1) that unless $k_0 \gg 1$, r is a biased estimate of s. As k_0 decreases toward unity there is a tendency for r to overestimate s. Fluctuations whose effect is to make r greater than s are more likely than those with the contrary effect. This tendency was noted by Cranshaw *et al.*,[11] who suggested the expression $r[1-1/(2k_0)]^{1/2}$ as an approximation for $\langle s\rangle$. For k_0 values down to 1.5 the approximation is fairly good. Below that it breaks down because of an opposite tendency: For $k_0 \lesssim 0.5$, r tends to underestimate s. Such values of k_0 imply amplitude values so small as to be reckoned unlikely even if s were zero. The amplitude is deviant no matter what one assumes regarding s. Hence such a result has no power to discriminate against s values that are sufficiently small.

This work was begun during the writer's tenure of a British Science Research Council fellowship at Leeds University, and benefitted from stimulating discussions with Dr. Bryan Mercer, Dr. R. J. O. Reid, and Professor J. G. Wilson. Above all, important contributions by Dr. Alan A. Watson, both in discussion at Leeds and in later communications, are gratefully acknowledged.

*Research supported in part by the National Science Foundation.

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[3]Lord Rayleigh, Phil. Mag. 10, 73 (1880).

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[8]It is assumed throughout that the value of s is not too near the maximum possible value of 2. If it were, fluctuation effects would have little or no importance.

[9]An equation structurally identical to (3) has been given by S. Sakakibara [J. Geomagn. Geoelec. 17, 99 (1965)]. But her expression purports to be the s distribution: The variables corresponding to s and r have their definitions interchanged. It is true that in the small-fluctuation limit, Eqs. (3) and (7) transform into each other by interchange of s and r. That is by no means the case, however, when fluctuations are significantly large.

[10]An early criticism of that practice was given by K. Greisen, *Progress in Cosmic Ray Physics* (Interscience, New York, 1956), Vol. 3, Chap. 1. He commented that unless $k_0 \gg 1$, the "probability-of-error distribution" $(p_r?, p_s?)$ is distinctly non-Gaussian, and pointed out that unless $k_0 \gg 1$ the (asymptotic) "standard error" cannot be assigned its usual significance in terms of confidence limits.

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β-Decay Asymmetries in Polarized ^{12}B and ^{12}N and the G-Parity–Nonconserving Weak Interaction

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(Received 18 March 1975)

The decay asymmetries ($\mathfrak{a}$) in polarized ^{12}B and ^{12}N have been measured as a function of β-ray energy (E). The coefficients $\alpha_{\mp}$, in a form of $\mathfrak{a} = \mp P(p/E)(1+\alpha_{\mp}E)$, have been determined to be $\alpha_-(^{12}\mathrm{B}) = +(0.31 \pm 0.06)$%/MeV and $\alpha_+(^{12}\mathrm{N}) = -(0.21 \pm 0.07)$%/MeV. The experimental value, $\alpha_- - \alpha_+ = (0.52 \pm 0.09)$%/MeV, is larger than the prediction of conserved-vector-current theory, $(\alpha_- - \alpha_+)_{\mathrm{CVC}} \simeq 0.27$%/MeV, and in favor of the existence of the second-class induced-tensor current.

Because of the parity nonconservation in the weak interaction, β rays are emitted asymmetrically from polarized nuclei. Conserved-vector-current (CVC) theory predicts, as a result of the weak magnetism, a dependence of the decay asymmetry on the β-ray energy as a higher-order effect. The G-parity–nonconserving induced-tensor current in the framework of $V-A$ theory, if it ever plays a role, can also affect the asymmetry's energy dependence. In order to investigate such effects, the present experiment has been undertaken.

The ^{12}B and ^{12}N decays are suitable for the present purpose: Firstly, they constitute a pair of mirror β decays and their transition energies are high. Secondly, as we have shown previously,[1,2] the polarized nuclei can be produced with adequate yields through the reactions $^{11}B(d,p)^{12}B$ and $^{10}B(^3He,n)^{12}N$, and the polarization direction can be reversed by use of a NMR technique.

