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[8]$|C_{1,\rho^0}|$, $|C_{1,\omega_8}| \ll 1$, and $|C_{1,\rho^0}|$, $|C_{\rho^0,\omega_8}| \ll 1$ because of the smallness of color-singlet-octet mixing and of color-isospin violation, respectively. Because of these simultaneous suppressions of C_{1,ρ^0}, it may be that $|C_{1,\rho^0}| \ll |C_{1,\omega_8}|, |C_{\rho^0,\omega_8}|$. This has been pointed out to the authors by V. Rittenberg.

SU(4) Symmetry and Nonleptonic Decays*

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I derive the new relation $\Lambda_-^0:\Sigma_0^+:\Xi_-^- = 1:-\sqrt{3}:-2$ for the S-wave amplitudes of the hyperon nonleptonic decays, assuming SU(4) 20-plet dominance for the weak interaction of current-current type. I also discuss the nonleptonic decays of charmed mesons.

The purpose of this Letter is to discuss the nonleptonic decays of the hyperons and charmed mesons, by assuming the validity of the mechanism proposed by Glashow, Iliopoulos, and Maiani[1] (GIM mechanism), which can suppress the strangeness-changing neutral current in semileptonic decays. The weak current is of the form[1,2]

$$J_\mu = \bar{u}\gamma_\mu(1+\gamma_5)(\cos\theta d + \sin\theta s) + \bar{c}\gamma_\mu(1+\gamma_5)(\cos\theta s - \sin\theta d), \qquad (1)$$

where θ is the Cabibbo angle and u, d, s, and c represent quark fields of fractional charges. The strong interaction possesses approximate SU(4) symmetry. I assign meson multiplets to $\underline{15}\oplus\underline{1}$ and the baryon multiplet $(\frac{1}{2}^+)$ to $\underline{20}$,[3] as usual.

Nonleptonic decays occur through the current-current interactions which may be mediated by weak bosons. The bilinear form contains two parts[4] belonging to a 20-dimensional[9] and an 84-dimensional SU(4) representations; denote them by traceless tensors $T_{[cd]}{}^{[ab]}$ and $T_{\{cd\}}{}^{\{ab\}}$. A generalization of the SU(3) octet dominance is the SU(4) 20-plet dominance,[4] since the SU(3) decompositions are as follows: $\underline{20}\supset\underline{8}$ and $\underline{84}\supset\underline{27}\oplus\underline{8}\oplus\underline{1}$ for $\Delta C=0$. Thus I assume the $\underline{20}$-plet dominance throughout this Letter.

Now it is straightforward to discuss the nonleptonic decays of the baryons. We can easily derive[10]

$$\Lambda_-^0:\Sigma_0^+:\Xi_-^- = 1:-\sqrt{3}:-2, \qquad (2)$$

for the S-wave amplitudes. [I only consider CP-conserving interactions.] The derivation is analogous to that of the Lee-Sugawara relation for the S-wave amplitudes in the SU(3) symmetry. I shall sketch the derivation. The parity-nonconserving part of the matrix element of the nonleptonic decay has the following form[11]:

$$\begin{aligned} &a(T_{[kl]}{}^{[ij]}\bar{B}_{[nm]}{}^{k}B_i{}^{[nm]}M_j{}^{l} - T_{[ij]}{}^{[kl]}\bar{B}_{[nl]}{}^{i}B_k{}^{[nm]}M_m{}^{j})\\ &+b(T_{[kl]}{}^{[ij]}\bar{B}_{[im]}{}^{n}B_j{}^{[km]}M_n{}^{l} - T_{[ij]}{}^{[kl]}\bar{B}_{[km]}{}^{j}B_n{}^{[im]}M_l{}^{n})\\ &+c(T_{[kl]}{}^{[ij]}\bar{B}_{[ij]}{}^{l}B_m{}^{[kn]}M_n{}^{m} - T_{[ij]}{}^{[kl]}\bar{B}_{[kn]}{}^{m}B_l{}^{[ij]}M_m{}^{n})\\ &+d(T_{[kl]}{}^{[ij]}\bar{B}_{[ij]}{}^{m}B_m{}^{[kn]}M_n{}^{l} - T_{[ij]}{}^{[kl]}\bar{B}_{[kn]}{}^{m}B_m{}^{[ij]}M_l{}^{n}), \end{aligned} \qquad (3)$$

where I have dropped the reference to the spinor structure. This leads to the final expressions for the s-wave amplitudes,

$$\Lambda_-{}^0 = -(1/\sqrt{6})(a-d)T,$$
$$\Sigma_0{}^+ = +(1/\sqrt{2})(a-d)T,$$
$$\Xi_-{}^- = +\sqrt{\tfrac{2}{3}}\,(a-d)T, \qquad (4)$$

as well as the $\Delta I = \frac{1}{2}$ rule. Here $T \equiv T_{21}{}^{31} = -T_{24}{}^{34}$. Thus we get the relation (2), which includes the Lee-Sugawara relation.

