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Strong Polarization of the 1*p*-Shell Core in the Lowest 0^+ States of ^{24}Mg and ^{28}Si†

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Inelastic α scattering to the first excited 0^+ states in ^{24}Mg and ^{28}Si has been measured at $E_\alpha = 23.5$ MeV and analyzed in a microscopic model. Strong evidence for the need of including $1p_{1/2}$ hole components in the wave functions of the lowest 0^+ states was found. Deduced $E0$ matrix elements are in surprisingly good agreement with results from electron scattering.

In most recent shell-model calculations for nuclei in the middle of the $2s1d$ shell it is assumed that the ^{16}O core is closed.[1,2] However, nuclei in this region are known to be strongly deformed; thus, the validity of this assumption is open to question. Often, polarization of the core can be determined from a measurement of single-particle-transfer transitions which are forbidden by the simple shell model, e.g., stripping to hole states. However, this method cannot be used to determine a possible $1p$ core polarization in the middle of the $2s1d$ shell where $1p$ hole and $2p$ particle states appear to be mixed,[3] since $2p$ particle stripping is strongly enhanced over $1p$ hole stripping and $1p$ and $2p$ stripping do not lead to significantly different angular distributions.

Another tool for determining core polarization is the study of inelastic monopole transitions induced by strongly absorbed particles.[4] Here the angular distributions of the inelastically scattered particles are extremely sensitive to the radial form factors. Because these form factors are significantly different for $1p$ hole and $2p$ particle components, a study of these monopole transitions may allow a determination of $1p$ core polarization.

In the present work monopole transitions were measured in inelastic scattering of α particles on ^{24}Mg and ^{28}Si. These nuclei, which are assumed to have prolate as well as oblate shapes,[5] were chosen to look for a possible dependence of the monopole transitions on the sign of the nuclear deformation.

The experiments have been performed with use of a 23.5-MeV α beam from the University of Minnesota's model MP tandem Van de Graaff accelerator. Scattered α particles were detected by position-sensitive solid-state detectors placed along the focal plane of an Enge split-pole magnetic spectrometer. Targets of enriched ^{24}Mg and $^{28}SiO_2$ approximately 50 μg/cm^2 thick were used. Special care was taken in focusing and collimating the α-particle beam to reduce spectral background, so that the cross sections both at the deep minima of the angular distributions and at far forward angles could be measured. The experimental data are shown in Fig. 1. Absolute cross sections to an accuracy of 15% have been derived from a comparison of the yield of elastically scattered α particles with optical-model predictions. The curves shown in Fig. 1 refer to distorted-wave Born-approximation calculations using microscopic form factors to be discussed below.

For the calculation of the microscopic form factors the nuclear wave functions were assumed to be of the form

$$|0^+j\rangle = a_j|\mathrm{sph}\rangle + \sum_k b_{jk}|\mathrm{p\text{-}h}\rangle_k \,, \tag{1}$$

where $|\mathrm{sph}\rangle$ is the spherical component, $|\mathrm{p\text{-}h}\rangle_k$ are particle-hole components, and j is the state index. With use of the orthogonality of the wave functions, $a_f a_i + \sum_k b_{fk} b_{ik} = 0$, the inelastic monopole form factor can be written

$$F(r) = \langle 0^+f|V_0(r)|0^+i\rangle = \sum_k b_{fk} b_{ik} [\langle \mathrm{p}|V_0(r)|\mathrm{p}\rangle - \langle \mathrm{h}|V_0(r)|\mathrm{h}\rangle]_k \,. \tag{2}$$

For strongly deformed nuclei a large number of particle-hole components is expected, many of which may have the same radial form factor $[F(r)]_c$ as indicated by the index c. For this reason and because the shapes of the angular distributions depend only on the radial form of Eq. (2) we define spectroscopic amplitudes S_c by using the angular momentum matrix elements M_p, M_h, and M_sph given by Satchler[6]:

$$S_c = \sum_{k_c} b_{fk_c} b_{ik_c} M_{k_c} \,. \tag{3}$$

Here $M = M_\mathrm{p} - M_\mathrm{sph}$ if the l and j quantum numbers of the particle and spherical configurations are equal and $M = M_\mathrm{p}$ if the l and j quantum numbers of the hole and spherical configurations are equal. The summation over k_c includes only particle-hole components $|\mathrm{p\text{-}h}\rangle_k$ which have the same radial structure $[F(r)]_c$. Because the spectroscopic amplitude S_c contains the matrix elements M_{k_c} no explicit information on the spherical component $a_f a_i = -\sum_k b_{fk} b_{ik}$ can be obtained. Use of the above definitions leads to the monopole form factor

$$F(r) = \sum_c S_c [F_\mathrm{p}(r) - F_\mathrm{h}(r)]_c + \epsilon F_x(r) \,, \tag{4}$$

where $F_\mathrm{p}(r)$ and $F_\mathrm{h}(r)$ are radial integrals for particle and hole components, respectively. The last term in Eq. (4), $\epsilon F_x(r)$, accounts for particle-hole components which are not considered explicitly, essentially contributions from higher shells. Its radial dependence may be approximated by the form factor of the spherical component.

