Observation of Nonlinear Landau Damping of Broad-Band Plasma Oscillations

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Nonlinear damping of a broad-band spectrum of plasma waves has been observed. Frequencies lower than the spectrum are growing from background noise. A shift of the entire spectrum towards lower frequencies results. The evolution of the spectrum is in good agreement with the predictions of weak-turbulence theory. Measurements of the electron velocity distribution are presented.

Nonlinear damping of monochromatic waves, electron Bernstein waves,[1] electron-plasma waves,[2] and ion-acoustic waves[3] has been observed. In this Letter we report experimental observations of nonlinear damping and growth of a broad-band spectrum of electron-plasma waves excited in a plasma column, and a shift of the entire spectrum towards lower frequencies due to an unstable growth of the background noise (i.e., thermal and source noise). Results of measurements of the wave spectrum and the electron velocity distribution are presented and discussed.

The experiments were performed on a collisionless plasma column.[4] An argon plasma from a PIG discharge drifts freely down a uniform magnetic field (1.3 kG) which can be considered as infinite in intensity for the plasma densities used (1.1×10^8 electrons/cm^{-3}). The electron temperature is 3.2 eV and the background pressure is of the order of 10^{-5} Torr.

A broad-band noise spectrum is injected into the plasma column by means of a probe. The central frequency of the spectrum is near the mean plasma frequency and the relative width $\Delta\omega/\omega$ is 0.3. The correlation length is of the order of 6 cm. The amplitude of the electric field in the plasma is such that the weak-turbulence parameter $\alpha = \mathcal{E}_F/nkT \leq 0.05$, where $\mathcal{E}_F$ is the field energy density. The group velocity $v_g = 1.5 \times 10^8$ cm/sec is almost constant over the width of the spectrum while the phase velocity varies from 2.7×10^8 to 2.3×10^8 cm/sec.

The evolution of the spectrum along the plasma column, shown in Fig. 1 for a fixed value of the initial-wave energy density, is measured by connecting a movable receiving probe to a spectrum analyzer through a borad-band amplifier. For a fixed distance from the emitting probe, the evolu-

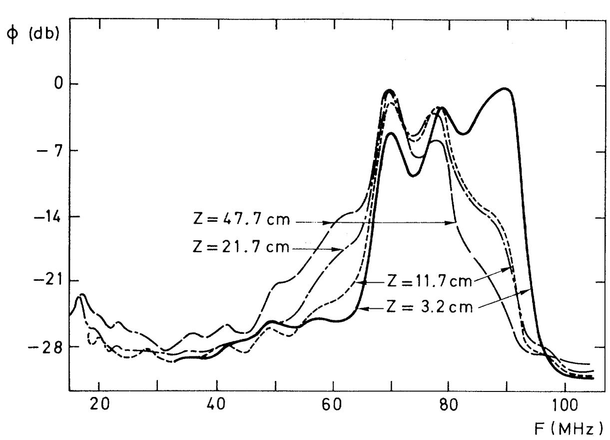

FIG. 1. Evolution of the complete frequency spectrum over the distance. Experimental conditions: $n_0 = 1.1 \times 10^8$ electrons/cm^3; $B_0 = 1.3$ kG; $T_e = 3.2$ eV; $\alpha = 0.05$.

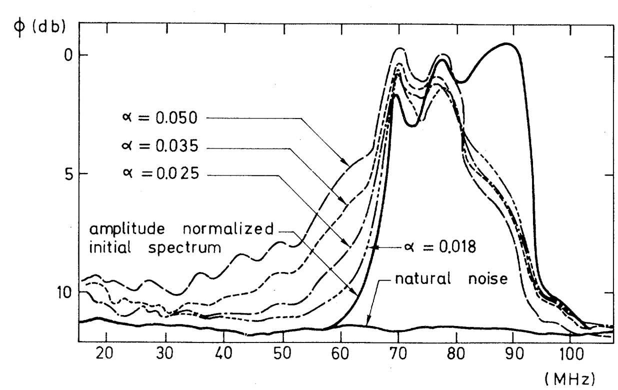

FIG. 2. Evolution of the complete frequency spectrum with wave energy. Experimental conditions: $n_0 = 1.1 \times 10^8$ electrons/cm^3; $B_0 = 1.3$ kG; $T_e = 3.2$ eV.

tion of the spectrum as a function of initial wave energy density is plotted in Fig. 2. In both cases the high-frequency part is heavily damped. Spatial growth for frequencies lower than those of the spectrum occurs. The growth rate depends on the wave energy density in the injected signal. Such results have the features predicted by nonlinear Landau damping theory. Test-wave experiments confirm that the spectrum evolution is due to nonlinear Landau damping; two test waves $(\omega_1,k_1;\omega_2,k_2)$ are injected into the spectrum and the beat wave number, k_2-k_1, is measured.

