is the rotational constant of H_2; and V_c, which is the crystal field along the c axis and which is expected to be less than 0.01 cm^{-1}.[8] We have treated $\Gamma\rho$ and Γ^2/B as free parameters because we suspect that they may be strongly renormalized by zero-point motion. However, the value $\Gamma\rho = -0.012$ cm^{-1} which gives the best fit to the spectrum is unphysical and strongly suggests that the theory is in some way incomplete. Consequently, in collaboration with A. B. Harris, we have begun a comprehensive study of the other possible interactions which may influence the pair levels. Some of these are coupling to phonons, three-body interactions involving ϵ_0 and ϵ_2, dielectric screening effects, and admixture of higher J states into the pair wave functions.[9]

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Pre-equilibrium and Heat Conduction in Nuclear Matter

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A model for pre-equilibrium nucleon emission is formulated in terms of diffusion of heat in nuclear matter. A strong asymmetry in the angular distribution of evaporation products is predicted.

Macroscopic concepts in nuclear physics are a topic of high current interest. In heavy-ion physics, e.g., viscosity and "friction" are introduced in order to describe fusion reactions.[1] On the other hand the conventional statistical model[2] which assumes thermodynamical equilibrium (another macroscopic concept) in certain processes has recently been generalized in order to take into account pre-equilibrium behavior.[3] As far as we can gather no link has yet been established between pre-equilibrium phenomena and transport coefficients although it is clear that when two heavy ions collide, the rate at which equilibrium (if any) is reached is intimately connected with transport properties of nuclear matter (n.m.). Energy dissipation involves not only viscosity but also heat conductivity κ. It is the purpose of this paper to formulate a new approach to pre-equilibrium phenomena in which a natural link between heat conductivity of n.m. and pre-equilibrium phenomena is established and to suggest how κ can be measured experimentally. (A similar suggestion in high-energy physics has been made recently.[4])

It turns out that once the existence of κ is taken into account a new and surprisingly very large effect in the angular and energy distribution of decay products of nuclear reactions can be predicted, the investigation of which may lead to new insight into the problem of nuclear structure and pre-equilibrium behavior. It permits us among other things for the first time to follow experimentally the localization and propagation of heat in nuclear matter.

It is believed at present that an appreciable fraction of particle emission occurs before thermodynamical equilibrium is established.[3] The time evolution of this emission process has been described so far either by following the fate of a single nucleon in the nucleus (e.g., the cascade model[5]), or by discussing the time evolution of the exciton number (the exciton model[6]), or by combining these two approaches (the hybrid model[7]). None of these models predicts satisfactorily the angular distribution of pre-equilibrium nucleons. Furthermore, common to these models is the tendency to reduce by a statistical argument (Monte-Carlo, master equation) the problem to nucleon-nucleon scattering. On the other hand, since equilibrium is a thermodynamical concept it seems natural to try to investigate the evolution towards equilibrium by a thermodynamical model while the parameters of the model like κ are computed in a microscopic way.

Such an approach is suggested below in terms of the classical diffusion equation for the temperature density τ:

$$\rho\frac{\partial w}{\partial t}=c_p\frac{\partial \tau}{\partial t}=\operatorname{div}\left(\frac{\kappa}{\rho}\operatorname{grad}\tau\right), \qquad (1)$$

where w is the heat density, ρ the density of n.m., and c_p the specific heat at fixed pressure. We believe that the use of this equation corresponds in letter and in spirit to the macroscopic approach to nuclear phenomena advocated so far. This approach seems to be simple and intuitively transparent. In a grazing collision with momentum transfer q to the target, one can expect a local excitation ("hot spot") on the surface of the target if $|\vec{q}| \gg R^{-1}$, where R is the target radius (we use units $\hbar = c = k_B = 1$, where k_B is the Boltzmann constant). For mass number $A = 200$ this condition is satisfied already if $|\vec{q}| \gg 25$ MeV/c. A necessary condition for peripheral reactions is small-angle scattering of the projectile, i.e., $|\vec{p}_i| \gg |\vec{q}|$, where p_i is the incident momentum. (The same condition also ensures that the angular momentum transfer remains small.)

There exist essentially two ways in which this local excitation will be dissipated in the target: (i) A direct reaction in which a single nucleon will absorb the four-vector q. This would correspond to a knock-out process and is not of interest to us. The events corresponding to this process could be eliminated by measuring $\vec{q}$ in coincidence with the evaporation spectrum and by associating the direct products with the nucleons which have their momentum in the $\vec{q}$ range. (ii) Multiple scattering, through which the excitation will be shared by many nucleons. It is here that the diffusion equation should be applicable since Eq. (1) can describe the process of energy dissipation in n.m. once the direction of $\vec{q}$ has been forgotten by the target. Process (i) takes place on a time scale t_0 which is smaller than the time scale t_D of (ii). Essentially, $t_0 \leq \lambda/\bar{v}$, where λ is the mean free path of a nucleon in the nucleus and $\bar{v}$ is its average velocity.