For the effective interaction a nucleon-nucleon interaction of Gaussian form was used with a range of 1.6 fm leading to a volume integral of 446 MeV fm^3; this is consistent with interactions describing few-nucleon problems.[7] This interaction was folded with a Gaussian α-particle internal wave function of Koepke *et al.*[8] The details of the microscopic calculations are similar to those of Ref. 4. There it was also shown that the differential cross sections are not sensitive to the bound-state geometry. Optical parameters of volume Woods-Saxon shape have been derived from a fit to elastic scattering data: $V_0 = 181.1$ MeV, $r_0 = 1.402$ fm, $a_0 = 0.606$ fm, $W_v = 13.91$ MeV, $r_w = 1.30$ fm, and $a_w = 0.657$ fm. It should be noted that small changes of the optical parameters do not affect the angular distribution in the forward-

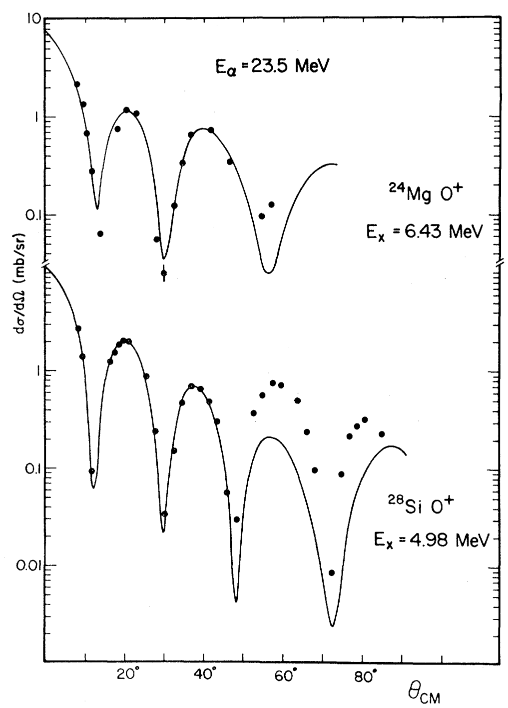

FIG. 1. Differential cross section for inelastic α scattering to first excited 0^+ states in ^{24}Mg and ^{28}Si. The solid lines are the result of distorted-wave Born-approximation calculations using microscopic form factors.

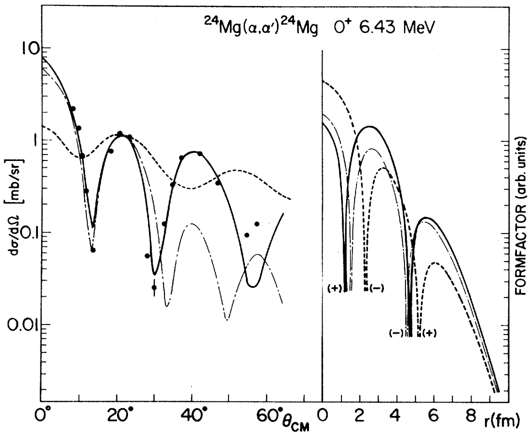

FIG. 2. Dependence of the shape of the angular distributions and the form factor on the shell-model configuration. Broken line: $(1d_{5/2})(2s_{1/2})^{-1}$ particle-hole components only. Dash-dotted line: $(1d_{5/2})(2s_{1/2})^{-1}$ and $(1p_{1/2})(1d_{5/2})^{-1}$ components. Solid line: best fit using particle-hole components given in Table I.

angle region; only at angles larger than 50° are the inelastic cross sections more sensitive to changes in the optical parameters.

Within the $2s1d$ shell, only two particle-hole form factors $F_p(r) - F_h(r)$ contribute, the $(1d_{5/2}) \times (2s_{1/2})^{-1}$ and the $(1d_{5/2})(1d_{3/2})^{-1}$ form factors. From shell-model calculations[1] one expects dominant $(1d_{5/2})(2s_{1/2})^{-1}$ particle-hole contributions. The corresponding angular distribution (Fig. 2, dashed line) fits the data only very poorly. Adding only a $(1d_{5/2})(1d_{3/2})^{-1}$ component does not improve the fit. Again a rather structureless angular distribution was obtained. Reproduction of the deep minima observed experimentally requires a sizable $1p_{1/2}$ hole component in the wave functions. The dashed-dotted line in Fig. 2 is the result of a superposition of $(1d)(2s)^{-1}$ and $(1p)(1d)^{-1}$ components. To arrive at the best-fit curve (solid line in Fig. 2) also a large $(1d_{5/2})(1d_{3/2})^{-1}$ component was needed. Significant differences have been found in the structure of the angular distribution for ^{24}Mg and ^{28}Si (Fig. 1) resulting in a smaller $(1d_{5/2})(2s_{1/2})^{-1}$ particle-hole component for ^{28}Si than for ^{24}Mg (Table I). These differences can be explained in the deformed shell model by an assumption of a prolate shape for ^{24}Mg and oblate shape for ^{28}Si. In such a description the ^{24}Mg ground-state wave function contains comparable $2s_{1/2}$ and $1d_{3/2}$ components whereas for ^{28}Si much larger $1d_{3/2}$ components are found.

