compounds.[10,11] A contribution of acoustic SM in the gap region would then increase the height of the first peak at 44 meV, but would leave the qualitative featrues of $g^s(\omega)$ unchanged.

We have verified the strong contribution of acoustic SM at low energies and proved the existence of SM in the acoustic-optical bulk gap, corroborating experimentally the results of many theoretical investigations.[1-6] Parallel to further experimental efforts, it would be interesting to investigate theoretically the SM of TiN using an appropriate model,[11] considering especially the following new questions: What are the conditions for a nonanomalous dispersion of the Rayleigh SM branch and for the existence of acoustic SM above the acoustic bulk bands?

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Cyclotron Resonance of Localized Electrons on a Si Surface

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Cyclotron resonance of electrons in an inversion layer on a Si (100) surface is studied in the regime of low electron densities ($n_s < 10^{12}$ electrons/cm^2). A distinct and sample-dependent shift of the resonance to lower magnetic field is observed as n_s is decreased and electron localization is expected to occur. We give a simple interpretation of the effect in terms of electronic states bound in a harmonic-oscillator potential.

Cyclotron resonance of electrons in an inversion layer on the surface of Si has been observed recently.[1-3] Such experiments are proving a successful tool for the investigation of the proper-

ties of the two-dimensional electron gas induced at the semiconductor surface by an electric field. Cyclotron-resonance signals have been reported[3] on the (100) Si surface for a wide range of surface electron densities, down to as low as 0.4 $\times 10^{12}$ electrons/cm^2.

In the regime of low densities of conduction electrons at the Si-SiO_2 interface, both Mott[4] and Stern[5] have suggested that because of potential fluctuations at the Si surface there may occur localization of the electronic states. These potential fluctuations are thought to arise from immobile charges in the oxide or at the interface, as well as from surface irregularities. As a result, conduction at low temperatures is by a variable-range hopping mechanism. The mobility measurements of Pepper and co-workers[6-8] and Tsui and Allen[9] confirm the expected electron localization whenever the density n_s is sufficiently small ($n_s < 10^{12}$ electrons/cm^2). Earlier experiments of Fang and Fowler,[10] as well as the more recent work of Komatsubara *et al.*[11] and Tidey and Stradling,[12] all find a plausible explanation for the observed mobilities in terms of bound electronic states. The latter two experiments observe definite binding energies below the conduction-band tail. All the experiments show that with increasing surface electron density the activation or binding energy diminishes rapidly until one has metallic conduction at about 1×10^{12} electrons/cm^2.

We want to report here results of cyclotron-resonance studies at very low electron densities in which distinct and sample-dependent shifts of the cyclotron-resonance peak have been observed. Figure 1 shows the experimental data for a particular sample. The transmitted power is recorded as a function of the applied magnetic field for a range of relevant gate voltages. The experimental apparatus and techniques are essentially the same as in our previous work.[1-3] With decreasing gate voltage V_g the cyclotron-resonance absorption maximum is found to shift first upward in field. After the initial small increase in the resonance field a pronounced shift to lower fields together with an increase of relative linewidth $\Delta H/H$ is observed. The variation of the resonance field with the surface electron density n_s is found to differ from sample to sample as shown in Fig. 2. For the sample listed as $A1$ no decrease of the resonance field was discernible down to the very lowest electron concentrations at which the signal could be observed. Samples listed as $D3$ and $D5$ both show the resonance

