reversal corresponding to hybridization. One obvious discrepancy is the extra breadth of the upper theoretical peak; however, in view of the oversimplified nature of the theory, the agreement is quite satisfactory.

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Critical Dynamics of Isotropic Antiferromagnets in $4-\epsilon$ Dimensions

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The critical dynamics of a model isotropic antiferromagnet is studied in $4-\epsilon$ dimensions above and at T_{N}. The renormalization-group, characteristic-frequency exponent, and shape function are determined to order ϵ. In particular we find fluctuation-induced peaks in the shape function at and above the Néel temperature. We find results in agreement with dynamical scaling.

In this Letter we discuss a model for the critical dynamics of an isotropic Heisenberg antiferromagnet. In contrast to the ferromagnet studied earlier,[1,2] where the conventional theory holds only for $d>6$, the antiferromagnet has simple critical dynamics for $d>4$. We therefore use renormalization-group (RNG) techniques to analyze our model and perform a self-consistent calculation to $O(\epsilon)$ ($\epsilon=4-d$) for $T \geq T_{\mathrm{N}}$.

We have calculated the dynamic correlation function for the staggered magnetization (a nonconserved order parameter) extracting the characteristic frequency and the associated dynamical index Z_N. We find, to $O(\epsilon)$, $Z_N=2-\epsilon/2$ in agreement with the RNG calculation of Halperin, Hohenberg, and Siggia.[3] In three dimensions where $\epsilon=1$, we have $Z_N=3/2$ in agreement with dynamical-scaling predictions to within η corrections. Our calculation also gives an analytical form for the order-parameter correlation function and allows us to extract the shape function $f_{q\xi}{}^N(\omega/\omega(q,\xi))$, where $\omega(q,\xi)$ is the characteristic frequency and ξ is the correlation length. We find peaks in $f_x{}^N(\nu)$ $[x=q\xi,\ \nu=\omega/\omega(q,\xi)]$, at and above T_N, similar to those observed in the isotropic antiferromagnet $RbMnF_3$ by neutron scattering.[4] The behavior of $f_x{}^N(\nu)$ is shown in Fig. 1 for $\epsilon=1$. The shape function exhibits the following behavior:

(1) In the relaxational regime[5] $x\ll 1$, $f_x{}^N(\nu)$ is a Lorentzian centered about $\nu=0$.

(2) As x increases and we move towards the critical regime $x\gg 1$ we find that two fluctuation-induced peaks appear in the spectrum displaced symmetrically about the origin. For $\epsilon=1$ these two peaks first appear for $x\sim 2$.

(3) The position of the peaks moves continuously outward from the origin until $x\sim 15$ where the position of the peaks attains a limiting value $\nu_M=\pm 0.45$.

(4) The height of the peaks relative to the value of $f_x{}^N(0)$ increases as x increases reaching a maximum of 15% for $x\gg 1$.

(5) For $\epsilon\ll 1$ $f_x{}^N(\nu)$ is a Lorentzian centered abour $\nu=0$ for all values of x. In the critical regime the two-peak structure first appears for $\epsilon\sim 0.45$.

The model we have used in our calculation belongs to the same family of models as used by Ma and Mazenko[2] to study the critical dynamics of isotropic ferromagnets. If we neglect cou-

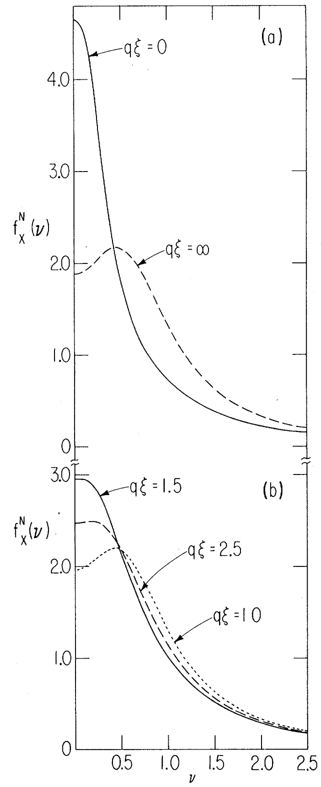

FIG. 1. A plot of the scaling function $f_{q\xi}{}^N(\nu)$ for $\epsilon=1$ (a) in the relaxational regime $q\xi=0$ and the critical regime $q\xi=\infty$ and (b) for intermediate values of $q\xi=1.5$, 2.5, and 10.