In order to compare the experimental results with theory, an expression for the spectrum evolution, taking the finite geometry into account, has been derived. For this purpose the Vlasov equation, after linearization with respect to the distribution function, is solved up to the third order[5] and the result is put into Poisson's equation to give

$$(\nabla_T{}^2-k^2)\Phi(k,\omega,m,r)-k^2N(r)[\epsilon_\infty{}^{(1)}(k,\omega)-1]\Phi(k,\omega,m,r)$$
$$-k^2N(r)\int\frac{d\omega'}{2\pi}\int\frac{dk'}{2\pi}\epsilon^{(2)}(\omega,\omega')\Phi(k',\omega',m',r)(k-k',\omega-\omega',m-m',r)$$
$$-k^2N(r)\int\frac{d\omega'}{2\pi}\frac{dk'}{2\pi}\frac{d\omega''}{2\pi}\epsilon^{(3)}(\omega,\omega',\omega'')\Phi(k-k'-k'',\omega-\omega'-\omega'',m-m'-m'',r)$$
$$\times\Phi(k',\omega',m,r)\Phi(k'',\omega'',m'',r)=0, \qquad (1)$$

where the notation is standard,[6,7] $N(r)$ is the reduced density, and $\epsilon_\infty{}^{(1)}$, $\epsilon^{(2)}$, and $\epsilon^{(3)}$ are, respectively, the linear and nonlinear dielectric constants at the center of the plasma. The differential equation

$$[(\nabla_T{}^2-k^2)+N(r)k^2\lambda]\psi(r)=0$$

is closely related to the linear part of Eq. (1). Orthonormal solutions $\psi_{km\nu}(r)$ of this equation exist for certain values of the separation constant $\lambda_{m\nu}(r)$. By use of the expansion of $\Phi(k,\omega,m,r)$ in radial eigenmodes,

$$\Phi(k,\omega,m,r)=\textstyle\sum_\nu\varphi(k,\omega,m,\nu)\psi_{km\nu}(r),$$

and by use of the orthogonality of the eigenfunctions $\psi_{km\nu}(r)$ we find

$$\epsilon_L(k,\omega,m,\nu)\varphi(k,\omega,m,\nu)$$
$$+\int\frac{d\omega'}{2\pi}\int\frac{dk'}{2\pi}\sum_{m',\eta,\mu}\epsilon^{(2)}(\omega,\omega')\varphi(k',\omega',m',\mu)\varphi(k-k',\omega-\omega',m-m',\eta)G_1$$
$$+\int\frac{d\omega'}{2}\int\frac{dk'}{2}\int\frac{d\omega''}{2}\int\frac{dk''}{2}\sum_{\substack{m',m''\\ \mu',\nu,\chi}}\epsilon^{(3)}(\omega,\omega',\omega'')\varphi(k-k'-k'',\omega-\omega'-\omega'',m-m'-m'',\mu)$$
$$\times\varphi(k',\omega',m',\nu)\varphi(k'',\omega'',m'',\chi)G_2=0,$$

where G_1 and G_2 are overlap integrals of radial eigenmodes and where $\epsilon_L(k,\omega,m,\nu)$ is the dielectric constant in finite radial geometry,

$$\epsilon_L(k,\omega,m,\nu)=\epsilon_\infty{}^{(1)}(k,\omega)+\lambda_{m\nu}(k)-1.$$

This leads to a usual integrodifferential equation for the spectrum evolution, taking account of finite geometry,

$$\frac{\partial}{\partial x}\langle\varphi\varphi^*(x,\omega,m,\nu)\rangle=-2\Gamma^L(x,\omega,m,\nu)\langle\varphi\varphi^*(x,\omega,m,\nu)\rangle$$
$$+\sum_{m',\mu'}\int\frac{d\omega'}{2\pi}W(\omega,\omega')\langle\varphi\varphi^*(x,\omega',m',\mu')\rangle\langle\varphi\varphi^*(x,\omega,m,\nu)\rangle,$$

with

$$W(\omega,\omega')=-\frac{2\pi q^4}{m_e{}^3\epsilon_0}\frac{k'^2(k-k')^4}{[(\partial/\partial k)\epsilon_{LR}(k,\omega,m,\nu)](k\omega'-k'\omega)^4}\frac{\omega-\omega'}{|\omega-\omega'|}\left(\frac{\partial f_0}{\partial v}\right)g,\quad v=(\omega-\omega')/(k-k'),$$