After t_0 the number of scattering processes in the nucleus increases very fast so that the reaction should be described in a statistical way.[3] The advantage of the diffusion-equation approach is that it describes the space-time evolution until equilibrium is reached. In this way the link between transport phenomena and pre-equilibrium states is clearly exhibited.

By measuring the temperature distribution of the excited nucleus via its decay products one can in principle obtain the information about κ that one is interested in. Indeed, it is reasonable to assume that the energy distribution of the nucleons emitted by the excited target is determined by the temperature distribution at the surface of the nucleus which for reasons of simplicity is assumed to be spherical. In a grazing collision the excitation of the target is asymmetric (cf. Fig. 1) and the initial condition with which Eq. (1) has to be solved will reflect this asymmetry:

$$\tau = T_i(q_0)\delta(\vec{r}-\vec{a}) \text{ at } t=0, \qquad (2)$$

where $\vec{a}$ is the vector of the "hot spot" and T_i

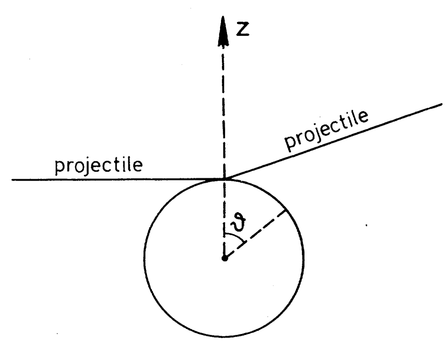

FIG. 1. Grazing collision on a spherical nucleus. The projectile is assumed only to "touch" the surface of the target. θ is the emission angle with respect to $-\vec{q}$ of a nucleon from the excited nucleus in the frame $\vec{q}=0$.

is the initial temperature determined[8] by the energy transfer q_0. (The δ-like excitation is of course an oversimplification dictated here only by mathematical convenience.) Then, solution τ of Eq. (1) will depend on θ (cf. Fig. 1). Moreover, since τ will also depend on the heat conductivity which enters as a parameter, Eq. (1), it follows that by measuring the θ dependence of τ one might be able to get insight into the value of κ.

In order to solve Eq. (1) we have to specify also a boundary condition for τ. In our case this condition should read

$$\partial\tau/\partial r \sim j_r^{\,w} = 0 \text{ at } r = R, \tag{3}$$

where $\vec{j}^{\,w}$ is the heat-flux density. This condition is obviously fulfilled as long as no particle has been emitted by the nucleus. It is reasonable to assume that after the emission of a particle the temperature gradients become so small that the resulting temperature field is homogeneous and no significant diffusion takes place anymore. A more general boundary condition, $\partial\tau/\partial r \neq 0$, does not introduce important changes in the form of the solution.[4,9]

Equations (1)–(3) define our diffusion problem. Given its solution $\tau(r,\theta,t)$ one can calculate the particle spectra in the direction of the $-\vec{q}$ axis which we identify with the z axis of Fig. 1, and opposite to it.[10] The difference of these spectra is what we call asymmetry and what we suggest be measured. Before discussing the form of the solution we have to face the problem of κ and c_p for n.m. Transport coefficients for n.m. can in principle be computed once the wave function of the nucleus is given. In practice, however, this is a very difficult task if one considers a realistic nuclear model and this problem is under investigation. For a Fermi gas, which might present a reasonable picture at least for high-excitation energies, the thermal conductivity is given by the expression[11]

$$\kappa = \frac{7}{48\pi 2^{1/2}} \frac{\epsilon_F^{\,3/2}}{m^{1/2}} \frac{1}{T} \frac{1}{Q}, \tag{4}$$

where ϵ_F is the Fermi energy, m the nucleon mass, T the temperature, and Q is of the order of 2×10^{-26} cm^2. The specific heat is[12]

$$c_p = \tfrac{1}{2}\pi^2 T/\epsilon_F. \tag{5}$$

As long as κ and c_p are T dependent Eq. (1) is a nonlinear equation and its solution can probably be found only by numerical integration. At present, however, we are primarily interested in the qualitative features of the phenomenon and for this purpose an analytic expression would be extremely useful. Fortunately enough, such a solution exists if κ and c_p are assumed to be constants. As mentioned above, the exact T dependence of κ and c_p for n.m. is not known, and in absence of better knowledge we will assume in applications that in a first approximation expressions (4) and (5) can be used to calculate average values in the T range of interest,

$$\langle\kappa\rangle = (T_i - T_f)^{-1}\int_{T_f}^{T_i} \kappa(T')\,dT'; \qquad \langle c_p\rangle = (T_i - T_f)^{-1}\int_{T_f}^{T_i} c_p(T')\,dT'. \tag{6}$$

Here T_f corresponds to the threshold energy for nucleon emission (e.g., for $A=200$, $T_f \simeq 0.7$ MeV).