The decay asymmetry $\mathcal{A}$ is defined by

$$\mathcal{A} = [W(0) - W(\pi)]/[W(0) + W(\pi)], \tag{1}$$

where $W(\theta)$ is the angular distribution of β rays at a polar angle θ with respect to the nuclear polarization. Theoretically, the distribution $W(\theta)$ in the ^{12}B and ^{12}N decays $(I^\pi, T, T_Z; 1^+, 1, \mp 1 \to 0^+, 0, 0)$ is expressed, in the impulse approximation, as[3]

$$\begin{aligned} W(\theta) \cong & (1 \pm \tfrac{8}{3}aE) \\ & \mp P(p/E)(1 \pm \tfrac{10}{3}aE \mp \tfrac{2}{3}bE)P_1(\cos\theta) \\ & \pm A\tfrac{2}{3}(aE - bE)P_2(\cos\theta), \end{aligned} \tag{2}$$

where the $P_n(\cos\theta)$ are Legendre polynomials, P and A are the nuclear polarization and the alignment,[4] and p and E are the momentum and the energy of the β ray. The upper sign refers to the ^{12}B (β^-) decay, and the lower sign to the ^{12}N (β^+) decay. In the formula, only terms linearly dependent on the momentum transfer are retained except for the allowed moment. The coefficients a and b are given by form factors f_i as

$$\begin{aligned} a &= \mathrm{Re}(f_A g_W^*)/|f_A|^2, \text{ with } g_W \cong f_W - f_V/2M, \\ b &= \mathrm{Re}(f_A g_T^*)/|f_A|^2, \text{ with } g_T \cong f_T \pm f_A/2M, \end{aligned} \tag{3}$$

where f_V, f_A, f_W, and f_T are the vector, axial-vector, weak-magnetism, and induced-tensor form factors, respectively,[5] and M is the nucleon mass. Thus the asymmetry $\mathcal{A}$ is approximately given by

$$\mathcal{A} \cong \mp P(p/E)[1 + \alpha_\mp(1 - A)E], \quad \text{with } \alpha_\mp = \pm\tfrac{2}{3}(a - b). \tag{4}$$

The induced-tensor current is classified as "second class," based on the transformation property under the G-parity operation by Weinberg.[6,7] The second-class current has been recently discussed in relation to the ft-value asymmetries in mirror β decays,[8,9] in which difficulties were due to unknown nuclear-structure effects. On the other hand, the effect on the β-decay asymmetry $\mathcal{A}$ is free from ambiguities due to the nuclear matrix elements at least in the lowest order, and suitable to search for the second-class current.

Polarized ^{12}B or ^{12}N nuclei were produced through the nuclear reaction $^{11}B(d,p)^{12}B$ or $^{10}B(^3He,n)^{12}N$, with use of the 4-MV Van de Graaff generator of Osaka University. The target of ~ 100 $\mu g/cm^2$ in thickness was prepared by evaporation of enriched ^{11}B ($\gtrsim 97.2\%$) or ^{10}B ($\gtrsim 92.2\%$). The incident particle energy and the product recoil angle (θ_R) were selected so as to optimize the nu-