plings to the energy[6] we find that the staggered magnetization density $\vec{N}(x,t)$ and the magnetization density $\vec{M}(x,t)$ satisfy the coupled equations of motion

$$\frac{d\vec{N}(x,t)}{dt}=-\lambda_{NM}\vec{N}\times\vec{H}_M-\lambda_{NN}\vec{M}\times\vec{H}_N-\Gamma_N\vec{H}_N+\vec{\eta}_N, \tag{1a}$$

$$\frac{d\vec{M}(x,t)}{dt}=-\lambda_{MM}\vec{M}\times\vec{H}_M-\lambda_{MN}\vec{N}\times\vec{H}_N+\Gamma_M\nabla^2\vec{H}_M+\vec{\eta}_M\ ; \tag{1b}$$

where the λ's are mode-mode coupling constants, Γ_N and Γ_M are the "bare" transport coefficients, and $\vec{H}_N$ and $\vec{H}_M$ are local fields given by

$$\vec{H}_N(x,t)=-\delta F\{\vec{N},\vec{M}\}/\delta\vec{N}(x,t),\quad \vec{H}_M(x,t)=-\delta F\{\vec{N},\vec{M}\}/\delta\vec{M}(x,t),$$

with $F\{\vec{N},\vec{M}\}$ the Landau-Ginzburg free-energy functional for the system. From the work of Alessandrini, de Vega, and Schapasnik,[7] it is known that in calculating the statics one can decouple $\vec{N}$ and $\vec{M}$ and write

$$F\{\vec{N},\vec{M}\}=\tfrac{1}{2}\int d^dx\,[r_0\vec{N}^2+(\nabla\vec{N})^2+\tfrac{1}{2}u(\vec{N}^2)^2+r\vec{M}^2+c(\nabla\vec{M})^2], \tag{2}$$

where $r_0=T-T_N$, and c, r, and u are positive constants. All wave numbers are taken to be less than a cutoff Λ. Since $\vec{M}$ is not critical at T_N we do not need an M^4 term. For simplicity we have not explicitly shown couplings to external fields; $\vec{\eta}_N(x,t)$ and $\vec{\eta}_M(x,t)$ are Gaussian random-noise sources satisfying

$$\langle \eta_{i\alpha}(x,t)\rangle = 0,$$

where $\alpha = M$ or N and $i = x$, y, or z, and

$$\langle \eta_{i\alpha}(x,t)\eta_{i\beta}(x',t')\rangle = 2\hat{\Gamma}_\alpha \delta_{ij}\delta_{\alpha\beta}\delta(x-x')\delta(t-t'),$$

where $\hat{\Gamma}_M = -\Gamma_M\nabla^2$ and $\hat{\Gamma}_N = \Gamma_N$. The model described above is similar to one discussed by Kawasaki.[8] The mode-mode coupling terms in Eqs. (1) follow directly from the discussion of Ref. 2, Sect. II, if we choose

$$Q_{\alpha\beta}{}^{ij}(r,r') = \lambda_{\alpha\beta}[\Psi_\alpha{}^i(r), \Psi_\beta{}^j(r')],$$

with $\Psi_\alpha{}^i(r) = N^i(r)$ for $\alpha = N$, $M^i(r)$ for $\alpha = M$ and if we use the standard commutation relations for $\vec{N}$ and $\vec{M}$.[3]

We have carried out a detailed RNG analysis of Eqs. (1) by following the techniques developed in Ref. 2. The details of our work will be presented elsewhere. Here we note some of the main results. We find that the static parameters r_0, u, r, and c transform among themselves as in the original Wilson theory[9] (this is in agreement with the theorem proved in Ref. 2 that the statics generated by equations of motion of this general form are the same as the statics generated by F); moreover we find that $R = r/c\Lambda^2$ scales to infinity[7] as the fixed point is approached. The coupling constants λ_{NN} and λ_{MM} do not enter into the calculation to $O(\epsilon)$. The dimensionless variables $f_{NM} \equiv \lambda_{NM}{}^2/\Gamma_N\Gamma_M$ and $a \equiv r\Gamma_M/\Gamma_N$ transform among themselves under the group and determine the dynamical fixed points. The RNG equations for these quantities have three fixed-point solutions but only one of them is stable. In summary we find one stable fixed point corresponding to

$$r_0{}^* = -5\epsilon\Lambda^2/22, \quad R^* \to \infty, \quad u = \epsilon/11K_4, \quad f_{MN}{}^* = \epsilon/K_4, \quad a^* = \tfrac{1}{3},$$

where $K_4 = (8\pi^2)^{-1}$.