With these approximations Eq. (1) reduces to the classical diffusion equation

$$\partial\tau/\partial t = \chi\Delta\tau, \tag{7}$$

where $\chi = \langle\kappa\rangle/\rho c_p$. This equation with conditions (2) and (3) can be solved exactly[13] by separating the variables t, r, and θ [because of (2) the problem has φ symmetry]. The solution reads

$$\tau(r,\theta,t) = \frac{3T_i(q_0)}{4\pi R^3} + \frac{T_i(q_0)}{2\pi}(rR^5)^{-1/2}\sum_{n=0}^{\infty}(2n+1)P_n(\cos\theta)\sum_{l=1}^{\infty}\frac{Z_{nl}^{\,2}J_{n+1/2}(R^{-1}rZ_{nl})\exp[-\chi Z_{nl}^{\,2}R^{-2}t]}{[Z_{nl}^{\,2}-n(n+1)]J_{n+1/2}(Z_{nl})}, \tag{8}$$

where $J_{n+1/2}(Z)$ are the Bessel functions of fractional order and Z_{nl} is the lth positive zero of the equation

$$(d/dZ)j_n(Z) = 0, \tag{9}$$

where j_n are the spherical Bessel functions.

To relate the θ dependence of τ to an observable asymmetry we assume that nucleon emission takes place from the surface of the excited nucleus and that it can be described in a thermodynamical way as in nuclear equilibrium calculations.[2] The emission probability per time unit, P, of a nucleon of kinetic energy ϵ from a nucleus of temperature T is, in the rest frame of the excited nucleus ($\vec{q}=0$), given by

$$P(\epsilon, T) \sim \sigma_{inv}\epsilon \exp[-(\epsilon+S)/T], \quad (10)$$

where σ_{inv} is the cross section for the inverse process and S is the nucleon separation energy.

In the energy range we are interested in σ_{inv} does not depend appreciably[2] on ϵ. The θ dependence of T implies that P will also depend on θ (measured in the system $\vec{q}=0$) and this is the essence of the asymmetry effect. From the foregoing it follows that the asymmetry effect implies the measurement, event by event, of (1) the direction and value of $\vec{q}$ in coincidence with (2) the evaporation products in peripheral reactions.[14] So far, however, only aspects (2) have been studied. That is why a direct comparison of the predictions of this paper with existing data is not possible. Nevertheless some qualitative features seem to emerge already. Since T depends only on q_0 (via T_i) and not on p_i (incoming momentum) it follows that in this model the cross section for pre-equilibrium emission does not depend on p_i. This is in agreement with what is known so far from experimental observation.[3]

By integrating Eq. (10) over angles we get the energy distribution of the evaporated nucleons. Since in our mechanism the evaporation spectrum gets also contributions from higher temperatures than those considered in the conventional statistical model it is clear that the contribution to the spectrum of higher ϵ is enhanced in comparison with the equilibrium spectrum. This again is confirmed by experiment. A more detailed discussion of these points will be given elsewhere. The asymmetry will manifest itself among others in (i) a characteristic angular dependence of the emission cross section and (ii) the θ dependence of the average kinetic energy $\bar{\epsilon}$ of particles emitted. As expected $\bar{\epsilon}$ is larger for nucleons emitted from the hemisphere which contains the "hot spot." In order to compute $\bar{\epsilon}$ we consider

$$\langle \epsilon(T(R,\theta,t))\rangle = \frac{\int_0^{\epsilon_{max}} \epsilon P(\epsilon,T)\,d\epsilon}{\int_0^{\epsilon_{max}} P(\epsilon,T)\,d\epsilon} = 2T(R,\theta,t)\,\frac{1-(1+\eta+\frac{1}{2}\eta^2)\exp[-\eta]}{1+(1+\eta)\exp[-\eta]}, \quad (11)$$

where $\eta = \epsilon_{max}/T$. $\bar{\epsilon}$ can be obtained from $\langle\epsilon\rangle$ by averaging over the time interval during which pre-equilibrium emission takes place, i.e.,

$$\bar{\epsilon}(\theta) = \frac{1}{t_D - t_0}\int_{t_0}^{t_D} dt \langle \epsilon(T(R,\theta,t))\rangle . \quad (12)$$

[We recall that $t_0 \lesssim \lambda/\bar{v}$ and $t_D \simeq (\chi/R^2)^{-1}$.]

