R. Koontz, E. Taylor, L. Trudell, and the entire SLAC operations staff. We also wish to thank C. W. Tu for his assistance during the early stages of the experiments.

*Research (Yale Report No. COO-3075-90) supported in part by the U. S. Energy Research and Development Administration under Contract No. AT(11-1) 3075 (Yale) and Contract No. AT(04-3)515 (SLAC); the German Federal Ministry of Research and Technology and the University of Bielefeld; and the Japan Society for the Promotion of Science.

†Also at Nagoya University, Nagoya, Japan.

‡Also at Los Alamos Scientific Laboratory, Los Alamos, N. Mex. 87544.

§Also at University of Bielefeld, Bielefeld, West Germany.

‖Also at Yale University, New Haven, Conn. 06520.

[1]F. J. Gilman, SLAC Report No. 167, 1973 (unpublished), Vol. 1, p. 71.

[2]S. M. Berman and J. R. Primack, Phys. Rev. D 9, 217 (1974).

[3]SLAC Users Handbook (unpublished), Sect. D. 3

[4]M. J. Alguard *et al.*, in *Proceedings of the Ninth International Conference on High Energy Accelerators, Stanford, California, 1974,* CONF 740522 (Stanford Linear Accelerator Center, Stanford, Calif., 1974), p. 313.

[5]Calculations by W. P. Lysenko and R. H. Helm place an upper limit of 2.8% on the depolarization of the electron beam during acceleration to high energy; see also R. H. Helm and W. P. Lysenko, SLAC Report No. SLAC-TN-72-1, 1972 (unpublished).

[6]H. Frauengelder and A. Rossi, in *Methods of Experimental Physics,* edited by L. C. L. Yuan and C. S. Wu (Academic, New York, 1963), Vol. 5, Pt. B, p. 214.

[7]D. M. Schwartz, Phys. Rev. 162, 1306 (1967).

[8]A. M. Bincer, Phys. Rev. 107, 1434 (1957).

[9]See, for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), p. 140.

[10]H. L. B. Gould and D. H. Wenny, Elec. Eng. (Amer. Inst. Elec. Eng.) 76, 208 (1967).

[11]The χ^2 statistic was evaluated for each run, in which 25 individual measurements of the asymmetry were combined to form a weighted mean. The χ^2 values for all the runs are in good agreement with the theoretical χ^2 distribution. Thus no evidence exists for nonstatistical fluctuations or drifts in monitors. In addition the false asymmetry formed from adjacent mini-run pairs of the same sign gave a result consistent with zero.

[12]L. L. DeRaad, Jr., and Y. J. Ng, Phys. Rev. D 11, 1586 (1975).

New Class of Bound-State Solutions in Field Theory

Paul Langacker*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 1974
(Received 5 May 1975)

It is suggested that bound states can emerge in field theory as alternate solutions to the Bethe-Salpeter equation, not corresponding to the Neumann-series (perturbation-theory) solution. These new solutions are asymptotically similar to elementary-particle solutions and imply nonperturbative anomalous dimensions in the Wilson operator-product expansion. For Goldstone bosons in standard quark models as well as for certain solvable ladder models, these are the only bound-state solutions.

It is not at all clear that renormalizable field theories possess any bound states. The Bethe-Salpeter equation[1] (BSE) in the ladder approximation (Fig. 1) can sometimes be solved exactly[2-4] if one ignores the mass of the exchanged particle. These calculations yield branch points rather than Regge poles[5] for the t-channel partial-wave amplitudes at $q_\mu = 0$. Perturbation calculations[6] of the same class of diagrams indicate that these branch points are fixed.[7] This is disturbing because the Schrödinger equation, which possesses bound states and moving Regge poles, can be de-

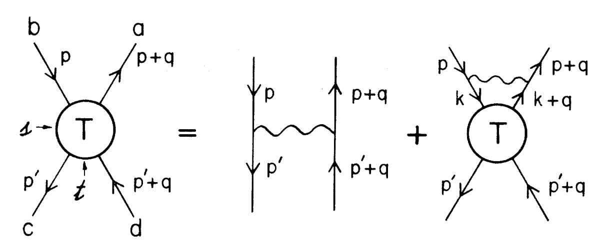

FIG. 1. The Bethe-Salpeter equation (BSE) for $T(p, p', q)$ in the ladder approximation. A bound state corresponds to a pole in T at $q^2 = m_B^2$. The bound-state vertex function $\varphi(p, q)$ satisfies the homogeneous BSE.

rived[1,8] from the instantaneous approximation to the ladder-model BSE.