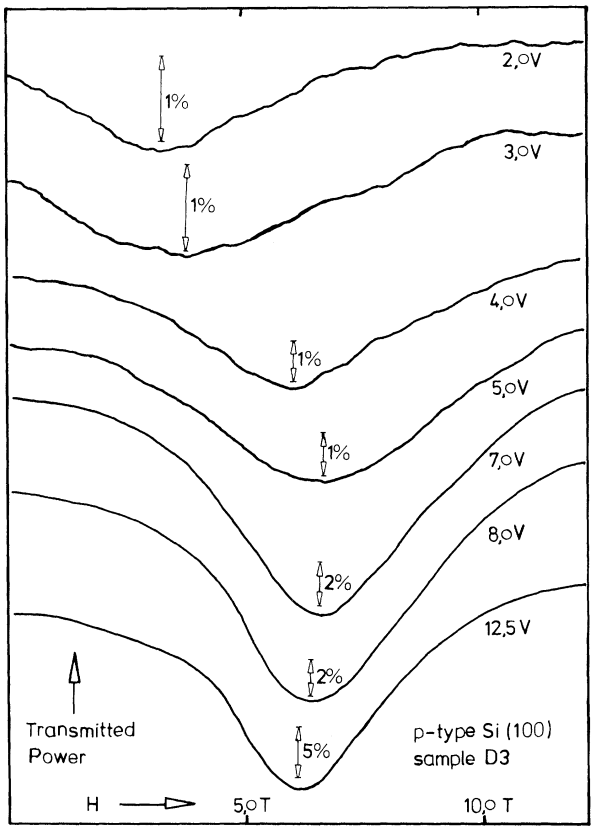

FIG. 1. Recorder tracings of transmitted power as a function of magnetic field applied normal to the surface of the sample. The sample is boron doped ($N_A \approx 1.3 \times 10^{15}$ cm^{-3}), p-type material with a thermally grown oxide 2330 Å thick. The experimental frequency is 890.7 GHz; the temperature 4.2°K. The voltages applied to the gate electrode are as indicated. Absorption is in terms of the relative change at the detector.

to shift to low fields for the same range of n_s. The relation of n_s to gate voltage V_g is determined from an extrapolation of the quantum oscillations that are observed in cyclotron resonance at high gate voltages.[13,14]

Every one of the samples that we have studied shows a distinct and significant increase of the resonance field H_{res} as n_s is decreased below 1 $\times 10^{12}$ electrons/cm^2. Moreover, within experimental uncertainty the initial rise of H_{res} is the same for all samples. This observation leads us to believe that the shift to higher field is an intrinsic effect, one that is characteristic of the ideal SiO_2-Si interface and that could possibly be described as a rise in the effective mass. A comparison with the mass increase observed in Shubnikov–de Haas measurements,[15] however, shows significant disagreement. The Shubnikov–

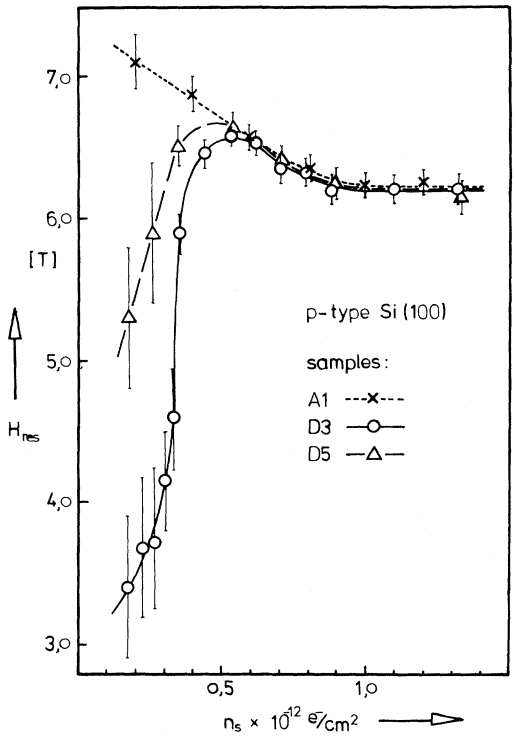

FIG. 2. Cyclotron-resonance field H_{res} versus surface electron density n_s for three of the samples that have been studied. The error bars represent an estimate of uncertainty in the resonance-peak position based on the signal-to-noise ratio and including the possibility of laser drift. Traces were averaged over repeated runs. The relative shift is known with considerably greater accuracy than the error bars reflect.

de Haas measurements give a rise of the effective mass m^* from $0.210m_0$ at an electron density $n_s = 3\times10^{12}$ electrons/cm^2 to $0.225m_0$ at $n_s = 0.7\times10^{12}$ electrons/cm^2. The cyclotron-resonance experiments yield an increase of m^* from about $0.197m_0$ to $0.218m_0$ when we decrease the electron density from 1×10^{12} electrons/cm^2 to 0.4×10^{12} electrons/cm^2.[3]

The strong downward shift of H_{res} is evidently sample dependent. The strongest shift was observed for samples with low electron mobilities (such as $D3$ and $D5$), while the very best samples, with the narrowest cyclotron-resonance lines (for example $A1$), do not show a decrease in H_{res} down to the very lowest electron densities. As the line shifts to lower fields the relative linewidth $\Delta H/H_{res}$ is found to increase.

