wave at frequency $\omega_0-\omega_p$, growing from the noise [Fig. 1(b)], beat, thus exciting, via $\vec{V}\times\vec{B}$ forces in the longitudinal direction [Fig. 1(c)], a resonant oscillation in the ambient plasma. In the course of this oscillation, the electrons undergo a density perturbation Δn, while at the same time they quiver transversely (with velocity v) in the electric field of the electromagnetic waves. The phase of the resulting incremental current $-ev\Delta n$ is such that it resonantly adds energy to the reflected wave at the expense of the pump wave.[5] As the amplitude of the plasma oscillation increases, the density perturbations steepen nonlinearly [Fig. 1(d)] and the incremental number density contains growing components at all the harmonics of the beat wave.

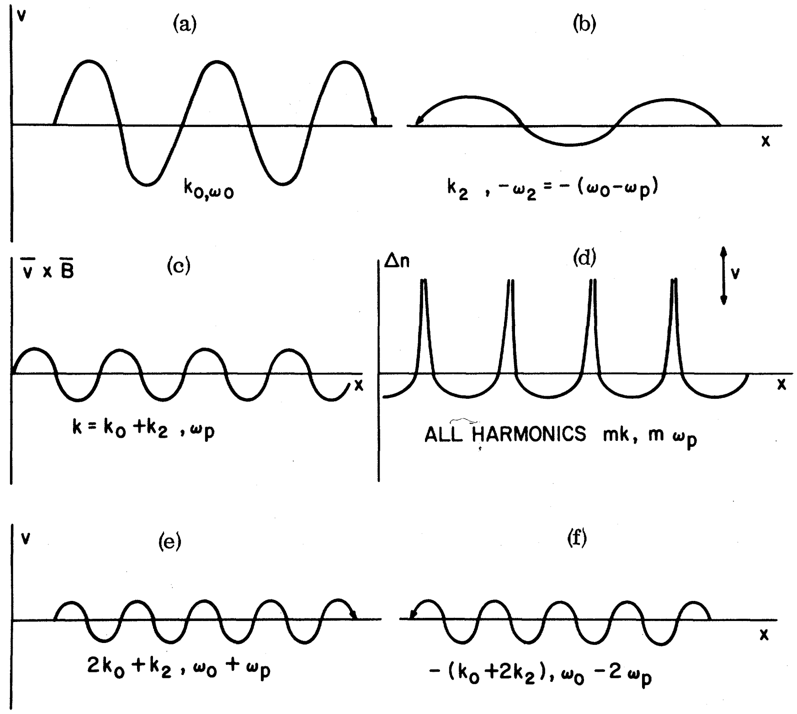

FIG. 1. The major waves present in stimulated Raman backscatter: (a) pump wave and (b) reflected wave quiver velocities (not to scale); (c) the longitudinal resonant $\vec{V} \times \vec{B}$ force due to the beat wave; (d) the incremental number density in the nonlinear plasma oscillation (the bunched electrons move transversely with V); (e) the first upper and (f) the first lower sidebands for $\omega_0 \gg \omega_p$ (at quarter-critical density the lower sidebands have zero or positive phase velocity).

The associated incremental currents then feed energy into electromagnetic sidebands which are separated from each other, and from the incident and scattered waves, by multiples of the beat wave number and the plasma frequency [Figs. 1(e) and 1(f)].

Interest in SRS has largely centered on the absolute instability[7,8] at its upper density limit (quarter-critical density $\omega_p = \frac{1}{2}\omega_0$) in inhomogeneous plasma.[14] It is recognized that the dynamics of quarter-critical plasma is significant to laser-pellet fusion because large ion disturbances, affecting the blowoff of the underdense plasma, occur there.[6,7] At quarter-critical density, SRS competes with an absorptive instability, two-plasmon decay.[4] If planned measurements of the scattered radiation at $\frac{1}{2}\omega_0$ and $\frac{3}{2}\omega_0$ in future pellet-irradiation experiments[15] are to be used in determining the winner of this competition, the radiation at these frequencies generated by each of these mechanisms must be predicted. A recent computer simulation by Biskamp and Welter[9] shows radiation reflected and transmitted in lines at multiples of $\frac{1}{2}\omega_0$, due to SRS in underdense inhomogeneous plasma.