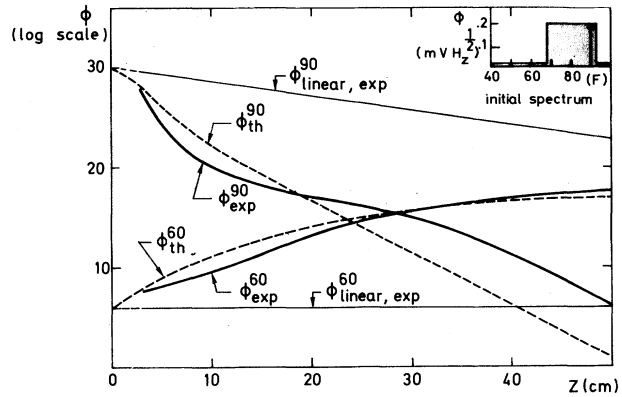

FIG. 3. Nonlinear growth and damping of two particular waves. Experimental conditions: $n_0 = 1.1 \times 10^8$ electrons/cm^3; $B_0 = 1.3$ kG; $T_e = 3.2$ eV; $\alpha = 0.032$.

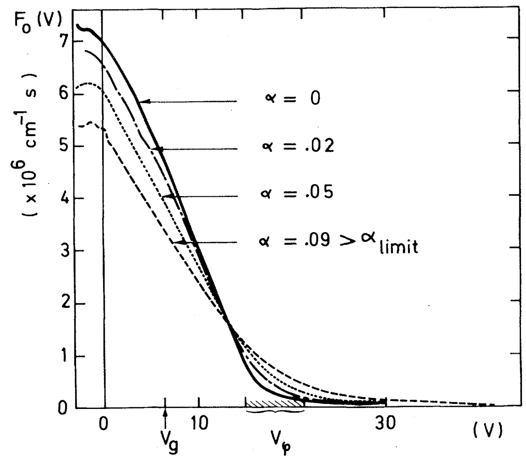

FIG. 4. Electron energy distribution evolution with wave energy. Experimental conditions: $n_0 = 1.1 \times 10^8$ electrons/cm^3; $B_0 = 1.3$ kG; $x = 56$ cm.

where the notation is standard and g is a numerical factor of the order of 0.16 involving overlap integrals of radial eigenmodes. An approximate value of $\epsilon_{LR}(k,\omega)$ can be derived from the constancy of the group velocity in the frequency range of the spectrum:

$$\epsilon_{LR}(k,\omega) = 1 - A/\omega - v_g k/\omega,$$

with $A = 2.15 \times 10^8$ sec^{-1}. Γ^L is deduced from linear propagation and the amplitude of the potential $\varphi(x,\omega,0,0)$ from power measurements and probe coupling coefficients. Then, for given initial conditions, the spectrum evolution is solved numerically. The amplitude of the spectrum is normalized to the measured value. Comparison between theory and experiment is shown in Fig. 3 for two components of the spectrum. The decay of the 90-MHz component and the growth of the 60-MHz component are in fairly good agreement with the theoretical curves.

The electron distribution function is measured with a multigrid electrostatic analyzer for different powers of the injected spectrum (Fig. 4). No significant localized disturbance in velocity space appears even around the group velocity of the waves. However the whole distribution is changed, remaining Maxwellian with a temperature increasing with the injected power. The change of the distribution evolves with the distance between the transmitter and the analyzer and this eliminates the trivial explanation of this evolution as being created by effects of the probe field. For these experiments, the autocorrelation length is of the order of the trapping length; consequently resonance broadening has to be taken into account.[5] For values used in the experiments, $\alpha \leq 0.05$, the resonance-broadening width in velocity space is of the order of or greater than the phase-velocity spread of the injected spectrum. Such a resonance broadening may explain the heating of the electron distribution function. However, because of the agreement between theoretical and experimental results on the spectrum evolution, resonance broadening does not seem to affect the basic features of nonlinear Landau damping as predicted by weak-turbulence analysis.