The undesirable solutions described above correspond to the Neumann-series, or perturbation-expansion, solution to the BSE. *I propose to abandon the Neumann series and look for alternative*[9] *but acceptable solutions to the BSE which do possess bound states.* I will, however, continue to use perturbation theory for the kernel and propagators (in principle bound-state effects could feed back into these quantities).

I first remark that if one ignores the question of the existence of bound states and simply looks for solutions to the homogeneous BSE in the asymptotic limit $-p^2$ and $-(p+q)^2 \to \infty$, with their ratio and q^2 fixed (or equivalently, taking $m_B{}^2$ and the internal masses to zero), then one finds at least two solutions.[1-3,10] For spin-zero bound states these are $\varphi^+(p,q) \sim (-p^2)^{-\epsilon}$ and $\varphi^-(p,q) \sim (-p^2)^{\epsilon-1}$, where $\epsilon = O(g^2)$ and g is the coupling constant. In asymptotically free theories[11,12] one can even justify the generalized ladder approximation (with momentum-dependent coupling constants). In this case the two solutions become $\varphi^+(p,q) \sim (\ln p^2)^{-A}$ and $\varphi^-(p,q) \sim (\ln p^2)^{+B}/p^2$, with A and B calculable. The + (−) solution will be called irregular (regular), because of its short-distance behavior in position space.

Older papers[1] have generally rejected the more singular irregular solutions by analogy with the quantum mechanical prescription. This argument is dubious, however, because the BSE is itself more singular at short distances than the Schrödinger equation.

I have found that in four models the only bound-state or Regge-pole solutions are asymptotically of the irregular type. Two models are in the ladder approximation: the pseudoscalar projection of the Abelian quark-gluon model and the renormalizable φ^3 theory in 6 dimensions ($\varphi_6{}^3$). The other two are for exact kernels: the Goldstone-boson solutions to Abelian and non-Abelian quark-gluon theories.

Consider first the ladder-model BSE for the γ_5 projection of T in the Abelian quark-gluon model, which is appropriate for pseudoscalar bound states and higher orbital excitations. Willey[2] has constructed the Neumann-series solution for zero gluon mass at the point $q_\mu = 0$. Defining $(\gamma_5)^{ab}{}_{ab}T_{cd} \times (\gamma_5)^{cd} \equiv 12ig^2F(p,p')$, Willey performs a Wick rotation to Euclidean space and makes an O(4) decomposition of the $p\cdot p'$ dependence of F. The O(4) partial-wave amplitudes are $F_n(u,v)$, where $u = -p^2/m^2$ and $v = -p'^2/m^2$. The BSE for F_n is then

$$F_n(u,v) = 2\pi^2 K_n(u,v) + \epsilon\int_0^\infty dw\,[w/(w+1)]\,K_n(u,w)F_n(w,v) \tag{1}$$

with

$$K_n(u,v) = (n+1)^{-1}(u_>)^{-1}(u_</u_>)^{n/2}, \tag{2}$$

where $\epsilon = 3g^2/(4\pi)^2$ and $u_>$ ($u_<$) is the larger (smaller) of u and v. Willey then converts (1) into the differential equation

$$\left(\frac{d^2}{du^2} + \frac{2}{u}\frac{d}{du} - \frac{n(n+2)}{4u^2} + \frac{\epsilon}{u(u+1)}\right)F_n(u,v) = -2\pi^2\frac{\delta(u-v)}{uv} \tag{3}$$

for which he constructs the particular solution $F_n{}^w(u,v) = -2\pi^2 Y_1{}^{(n)}(u_<)Y_2{}^{(n)}(u_>)$. The hypergeometric functions $Y_i{}^{(n)}$, which satisfy the homogeneous version of (3), have the asymptotic forms

$$Y_1{}^{(n)}(u) \underset{u\to\infty}{\longrightarrow} u^{(\nu-1)/2} \approx u^{n/2-\epsilon/(n+1)}, \tag{4}$$

$$Y_2{}^{(n)}(u) \underset{u\to\infty}{\longrightarrow} u^{-[(\nu+1)/2]} \approx u^{-[(n+2)/2-\epsilon/(n+1)]}, \tag{5}$$

where $\nu \equiv [(n+1)^2 - 4\epsilon]^{1/2}$. Willey's solution has no Regge poles, only branch points at $n = -1 \pm 2\epsilon^{1/2}$.

It is simple to construct an alternative Regge-pole solution to (3). It is

$$F_n(u,v) = F_n{}^w(u,v) + \gamma_n(u)\gamma_n(v)/(n-n_0), \tag{6}$$

where the position n_0 of the Regge pole is arbitrary and $\gamma_n(u) = C_1Y_1{}^{(n)}(u) + C_2Y_2{}^{(n)}(u)$. From (4) and (5) we see that $Y_1{}^{(n)}$ corresponds to the irregular asymptotic solution for spin n and $Y_2{}^{(n)}$ to the regular solution. However, the integral equation (1) has boundary conditions not incorporated into (3). One can easily show that the regular solution $Y_2{}^{(n)}$ fails to satisfy the BSE at the limit $u \to 0$ while $Y_1{}^{(n)}$ does satisfy it.[13] Hence, the only Regge-pole solution is of the irregular type.