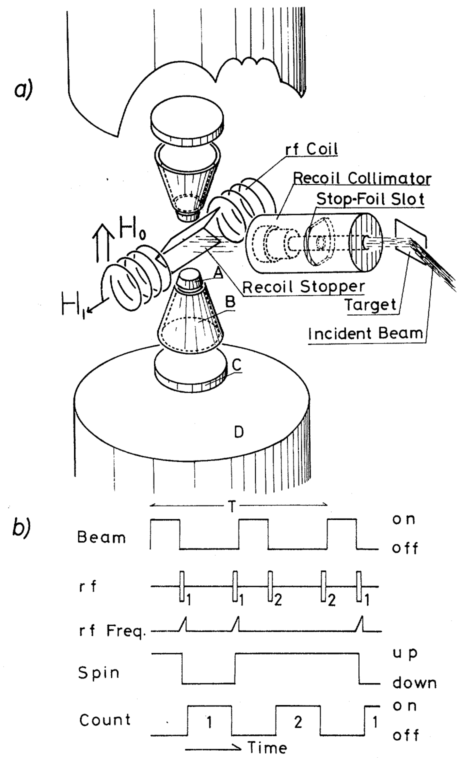

FIG. 1. (a) Schematic view of the experimental setup. The electromagnet poles for H_0, the vacuum chamber, the radiation shield around the target, etc. are abbreviated. Each counter telescope consisted of four plastic scintillators: A, B, C, and D. The counting logic was $A\bar{B}CD$. The last large counter D, which acted as an energy counter, was 17 cm in diameter and 20 cm in length. (b) Time-sequence program of activity production and β counting. The repetition period (T) was 120 msec for ^{12}B and 60 msec for ^{12}N. In the case of polarization reversal, the rf frequency was swept once to pass the NMR frequency. At the end of the β-counting period, the nuclear polarization was again reversed. At a time section for the original polarization direction, a simulated rf field at an off-resonance frequency was applied, to keep any possible disturbance symmetric.

clear polarization and the yield; $E_d = 1.5$ MeV, $\theta_R = 45°$, and $P \simeq 13\%$ for ^{12}B, and $E_{^3\mathrm{He}} = 3.2$ MeV, $\theta_R = 24.5°$, and $P \simeq 25\%$ for ^{12}N. The recoil nuclei were implanted, through a recoil collimator, into an Al foil of ~6 μm in thickness. The nuclear polarization was preserved by applying a static magnetic field $H_0 \simeq 2.2$ kG. β rays, emitted from the recoil stopper, were detected by two counter telescopes. A schematic view of the experimental setup is shown in Fig. 1(a). The time-sequence program of activity production and β counting was as shown in Fig. 1(b). After every other period of beam bombardment, the product nuclear polarization was reversed by use of the adiabatic fast passage in NMR. Typical net counting rates were ~200/sec for ^{12}B and ~10/sec for ^{12}N for each counter telescope. Altogether, four pulse-height spectra were accumulated from the up and down counters for both polarization directions. The background was estimated from spectra accumulated by closing the recoil collimator with a thin foil. Magnitudes of the background were a few parts in 10^{-3} compared to the main spectra at around 6 MeV and less at higher energies.

From the four kinds of spectra thus obtained, an energy-average asymmetry $\overline{\mathfrak{A}}$ was calculated, in which integrated intensities from ~6 MeV to the maximum energy of the β ray were used. The energy-dependent asymmetry $\mathfrak{A}(E)$ was obtained from the spectra of the up and the down counters separately, with use of $\overline{\mathfrak{A}}$ as the normalization factor. When the target and/or the recoil stopper were replaced, the asymmetry varied slightly because of a possible small variation in the relevant geometry. In order to take the average among data of different runs, a normalized asymmetry $\hat{\mathfrak{A}}(E)$ was introduced; $\hat{\mathfrak{A}}(E) = \mathfrak{A}(E)/\overline{\mathfrak{A}}$.

Calibration of the energy scale was made by utilization of the Kurie plot. Because the Kurie plot should be linear for the present case,[10] the energy scale could be determined so as to make the Kurie plot linear. Points of zero energy and the maximum energy were treated as fitting parameters. Parts of a spectrum near the high-energy end and also at lower energies were not included in the fitting. The peak energy of the β-ray spectrum was used as another calibration point. The reliability of the calibration procedure was checked by an auxiliary β-γ coincidence experiment: Two spectra corresponding to the ground-state transition and the branching to the 4.4-MeV level of ^{12}C were accumulated simultaneously. In this way, independent calibration points were obtained, and the uncertainty of the energy scale was estimated to be at most 6%.

Corrections applied on the data were the correction for the branching transition to the 4.4-MeV level of ^{12}C, which was important because of its reversed and different asymmetry compared with the main component, and the factor p/E. The radiative correction to the asymmetry was ignored, because it was theoretically estimated[11] to be very small ($\lesssim 10^{-4}$).

The normalized asymmetry $\hat{\mathfrak{A}}^c(E)$ after the corrections is shown in Fig. 2. The data were fitted by a linear function of E; $\hat{\mathfrak{A}}^c(E) = \beta(1 + \alpha_0 E)$, where α_0 and β are the fitting parameters, and α_0 corresponds to the term $\alpha_\mp(1 - A)$ in Eq. (4).