In conclusion, nonlinear Landau damping of broad-band plasma oscillations has been observed experimentally. The experimental results on the evolution of the spectrum agree with a theory which assumes a constant distribution function. However, we do not observe the localized disturbance of the electron distribution function which is predicted by this theory. Moreover the observed increase in temperature with injected power implies that a supplementary mechanism, such as resonance broadening, must be included in order to solve the problem in a consistent manner.

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Stochastic Acceleration by a Single Wave in a Magnetic Field

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The nature of a particle orbit in an electrostatic plasma wave is modified by a magnetostatic field, because there exists a *set* of resonant parallel velocities $(\omega + l\Omega)/k_z$. If the wave amplitude Φ_0 is sufficiently large, neighboring resonant regions overlap, and the particle motion becomes stochastic; the threshold condition is $k_z^2(e/m)\Phi_0|J_l(k_\perp\rho)| \approx \Omega^2/16$. As an application, a weakly damped intermediate-frequency ion-acoustic wave may be used to heat the tail of an ion distribution.

The character of the resonant interaction of a particle with an electrostatic wave can be qualitatively different in the presence or absence of an ambient magnetostatic field. In its absence, it is well known[1] that particles whose velocity (projected along the wave vector) differs from the wave phase velocity ω/k by less than the trapping half-width $2(e\Phi_0/m)^{1/2}$ may be trapped into orbits oscillating about the phase velocity at a bounce frequency $k(e\Phi_0/m)^{1/2}$. This behavior, whose short-term consequence is Landau damping, asymptotically limits the net damping and energy (or momentum) transfer of the wave to the resonant particles.

In a magnetized plasma, an electrostatic wave propagating at an oblique angle $\theta = \tan^{-1}(k_\perp/k_z)$ to the uniform field $B_0\hat{z}$ has a *set* of resonant parallel velocities $\{V_l\}$ which satisfy

$$\omega - k_z V_l = -l\Omega, \quad l = 0, \pm 1, \pm 2, \ldots, \tag{1}$$

where the left-hand side is the Doppler-shifted wave frequency and the right-hand side is a multiple of the gyrofrequency $\Omega = eB_0/mc$. As shown below, the trapping half-width at the lth resonance is

$$w_l \equiv 2|e\Phi_0 J_l(k_\perp\rho)/m|^{1/2}, \tag{2}$$

where ρ is the gyroradius of the particle. When the wave amplitude Φ_0 is so large that the trapping layers $(V_l \pm w_l)$ overlap, a particle can move from one resonance region to the next, executing a random walk in v_z space, so to speak. As a result, the mean net momentum transfer to the particles can be appreciably larger than expression (2) would indicate. In this paper we study the transition from "adiabatic"[2] particle trajectories, when Φ_0 is small, to "stochastic" trajectories, when Φ_0 is large. The motion of a particle in a magnetic field and a single oblique wave has previously been treated by Fredricks.[3] Analogous studies on cyclotron heating in a mirror field[4] and on "super-adiabaticity"[5] may be mentioned.

In the wave frame, moving at $(\omega/k_z)\hat{z}$ with respect to the plasma, the particle Hamiltonian is

$$H(\vec{r}, \vec{p}) = (\vec{p} - m\Omega \times \hat{y})^2/2m + e\Phi_0 \sin(k_z z + k_\perp x).$$

Two canonical transformations allow us to write the Hamiltonian as

$$H(z, p_z; \varphi, p_\varphi) = p_z^2/2m + \Omega p_\varphi + e\Phi_0 \sin(k_z z - k_\perp \rho \sin\varphi), \tag{3}$$

where $p_\varphi = mv_\perp^2/2\Omega$ is the canonical angular momentum of gyration, conjugate to the gyrophase φ, and $\rho \equiv (2p_\varphi/m\Omega)^{1/2}$ is the gyroradius. This Hamiltonian system has two degrees of freedom. Since (3) is independent of time, in the wave frame the energy of the particle is conserved.

To analyze (3), it is helpful to use a Bessel-function identity to write (3) as

$$H = p_z^2/2m + \Omega p_\varphi + e\Phi_0 \textstyle\sum_l J_l(k_\perp\rho)\sin(k_z z - l\varphi). \tag{4}$$