In the present analytical study of SRS in homogeneous plasma, we show that these anti-Stokes lines or sidebands are caused by nonlinear steepening of the resonant electron-plasma oscillations. We give a quantitative estimate of their intensity which is in fair agreement with numerical results for inhomogeneous plasma.[9] The methods used here yield physical and analytic insights necessary to the solution of the inhomogeneous problem. From the theoretical point of view, it is most interesting that the plasma oscillation can be described by a Lagrangian equation (in terms of t and initial particle position x_0) and the electromagnetic waves by an Eulerian equation (in terms of t and x), both of which have linear kernels, and that the exact solution of these equations differs from a linear solution (in which no distinction is made between x and x_0 as *independent* variables) by at most 12% at wave breaking. Dephasing terms, such as the relativistic corrections found by Rosenbluth and Liu[2] for the case of beat heating by co-propagating electromagnetic waves, or corrections due to transfer of energy to nonresonant modes (e.g., to the sidebands) do not have significant effect in the present case. The details of the calculation leading to this negative conclusion will be presented in a more complete paper.

Growth of the instability proceeds at approximately the linear rate until wave breaking, provided the pump wave (which we treat self-consistently) is intense enough. We calculate the minimum pump strength necessary for saturation to occur through wave breaking rather than pump depletion, and the associated pump transmission in the strong-pump case. The zero-order solution which gives these results can be used to obtain expressions for the amplitudes of the anti-Stokes lines. In homogeneous plasma these are "near" electromagnetic fields—their frequencies and wave numbers do not satisfy the dispersion relation for electromagnetic radiation. They can be expected to couple to both transmitted and reflected radiation in inhomogeneous plasma.[9] We find that the most intense sideband is one at frequency $\frac{3}{2}\omega_0$ which occurs in quarter-critical plasma. This near field has positive phase velocity and must thus be compared with the first transmitted line of Ref. 9.

Although the electrons move both longitudinally and transversely, the backscatter problem is essentially one-dimensional since the fields and plasma properties vary only in the direction of propagation of the electromagnetic wave (the x direction). The longitudinal force due to the electromagnetic waves is $(\vec{V}\times\vec{B})_x$; using $v=eA/mc$, where v is the transverse electron velocity and A the vector potential, we can write the equation governing the cold-plasma oscillations[2] as

$$\ddot{\delta}+\omega_p{}^2\delta=-v\,\partial v/\partial x. \tag{1}$$

Here $\delta(x_0,t)=x-x_0$ is the displacement of a particle from its equilibrium position (x_0) and the time derivative is Lagrangian (at constant x_0). Equation (1) holds for inhomogeneous plasma, provided the displacement is small compared with the scale length. In the homogeneous limit considered here the scale length is taken to be infinite. The electromagnetic wave obeys

$$(\partial^2/\partial t^2+\omega_p{}^2-c^2\partial^2/\partial x^2)v=\omega_p{}^2 v\,\partial\delta/\partial x, \tag{2}$$

where the right-hand side is the incremental electron current $-ev\Delta n_e$. To obtain the term in this form we have used the continuity equation $n_e\,\partial x/\partial x_0=N$, where N is the fixed background ion density, $\omega_p{}^2=4\pi Ne^2/m$. The time derivative here is Eulerian (at constant x).

Raman backscatter can be represented by taking the resonant parts v to be of the form

$$\tfrac{1}{2}[V_0T(t)\exp(ik_0x-i\omega_0t)+v_0R(t)\exp(ik_2x+\omega_2t)+\text{c.c.}],$$

where $\omega_2=\omega_0-\omega_p$, $v_0\ll V_0$, (k_0,ω_0) and (k_2,ω_2) satisfy $\omega^2=\omega_p{}^2+c^2k^2$, and R and T are slowly varying functions of time over a plasma period. The resonant part of the driver in Eq. (1) is then proportional to $\exp(i\lambda)$, where $\lambda=kx-\omega_pt=kx_0-\omega_pt+k\delta$ and $k=k_0+k_2$. The nonlinear occurence of the unknown δ in the driving term[2] results, even for $|k\delta|=1$, in small corrections (of order γ/ω_p where γ is the linear growth rate) to the sinusoidal variation $\delta\propto\exp(i\lambda_0)$, where $\lambda_0=kx_0-\omega_pt$. These corrections are among the dephasing terms mentioned above; they have no appreciable effect on the growth.