In order to determine the nuclear alignment A, an additional experiment was performed in the same condition, except that the recoil nuclei were implanted into a thin magnesium single crystal. In this case, the Zeeman splitting was perturbed by the quadrupole interaction due to the nuclear surroundings in Mg, and two separate NMR transitions were observed.[12] From changes in the β-decay asymmetry by the saturated NMR transition, the alignment as well as the polarization was determined.[13] The results were $A(^{12}\mathrm{B}) = +(0.03 \pm 0.02)$ and $A(^{12}\mathrm{N}) = \pm(0.03 \pm 0.03)$. The alignments were small at the present experimental conditions.

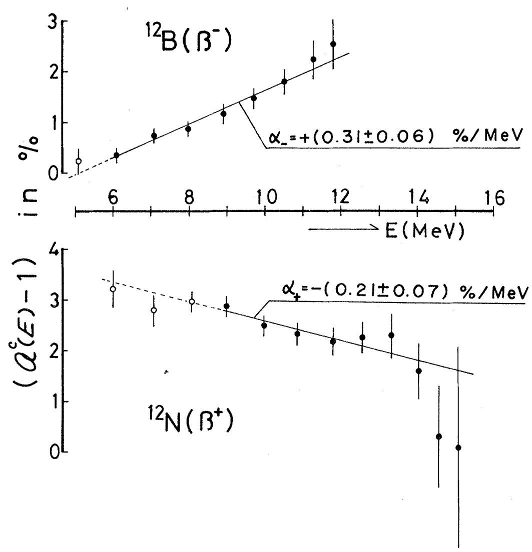

FIG. 2. Normalized asymmetry $\hat{\mathfrak{A}}^c(E)$. The attached error bars indicate the counting statistics only. The fitted lines are drawn. In the fittings, data which are denoted by open circles were not included, because those data were subject to large corrections due to the branching transition to the 4.4-MeV level of ^{12}C.

TABLE I. Summary of results and relevant data.

	β emitter	
	^{12}B	^{12}N
Decay properties		
(I^π, T, T_Z)	$(1^+, 1, -1) \to (0^+, 0, 0)$	$(1^+, 1, +1) \to (0^+, 0, 0)$
$E_\beta^{\max}$ (MeV)[a]	13.88	16.83
$T_{1/2}$ (msec)[a]	20.4	11.4
Branching ratio[b] (%)	1.27 ± 0.06[c]	2.10 ± 0.16[d]
Present results		
$\mp\bar{\alpha} \simeq P = a_{+1} - a_{-1}$	0.13	0.25
$A = 1 - 3a_0$	$+(0.03 \pm 0.02)$	$\pm(0.03 \pm 0.03)$
$\alpha_\mp$ (%/MeV)	$+(0.31 \pm 0.06)$	$-(0.21 \pm 0.07)$
$\alpha_\mp$ uncertainties (%/MeV)		
Counting statistics	0.041	0.051
Branching correction	0.03	0.04
Energy scale	$\lesssim 0.016$	$\lesssim 0.012$
Gain shift[e]	$\lesssim 0.015$	$\lesssim 0.011$
Background[f]	$\lesssim 0.02$	$\lesssim 0.005$
Alignment A	0.006	0.013

[a]F. Ajzenberg-Selove and T. Lauritzen, Nucl. Phys. A114, 1 (1968).
[b]Branching to the 4.4-MeV level ($I^\pi = 2^+$) of ^{12}C.
[c]D. E. Alburger, Phys. Rev. C 6, 1167 (1972).
[d]D. H. Wilkinson, Phys. Lett. 48B, 169 (1974).
[e]Between the spin-up and spin-down periods.
[f]Ambiguity of estimation for scattered electrons.

The coefficients $\alpha_\mp$ were thus obtained to be

$$\alpha_-(^{12}\text{B}) = +(0.31 \pm 0.06)\%/\text{MeV},$$

$$\alpha_+(^{12}\text{N}) = -(0.21 \pm 0.07)\%/\text{MeV}.$$

The uncertainties contributed to the present determinations are enumerated in Table I.