We have used these results to calculate the self-energy associated with the linear response function $G_N(q,\omega)$, defined by Eq. (2.32) in Ref. 2, in the scaling region. We therefore follow the discussion of Sect. VA of Ref. 2 and replace f_{MN}, u, a, and R by their fixed-point values and carry out perturbation theory to $O(\epsilon)$. We find

$$G_N(q,\omega) = \frac{\chi_N(q)\gamma(x,\nu)}{-i\nu + \gamma(x,\nu)}, \tag{3}$$

where

$$\chi_N{}^{-1}(q) = q^2 + \xi^{-2},$$
$$\omega(q,\xi) = \Gamma_N\Lambda^{\epsilon/2}q^{2-\epsilon/2}(1+x^{-2})^{1-\epsilon/4}(1+a^*)^{\epsilon/4}, \tag{4}$$
$$\gamma(x,\nu) = (a^* - i\nu)^{-\epsilon}\left(\frac{a^*}{1+a^*} - i\nu\right)^{3\epsilon/4} W(x,\nu),$$

and where

$$W(x,\nu) = 1 + \frac{\epsilon(1+a^*)}{2a^*x^2}\left[-t\Delta\ln\sigma + (t-\tfrac{1}{2}b)\ln(1+t-\tfrac{1}{4}b) + \frac{x^2a^*}{1+a^*}\ln\left(\frac{\bar{t}^2}{\tau}\right) - \frac{x^2a^*(1-a^*)}{(1+a^*)^2}\ln\tau\right],$$

$$t = \frac{-a^*(1-x^2)}{1+a^*} - \frac{i\nu(1+x^2)}{1+a^*}, \quad b = 4a^{*2}x^2/(1+a^*)^2, \quad \bar{t} = (1+x^2)(a^*-i\nu)/(1+a^*), \tag{5}$$

$$\tau = \frac{(1+x^2)}{(1+a^*)}\left(\frac{a^*}{1+a^*} - i\nu\right), \quad \Delta = (1+b/t^2)^{1/2},$$

and

$$\sigma = \frac{2t(1+t)(1+\Delta)+b}{2t(1+\Delta)-b}.$$

We have performed certain exponentiations in writing down this final form so that our results will be in agreement with the scaling predictions of the RNG.

The correlation function is related to the response function via the classical fluctuation-dissipation theorem

$$C_N(q,\omega) = (2/\omega)\,\mathrm{Im}G_N(q,\omega) \tag{6}$$

and the shape function plotted in Fig. 1 is related to $C_N(q,\omega)$ by

$$C_N(q,\omega) = \chi_N(q) f_x{}^N(\nu)/\omega(q,\xi). \tag{7}$$

The physical transport coefficient can be easily extracted from our results for G_N. We find

$$\tilde{\Gamma}_N = \lim_{q\to 0}\lim_{\omega\to 0}\chi_N(q)\omega(q,\xi)\gamma(x,\nu) = \frac{\Gamma_N(\Lambda\xi)^{\epsilon/2}}{a^{*\epsilon/4}(1+a^*)^{\epsilon/2}}\left\{1-\frac{a^*\epsilon}{(1+a^*)}\left[\ln\left(\frac{1+a^*}{a^*}\right)+1\right]\right\}. \tag{8}$$

We see that the mode-coupling terms lead to a qualitative change in the temperature dependence of the transport coefficient as $T \to T_\mathrm{N}$.

From continuity arguments we expect that the peaks we find at and above T_N will persist for temperatures below T_N. Eventually as $x = q\xi$ decreases, going from Halperin and Hohenberg's[10] region II to region I, there will be a crossover from these fluctuation-induced peaks to the Goldstone boson (spin wave) associated with the broken symmetry $\langle N(x,t)\rangle \neq 0$. We will investigate this crossover in future work.

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