For the ^{28}Si data also a $(1d_{5/2})(2p_{3/2})^{-1}$ configuration was needed to get a reasonable fit up to large angles. However, because of the usual problems with the description of α scattering at larger angles (compound nuclear contributions or other processes which give rise to backward-angle anomalies[9,10]) the significance of these components is questionable. In Table I spectroscopic amplitudes are therefore given for a best-fit description with and without the $(1d_{5/2})(2p_{3/2})^{-1}$ components in the wave functions.

The spectroscopic amplitudes of Table I have been used to derive $E0$ matrix elements $\langle 0_f^+ | r^2 \times | 0_i^+ \rangle$. These were found in very good agreement with results from electron scattering[11] (Table II).

The present results were also compared with spectroscopic amplitudes which we derived from the shell-model wave functions of Wildenthal and McGrory[1] (Table I). These wave functions yield only a $(1d_{5/2})(2s_{1/2})^{-1}$ contribution of any significant strength. Furthermore, since $1p_{1/2}$ hole components are not included in these calculations

TABLE I. Spectroscopic amplitudes S_c for inelastic monopole transitions in ^{24}Mg and ^{28}Si.

Nucleus	Configuration c	S_c(exp)[a]	S_c(exp)[b]	S_c(shell model)[c]
^{24}Mg	$(1d_{5/2})(2s_{1/2})^{-1}$	0.25		0.20
	$(1d_{5/2})(1d_{3/2})^{-1}$	−0.49		0.03
	$(1p_{1/2})(1d_{5/2})^{-1}$	0.14		···
^{28}Si	$(1d_{5/2})(2s_{1/2})^{-1}$	0.11	0.08	0.24
	$(1d_{5/2})(1d_{3/2})^{-1}$	−0.43	−0.43	0.01
	$(1p_{1/2})(2s_{1/2})^{-1}$	0.06	0.06	···
	$(1p_{1/2})(1d_{3/2})^{-1}$	0.06	0.06	···
	$(1d_{5/2})(2p_{3/2})^{-1}$	···	0.08	···

[a]Without $(1d_{5/2})(2p_{3/2})^{-1}$ component and $\epsilon = 0.06$ for ^{24}Mg and $\epsilon = 0$ for ^{28}Si [see Eq. (4)].

[b]With $(1d_{5/2})(2p_{3/2})^{-1}$ component and $\epsilon = 0$.

[c]Ref. 1.

TABLE II. $E0$ matrix elements (fm^2) for monopole transitions in ^{24}Mg and ^{28}Si.

Nucleus	Present result	$(e,e')^a$	Shell model^b
^{24}Mg	7.3	6.39 ± 0.39	1.47
^{28}Si	7.7	6.8 ± 0.4	0.92

[a]Ref. 11. [b]Ref. 1.

these wave functions cannot describe the deep structure of the angular distribution. The strong disagreement between experimental and calculated values of the $(1d_{5/2})(1d_{3/2})^{-1}$ particle-hole amplitudes is similar to the discrepancy observed with single-nucleon pickup reactions[12] which also indicates the presence of a large $1d_{3/2}$ particle amplitude. Also, the $E0$ matrix elements obtained in the truncated space of Ref. 1 are in poor agreement with the data (Table II).

In conclusion we believe to have demonstrated that the monopole inelastic cross sections allow a remarkably sensitive test of nuclear wave functions. In particular we find strong evidence for significant $1p$ hole and $1d_{3/2}$ particle components in the wave functions of ^{24}Mg and ^{28}Si. We were also able to derive $E0$ matrix elements which show remarkably good agreement with inelastic-electron-scattering results. Possible contributions from two-step processes are currently being investigated.

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Fluctuation Effects on Directional Data*

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The problem addressed is that of determining best values and confidence limits for the amplitude and phase of an unknown harmonic amplitude vector, given a single set of directions measured on a circle. The previously known solution, for the limiting case of small fluctuation, is inapplicable to the great majority of cosmic-ray anisotropy measurements. Solutions are given that are thought to be valid for all cases likely to occur in practice.

The following results were obtained in response to a specific need that arose recently in the study of cosmic-ray directionality above 10^{19} eV, as determined by air-shower observations.[1] It was found almost immediately that they are also useful for reanalyzing a considerable body of accumulated data pertaining to lower air-shower energies. The results will be developed in more detail, and applied to cosmic-ray data from many sources, in forthcoming publications. They are presented here in general form with the thought that applications may be found in other areas of physics and astronomy.

I consider as data a set of N directions $\psi_1, \psi_2, \ldots, \psi_N$ such that $0 \leq \psi \leq 2\pi$. I assume that the individual values ψ_i have negligible error and