The theoretical prediction for $\alpha_\mp$ according to CVC theory, without the second-class current, can be calculated by Eqs. (3) and (4) with $g_W = -\mu/2M$, and $g_T = \pm f_A/2M$, where $\mu \simeq \mu_p - \mu_n = 4.7$ is the transition magnetic moment: $\alpha_-^{\text{CVC}} \simeq +0.10\%/\text{MeV}$, and $\alpha_+^{\text{CVC}} \simeq -0.17\%/\text{MeV}$. The prediction due to the weak-magnetism term is symmetric between a pair of mirror β decays, and can be compared with the present experimental result in the form $\alpha_- - \alpha_+$: $(\alpha_- - \alpha_+)_{\text{CVC}} \simeq 0.27\%/\text{MeV}$ and $(\alpha_- - \alpha_+)_{\text{exp}} = (0.52 \pm 0.09)\%/\text{MeV}$. The experimental value is considerably larger than the prediction of the weak-magnetism term alone.[14] The effect on $\alpha_\mp$ due to the second-class induced-tensor current is also expected to be symmetric[15] as seen in Eqs. (3) and (4). Thus, the present result is in favor of the existence of the induced-tensor current of which the form factor may well be comparable to the weak-magnetism term.

The authors wish to thank Professor M. Morita and Professor S. Nakamura for their theoretical discussion. They are indebted to Professor A. Mizobuchi, Dr. T. Minamisono, and Dr. Y. Nojiri for their advice and cooperation in various stages of the experiment. Thanks are also due to Mr. Y. Takahashi for his technical assistance.

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[4]P and A are given by $P = a_1 - a_{-1}$ and $A = 1 - 3a_0$ for the present case, where the magnetic substate populations a_m are normalized as $a_1 + a_0 + a_{-1} = 1$.
[5]The form factors f_i ($i = V, W, S, A, T, P$) define the strength of the respective couplings in the general nu-

clear $V-A$ currents;

$$\langle |J_\lambda|\rangle = i\langle|\gamma_\lambda(f_V - f_A\gamma_5) + \sigma_{\lambda\nu}k_\nu(f_W + f_T\gamma_5) + ik_\lambda(f_S + f_P\gamma_5)|\rangle.$$

Under the assumption that CVC theory as well as partial conservation of axial-vector current theory holds, f_S equals 0 and $|f_P/f_A|$ is as small as ~1/20 for nuclear β decay.

[6]S. Weinberg, Phys. Rev. 112, 1375 (1958).

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[10]Strictly, the contribution from the weak magnetism and the radiative correction should be taken into account.

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[12]R. C. Haskell and L. Madansky, J. Phys. Soc. Jpn., Suppl. 34, 167 (1973).

[13]T. Minamisono *et al.*, J. Phys. Soc. Jpn. 30, 311 (1971); M. Hori *et al.*, J. Phys. Soc. Jpn., Suppl. 34, 161 (1973).

[14]The reliability of the impulse approximation used in our estimation can be seen from the CVC test experiment on ^{12}B-^{12}C-^{12}N; see, e.g., C. S. Wu, Rev. Mod. Phys. 36, 618 (1964), where the experimental results and the theoretical predictions are in agreement with ~20% uncertainties.

[15]The induced-pseudoscalar current is theoretically expected to have practically no effect on $\alpha_\mp$; M. Morita, to be published.

COMMENTS

Comments on the Observation of Nondivergent Radiation of Discrete Frequencies

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A high-resolution spectroscopic study has been made of the radiation emitted in the forward direction from cylindrical targets bombarded with electrons. The discrete energy bands previously reported by Das Gupta were not observed for photon energies from 4 to 200 keV.

In a recent Letter,[1] Das Gupta reported the observation of nondivergent beams of x rays produced by electron bombardment of cylindrical targets. He also reported the observation of discrete x-ray energy bands in the nondivergent beam spectra. Das Gupta ascribed these observations to an electron-photon parametric coupling that results as electrons move along the inside wall of the cylindrical target.[1,2] According to Das Gupta's model, which couples the electron DeBroglie wavelength with corresponding photon wavelengths, the energy of these bands is given by

$$T = 2m_0c^2/(n^2-1), \tag{1}$$

where n is an integer greater than 1.

With the objective of subjecting these heretofore unreported observations and conclusions to a more sensitive experimental test, we have attempted to detect x rays at some of the discrete