We propose that the downward shift of the resonance field is caused by electron localization in the fluctuating surface potential at the Si-SiO_2 interface. At the very low electron concentrations where we see the shifted resonance, electrons are assumed to occupy the surface in little puddles where the electrostatic potential energy has a local minimum. The depths of the potential wells are expected to be of order 10 meV, and their ranges of order 100 Å. The relevant magnetic length, i.e., the spread of the electrons in the lowest, $n=0$ Landau level, is $(\hbar/eH)^{1/2} \sim 100$ Å. For simplicity, and only in order to illustrate qualitatively the effect of electron localization on the cyclotron resonance, we take a particularly simple model for the surface potential well. We take a potential in the form of a trough, infinitely long in the y direction and varying parabolically with x as $\frac{1}{2}m^*\omega_0^2x^2$. For the kind of numbers we quoted before, ω_0 would be on the order of 10^{13} Hz and thus comparable with the experimental frequency. The magnetic field we represent with the vector potential in the Landau gauge, $\vec{A} = -H(0, x, 0)$. In the usual fashion the magnetic confinement of the electrons is thus in a parabolic potential of the form $\frac{1}{2}m^*\omega_c^2(x - x_0)^2$, where ω_c and x_0 are the usual parameters of the "electron-in-a-magnetic-field" problem. The solutions for the problem of the localized electrons in the magnetic field are a set of harmonic-oscillator levels equally spaced as $\hbar\omega_{res}$, with ω_{res} given by $\omega_{res} = (\omega_0^2 + \omega_c^2)^{1/2}$. This expression is strictly valid only for the assumption of a parabolic potential well infinitely extended in the x direction.

This simple argument contains the essential physics and predicts the downward shift of the resonance as observed in the experiment. We explain the variation of H_{res} with n_s in terms of the decreasing strength of binding (i.e., decreasing ω_0) of the electrons near the Fermi energy E_F as the density n_s increases. The latter statement is equivalent to the fact that in reality the fluctuating potential is quite nonparabolic. The increased linewidth observed in the experiments we attribute to inhomogeneous broadening from a statistical distribution of the binding potentials.

Our model to explain the observed resonance shifts is readily generalized to two dimensions and a more realistic model of the potential. This refinement and a more substantial treatment of our suggestion has just been completed by Mikeska and Schmidt.[16] The increased splitting of the Landau levels perturbed by a binding potential is a very general result. The effect of an attractive impurity Coulomb potential[17,18] or of the confinement of the electron in a thin slab[19] also leads to an increase of the Landau level separation.

Our considerations only involve the effects of long-range surface potentials and consequent inhomogeneous broadening. In the free-electron calculation for cyclotron resonance, lifetime broadening, i.e., homogeneous broadening, can also produce a significant shift of the transmission minimum whenever $\omega\tau < 1$. If, using the free-electron calculation, we attempt to fit our line shapes, we obtain for all data $\omega\tau > 1$. However, as Ando and Uemura[13] point out, the classical description is totally inadequate for the two-dimensional electron gas at low temperatures and in high magnetic field. Ando and Uemura have made a calculation involving lifetime broadening that also gives a shift in the resonance at low values of n_s. We find, however, that their model does not explain the observed shifts satisfactorily, because it implies a relation of linewidth and shift which is not found in the experiments.

We believe the present model for the effect of a magnetic field on bound electron states accounts for the recently reported observation of negative transverse magnetoresistance in a Si inversion layer.[20] The authors of the magnetoresistance study note explicitly that their effect is most pronounced when the conduction is by hopping between localized states. Our proposal is simply that the application of the magnetic field raises the electron levels in the binding potential well closer to the edge of that well. It acts to facilitate the hopping process by lowering the activation energy and thus improves the conductivity.

There are two interesting consequences of our proposed explanation that merit further discussion. For one, the experimental frequency-field relation for the cyclotron-resonance peak should not be linear but should go according to something like the square-root relationship cited earlier. The model also predicts that raising the temperature should reduce the influence of the binding potentials. Such frequency and temperature experiments are currently in progress.

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