If one neglects the exponential terms in Eq. (11), it follows that the asymmetry of the particle average energy is given by the temperature asymmetry

$$\delta_T = \frac{\bar{T}(\theta=0) - \bar{T}(\theta=\pi)}{\bar{T}(\theta=0) + \bar{T}(\theta=\pi)}, \quad (13)$$

where $\bar{T}$ is the time-averaged temperature at the nuclear surface. A quantitative study[4,10] of δ_T shows that it increases with χ^{-1}. This is easy to understand since the smaller χ the more time it takes to reach equilibrium and hence the more probable is pre-equilibrium decay. The temperature dependence of the coefficients c_p and κ manifests itself in a variation of χ with q_0 such that $\chi \sim q_0^{-1}$. Thus, the asymmetry increases monotonically with the energy transfer q_0.

Furthermore δ_T increases with A. This is a consequence of the fact that the asymmetry is a finite-size effect. To get an estimate of the order of magnitude of the effect we calculated δ_T for $A=100$, $q_0=30$ MeV; the mean free path λ is estimated to be $\lambda \simeq 2r_0$, $r_0 = 1.25$ fm. This corresponds[8] approximately to $T_i \simeq 8$ MeV. For this case, the resulting value for $\chi R^{-2} t_0$ is $\simeq 2.4\times10^{-2}$ leading to an asymmetry $\delta_T \simeq 0.73$, not far from the maximum of 1, while as long as the local excitation effect is neglected, $\delta_T = 0$. It follows that the propagation of heat in n.m. leads to new and important observable consequences. Given the implications of the effect discussed experimentalists are urgently invited to look into this problem.

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[10]We consider essentially two extreme cases of peripheral reactions: The momentum transfer $\vec{q}$ points towards the center of the nucleus or opposite to it. In the latter case, however, it seems reasonable to assume a direct character of the reaction instead of a diffusion-type process.

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Strong Polarization of the $1p$-Shell Core in the Lowest 0^+ States of ^{24}Mg and ^{28}Si†

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Inelastic α scattering to the first excited 0^+ states in ^{24}Mg and ^{28}Si has been measured at $E_\alpha = 23.5$ MeV and analyzed in a microscopic model. Strong evidence for the need of including $1p_{1/2}$ hole components in the wave functions of the lowest 0^+ states was found. Deduced $E0$ matrix elements are in surprisingly good agreement with results from electron scattering.

In most recent shell-model calculations for nuclei in the middle of the $2s1d$ shell it is assumed that the ^{16}O core is closed.[1,2] However, nuclei in this region are known to be strongly deformed; thus, the validity of this assumption is open to question. Often, polarization of the core can be determined from a measurement of single-particle-transfer transitions which are forbidden by the simple shell model, e.g., stripping to hole states. However, this method cannot be used to determine a possible $1p$ core polarization in the middle of the $2s1d$ shell where $1p$ hole and $2p$ particle states appear to be mixed,[3] since $2p$ particle stripping is strongly enhanced over $1p$ hole stripping and $1p$ and $2p$ stripping do not lead to significantly different angular distributions.

Another tool for determining core polarization is the study of inelastic monopole transitions induced by strongly absorbed particles.[4] Here the angular distributions of the inelastically scattered particles are extremely sensitive to the radial form factors. Because these form factors are significantly different for $1p$ hole and $2p$ particle components, a study of these monopole transitions may allow a determination of $1p$ core polarization.

In the present work monopole transitions were measured in inelastic scattering of α particles on ^{24}Mg and ^{28}Si. These nuclei, which are assumed to have prolate as well as oblate shapes,[5] were chosen to look for a possible dependence of the monopole transitions on the sign of the nuclear deformation.

The experiments have been performed with use of a 23.5-MeV α beam from the University of Minnesota's model MP tandem Van de Graaff accelerator. Scattered α particles were detected by position-sensitive solid-state detectors placed along the focal plane of an Enge split-pole magnetic spectrometer. Targets of enriched ^{24}Mg and $^{28}SiO_2$ approximately 50 μg/cm^2 thick were used. Special care was taken in focusing and collimating the α-particle beam to reduce spectral background, so that the cross sections both