To zero order, i.e., ignoring all dephasing terms, $k\delta$ has the form[2]

$$k\delta=D(t)\cos[\lambda_0+\varphi(t)]. \tag{3}$$

The condition for wave breaking, $|\partial\delta/\partial x_0|=1$, is equivalent to $D=1$. In Fig. 1(d), we plot the electron density $n=N(1-\partial\delta/\partial x)$, at wave breaking, as it appears in Eulerian coordinates, in the beat-wave frame. It is periodic in λ but not sinusoidal; thus $\partial\delta/\partial x$ contains all the harmonics of the beat wave:

$$\partial\delta/\partial x=\sum_1^\infty imF_m(t)\exp(im\lambda)+\text{c.c.} \tag{4}$$

The Fourier coefficients, $F_m=(2\pi)^{-1}\oint d\lambda\,k\delta\times\exp(-im\lambda)$, can be found to zero order from Eq. (3), Bessel's identity

$$\exp(i\alpha\sin\theta)=\sum J_q(\alpha)\exp(iq\theta), \tag{5}$$

and the relation $d\lambda/d\lambda_0=1+dk\,\delta/d\lambda_0$ ($k\delta$ is periodic in both λ and λ_0 since the points $k\delta=0$, respectively, coincide). The result is

$$F_n=(-i)^{n-1}n^{-1}J_n(nD)\exp(in\varphi). \tag{6}$$

It is the fundamental, F_1, that beats with the pump to cause growth of the reflected wave and beats with the latter to cause depletion of the pump.[5] The components with $m\neq 1$ beat with both electromagnetic waves to produce the sidebands.

Equations (2) and (4) imply that v must have the form

$$V=\tfrac{1}{2}\{\sum_{-\infty}^{\infty} V_n(t)\exp(i[(k_0+nk)x-(\omega_0+n\omega_p)t])+\text{c.c.}\}, \tag{7}$$

where the V_n are complex. To distinguish the electromagnetic waves we shall continue to write V_0T in place of V_0 and v_0R^* in place of V_{-1}. For $n=0$ and -1, we obtain from Eqs. (2) and (7) the slow-time behavior of T and R:

$$\epsilon\omega_p\dot{T}=-2\mu^2\gamma^2R^*F_1, \tag{8}$$

$$\epsilon\omega_p\dot{R}=2\gamma^2T^*F_1, \tag{9}$$

where

$$\epsilon=\tfrac{1}{4}k^2\omega_p^{-2}v_0V_0,\quad \gamma=\tfrac{1}{4}kV_0(\omega_p/\omega_2)^{1/2},\quad \mu^2=\omega_2\omega_0^{-1}v_0^2V_0^{-2}. \tag{10}$$

The electromagnetic waves have the exact first integral, expressing conservation of action,[1]

$$\tau^2+\mu^2\rho^2=1+\mu^2, \tag{11}$$

where τ and ρ are the magnitudes of T and R, respectively, assumed to be 1 at $t=0$.

Of the quadratic terms obtained from substitution of Eq. (7) on the right-hand side of Eq. (1) for the longitudinal displacement, only the resonant driver need be kept to zero order. Following Rosenbluth and Liu,[2] we convert this from a series in kx to one in kx_0 using Eq. (5). Then

$$(\dot{D}+i\dot{D}\varphi)=\epsilon\omega_p[RTJ_0(D)\exp(-i\varphi)+R^*T^*J_2(D)\exp(i\varphi)]. \tag{12}$$