The $\varphi_6{}^3$ theory is analogous. There are no Regge poles in the Neumann solution.[3] One can construct alternative solutions, but only of the irregular type.

One could argue that it is irrelevant that the regular solutions fail to satisfy the low-momentum part of the BSE in the ladder model, because the ladder approximation is unreliable there. It is conceivable that the full theory has regular solutions. *There is, however, one example of a bound state in a full theory, which, if it exists, must be of the irregular type: Goldstone bosons in non-Abelian quark-gluon theories or in Abelian theories with an eigenvalue condition.*

In the Abelian theory one can use renormalization-group techniques to show[14] that the fermion self-energy function $\Sigma(p)$ behaves asymptotically as $(-p^2/m^2)^{-\epsilon}$, where $\epsilon = 3g^2/(4\pi)^2 + O(g^4)$. This result is based on a completely unambiguous application of renormalized perturbation theory. Similarly, in non-Abelian theories[12] $\Sigma(p) \to [\ln(-p^2/m^2)]^{-A}$. In both cases the bare mass $m_0(\Lambda)$ of the fermion vanishes as $\Lambda \to \infty$, suggesting[15] a chiral symmetry. If this symmetry exists and is realized in the Goldstone manner (i.e., with nonzero physical fermion mass) then the axial-vector Ward identity requires[15] the existence of a massless bound-state Goldstone boson with a vertex function $\varphi(p)$ proportional to $\Sigma(p)$. Hence, $\varphi(p) \sim (p^2)^{-\epsilon}$ or $(\ln p^2)^{-A}$, the irregular solution.

Several comments are in order. (a) In the ladder-model examples there is no quantization condition on the pole position n_0. I believe this highly undesirable feature to be due to the approximations (zero-mass ladder exchange) and the special kinematic point ($q_\mu = 0$) of high O(4) symmetry. (b) The $n = 0$ irregular solutions in the ladder model or for Goldstone bosons are normalizable under the most popular prescription.[1,16] (c) These bound-state solutions are similar asymptotically to the vertex functions for elementary particles.

(d) Recently, several authors[10-12] have tried to use the Wilson operator-product expansion[17] (OPE) to rule out the irregular solutions. To see this, define the Bethe-Salpeter wave function $g(p, q)$ as

$$S_F(p+q)\varphi(p,q)S_F(p)$$
$$= g(p,q) \equiv \int d^4x\, e^{ip\cdot x}\langle 0|T(\psi(x)\overline{\psi}^*(0))|B(q)\rangle, \qquad (7)$$

where B is the bound state. For short distances one has[17] $T(\psi(x)\overline{\psi}(0)) \sim c(x)\overline{\psi}(0)\psi(0)$, so that $g(p,q)$ is given asymptotically as the transform of $c(x)$. If one computes[10-12] $c(x)$ perturbatively in terms of the divergences in the matrix element of $\overline{\psi}^*\psi$ between elementary particle states, then one finds that the regular solution is required.

However, the OPE is abstracted from the behavior of individual Feynman diagrams. Its validity in as nonperturbative a context as bound states is questionable, especially if bound states originate as alternative solutions to the BSE rather than from summing an infinite sequence of Feyn man diagrams (the Neumann series).

This problem can be resolved by use of Zimmermann's nonperturtative derivation[17] of the OPE. The first term is

$$T(\psi(x)\psi^*(0)) \xrightarrow[x\to 0]{} c_{ab}(x)F(0), \qquad (8)$$

where

$$F(0) \equiv \lim_{x\to 0} \frac{T(\psi(x)\psi^*(0))}{c_{ab}(x)} \qquad (9)$$

and $c_{ab}(x) \equiv \langle a|T(\psi(x)\psi^*(0))|b\rangle$. The states a and b are chosen as those for which $c_{ab}(x)$ is the most singular as $x \to 0$. By construction all matrix elements of $F(0)$ are finite. The previous applications[10-12] of the OPE to the bound-state problem have all implicitly assumed that a and b could be chosen as elementary-particle states so that $c_{ab}(x)$ could be computed perturbatively. We now see, however, that if the bound-state wave function is more singular than individual Feynman diagrams it is necessary to choose a as the vacuum and b as the bound state. Then the transform of $c_{ab}(x)$ is the Bethe-Salpeter wave function $g(p, q)$. One can say that *the leading term in the operator-product expansion has a large anomalous dimension that is due to the bound state itself* rather than from the ultraviolet divergences of perturbation theory.[18]

(e) The electromagnetic form factor of a bound-state pion behaves,[19] up to logarithms or perturbative anomalous dimensions, as $O(1)$ or $O(q^{-4})$ as $q^2 \to -\infty$ for irregular and regular wave functions, respectively. These calculations have nothing to do with such effects as the ρ meson, however. Perhaps the inclusion of bound-state effects in the vector vertex could yield rapidly varying form factors for the irregular solutions for large but finite $-q^2$.

(f) Cardy has recently shown[20] that the full BSE for the asymptotically free $\varphi_6{}^3$ theory is Fredholm. He has not, however, proven his claim that there are moving Regge poles in the Fredholm solution. Both the momentum transfer and the complex angular momentum enter the kernel in a complicated nonlinear manner. Fredholm theory tells us nothing about the singularity structure of the solution in these parameters. In a very interesting paper Lovelace[21] has shown the existence of an accumulation of regular solutions to the homogeneous BSE near $n=-1$ in φ_6^3. This is a special feature of $n=-1$, however. I also remark that although the Fredholm solution is the unique square-integrable solution, there may be alternative acceptable, but nonsquare integrable, solutions to the BSE.

(g) Another argument against the notion that strong coupling or asymptotic freedom must be invoked to solve the bound-state problem is that *the same difficulties occur in quantum electrodynamics: Nobody has ever shown the existence of positronium directly from the field theory.* Whether the irregular solutions proposed here yield positronium when they are continued away from the forward direction is a crucial unsolved problem.

It is a pleasure to acknowledge useful discussions with Professor Gino Segrè, Professor Heinz Pagels, Professor Ralph Amado, Professor Abraham Klein, and Professor M. A. B. Bèg.

---

*Work supported in part by the U. S. Energy Research and Development Administration.

[1]An excellent review is given by N. Nakanishi, Progr. Theor. Phys., Suppl. 43, 1 (1969).

[2]R. S. Willey, Phys. Rev. 153, 1364 (1967). Willey actually considers quantum electrodynamics but the Abelian quark model is similar.

[3]I. J. Muzinich and H.-S. Tsao, unpublished.

[4]C. G. Callan and M. L. Goldberger, unpublished.

[5]I use the term Regge pole for any expansion group, even in Euclidean space.

[6]See, for example, R. W. Brown *et al*., unpublished; J. D. Bjorken and T. T. Wu, Phys. Rev. 130, 2566 (1963).

[7]Superrenormalizable theories have Regge poles in the ladder model. See B. W. Lee and R. F. Sawyer, Phys. Rev. 127, 2266 (1962).

[8]This reduction may be unreliable. See A. Klein and T.-S. H. Lee, Phys. Rev. D 10, 4308 (1974), and references therein. The ladder approximation could be the source of the difficulties.

[9]Such a nonperturbative *Ansatz* is made in other contexts, such as spontaneous symmetry breaking with either elementary or bound-state scalars.

[10]C. G. Callan and D. J. Gross, unpublished. They consider the massless Yukawa theory, for which there is a second irregular solution which they did not write down.

[11]T. Appelquist and E. Poggio, Phys. Rev. D 10, 3280 (1974).

[12]K. Lane, Phys. Rev. D 10, 2605 (1974).

[13]The $Y_1^{(n)}$ homogeneous solutions were first discovered by J. S. Goldstein, Phys. Rev. 91, 1516 (1953). Willey (Ref. 3) was also aware of them. They were first taken seriously by H. Pagels, Phys. Rev. Lett. 28, 1482 (1972), and Phys. Rev. D 7, 3689 (1973), in the context of the $n=0$ Goldstone boson.

[14]S. Adler, Phys. Rev. D 5, 3021 (1972), and references therein.

[15]Pagels, Ref. 13. See also P. Langacker and H. Pagels, Phys. Rev. D 9, 3413 (1974).

[16]C. H. Llewellyn Smith, Nuovo Cimento 60A, 348 (1969).

[17]K. Wilson, Phys. Rev. 179, 1499 (1968). See also W. Zimmermann, *Lectures on Elementary Particles and Quantum Field Theory* (Massachusetts Institute of Technology Press, Cambridge, Mass., 1970).

[18]Lane, Ref. 12, has used the perturbative OPE argument to choose the regular Goldstone-boson solution and concluded that $\Sigma(p)$ must behave nonperturbatively. I reject this argument because it is the OPE and not $\Sigma(p)$ that is directly affected by bound-state effects.

[19]See, for example, Ref. 11.

[20]J. L. Cardy, Phys. Lett. 53B, 355 (1974).

[21]C. Lovelace, unpublished.