Equations (6), (8), (9), and (12) reduce to

$$\epsilon^2\omega_p^2\rho^2=\epsilon^2\omega_p^2+\gamma^2D^2, \tag{13}$$

$$\dot{D}^2=4J_1^2(D)D^{-2}(\epsilon^2\omega_p^2+\gamma^2D^2)(1-\mu^2\gamma^2D^2/\epsilon^2\omega_p^2), \tag{14}$$

where we have assumed no electrostatic noise ($D=0$) at $t=0$. A small-D expansion of Eq. (14) recovers the linear result $D\propto\sinh(\gamma t)$ so that $2J_1(1)\sim0.88$ is a measure of the decrease in the growth rate at wavebreaking ($D=1$) compared with the linear rate. The last factor in Eq. (14) is the pump strength τ^2. Growth continues up to wave breaking without pump depletion provided $\epsilon\omega_p>\mu\gamma$, i.e.,

$$V_0/c>\omega_p^{3/2}\omega_0^{-1/2}(ck)^{-1}, \tag{15}$$

and the transmission is then

$$\tau_s^2=(1-\omega_p^3/\omega_0k^2V_0^2). \tag{16}$$

This implies that an incident pump with $V_0/c<0.5(\omega_p/\omega_0)^{3/2}$ will not be transmitted through a plasma with $\omega_p\ll\omega_0$, and similarly for $V_0/c<6^{-1/2}\sim0.4$ through a quarter-critical plasma. Numerical results[6] show an incident intensity corresponding to $V_0/c\sim0.1$ to be depleted by 60% in passing through inhomogeneous plasma with density up to ~0.6 critical. The discrepancy is likely to be due to the difference between homogeneous and inhomogeneous plasma.

The sideband amplitudes (V_n, $n\neq0$ or -1) can be found from Eqs. (2), (4), (6), (7), and (9). The left-hand side of Eq. (2), after substitution of Eq. (7), is of the form $n(n+1)\Omega^2V_n$, where $\Omega^2=c^2k_0k_2+\omega_0\omega_2-\omega_p^2\geq\omega_p^2$. Because the right-hand side of Eq. (2) is proportional to ω_p^2, the sideband amplitudes are proportional to the small parameter $\nu=\omega_p^2/\Omega^2\leq1$ (numerical factors insure that the sideband amplitudes are small even at quarter-critical density where $\nu=1$). To first order in ν, we find

$$V_n=\tfrac{1}{2}i\nu[(n+1)^{-1}V_0TF_n+n^{-1}v_0R^*F_{n+1}],\quad n\neq0,-1. \tag{17}$$

Since transverse velocity is proportional to vector potential, a field V with frequency ω and wave number k has intensity relative to the *self-consistent* value of the pump field $\omega k|V|^2/\omega_0k_0V_0^2|T|^2$. If there is no dephasing, we can set $\dot{R}=\gamma R$ in Eq. (9). Then at quarter-critical density ($\nu=1$), the first upper sideband V_1 ($\omega=\tfrac{3}{2}\omega_0$, $k=2k_0$—the first transmitted anti-Stokes line of Ref. 9) has relative intensity

$$I_{3/2}=\tfrac{3}{4}J_1^2(1)[\tfrac{1}{4}+2J_2^2(2)(1-\tau^2)], \tag{18}$$

where τ is the transmission at wave breaking. This expression is valid in the strong-pump limit. For a pump strength which just depletes ($\tau = 0$), we find $I_{3/2} \sim 8.5\%$, compared with $\sim 15\%$ in the numerical results.[9]

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Superconducting Properties of the Singlet–Ground-State System (LaPr)Sn_3

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The depressions of the superconducting transition temperature T_c as a function of Pr concentration and the specific-heat jump ΔC at T_c as a function of T_c have been measured for the singlet–ground-state system (LaPr)Sn_3. The results are well represented by numerical calculations based on the Pr energy-level scheme determined from separate measurement of the Schottky heat-capacity anomaly and the Van Vleck paramagnetic susceptibility contributed by the Pr ions.

In a recent series of papers, Fulde and co-workers have developed a theory for the effect on superconductivity of paramagnetic rare-earth impurities with crystal-field–split energy levels.[1-5] According to this theory, the superconducting properties of the matrix are modified by two competing mechanisms. The first, a *depairing* mechanism, involves the usual conduction-electron–impurity spin-exchange interaction which can be operative even when the relevant impurity energy