The experimental values are[12] $\Lambda_-{}^0 : \Sigma_0{}^+ : \Xi_-{}^- = (+1.545 \pm 0.024) : (-1.568 \pm 0.142) : (-2.020 \pm 0.029)$ [unit $= 10^5 m_\pi{}^{-1/2}\,\mathrm{sec}^{-1/2}$]. I think that the agreement of my relation with the experiment is satisfactory, since the SU(4) symmetry is rather strongly broken. The relation as well as the $\Delta I = \frac{1}{2}$ rule determines the relative magnitudes and relative signs of all the [experimentally possible] S-wave amplitudes of the hyperon decays, except one amplitude, say $\Sigma_+{}^+$. For the P-wave amplitudes we have no relations other than the $\Delta I = \frac{1}{2}$ rule.

For the nonleptonic decays of charmed baryons there are too many relations to discuss them here. I will report them elsewhere.

Now I discuss the nonleptonic two-body decays of the charmed pseudoscalar mesons, D^0 ($c\bar{u}$), D^+ ($c\bar{d}$) and F^+ ($c\bar{s}$). The discussion in this case is not as straightforward as in the case of the baryon decays, since in the SU(4) limit a pseudoscalar meson cannot decay to two pseudoscalar mesons[13] for the same reason as the K^0 meson cannot decay to 2π in the SU(3) limit.[14]

I assume the SU(4) symmetry breaking of the form $S_4{}^4 + \delta S_3{}^3$ [δ is a numerical parameter]. I take into account the effect of the symmetry breaking perturbationally, including arbitrary orders.

For instance, the matrix elements including the first order of the breaking are

$$T_\mp = T_{[kl]}{}^{[ij]} S_m{}^l M_i{}^k M_j{}^n M_n{}^m - T_{[ij]}{}^{[kl]} S_l{}^m M_k{}^i M_n{}^j M_m{}^n$$
$$\mp (T_{[kl]}{}^{[ij]} S_m{}^l M_i{}^m M_j{}^n M_n{}^k - T_{[ij]}{}^{[kl]} S_l{}^m M_m{}^i M_n{}^j M_k{}^n). \qquad (5)$$

The effective nonleptonic Hamiltonians, T_- and T_+, transform as $\underline{20}$ and $\underline{45} \oplus \underline{45}^*$, respectively. Let us assume again the $\underline{20}$ dominance (or the $\underline{45} \oplus \underline{45}^*$ suppression). It should be noted that this assumption is independent from that of the $\underline{20}$ dominance for the current$\times$current. [I shall discuss later the possible effect of the $\underline{45} \oplus \underline{45}^*$ parts.]

First I discuss the contribution from the term which is proportional to $\cos^2\theta$. The results are as follows:

$$D^0 \to \bar{K}^0 \eta_1 \ [D_1 \to \eta K_2, D_2 \to \eta K_1],$$
$$D^+ \to \text{forbidden}, \quad F^+ \to \pi^+ \eta_1, \qquad (6)$$

where η_1 is the SU(4) $\underline{1}$ component of the pseudoscalar meson. Thus the decay rates depend on how much $\eta(550)$ and $\eta(960)$ contain the $\underline{1}$ component in them. Here D_1 and D_2 are CP even and odd, respectively.

We can include the effect of the higher orders of the breaking $(S_4{}^4)^m (S_3{}^3)^n$ (m and n are arbitrary nonnegative integers). Under the assumption of the $\underline{45} \oplus \underline{45}^*$ suppression, possible two-body decays are as follows:

$$D^0 \to \bar{K}^0 \eta, \quad D^+ \to \text{forbidden},$$
$$F^+ \to \pi^+ \eta, K^+ \bar{K}^0, \qquad (7)$$

where η represents either $\eta(550)$ or $\eta(960)$. For example, by the breaking $S_4{}^4 \times S_3{}^3$, decays $D^0 \to \bar{K}^0 \eta(550)$ and $F^+ \to \pi^+ \eta(550)$ occur through the $M_3{}^3$ component, not through η_1. In this case the decay rates depend on the magnitude of the breaking only.

Next I discuss the term proportional to $\cos\theta \times \sin\theta$. By the breaking $S_4{}^4$, the following decays are possible in the case of the $\underline{45} \oplus \underline{45}^*$ suppression:

$$D^0 \to \pi^0 \eta_1, \eta\eta_1, \quad D^+ \to \pi^+ \eta_1, \quad F^+ \to K^+ \eta_1. \qquad (8)$$

The breaking $S_3{}^3$ only cannot cause any two-body decays. If we include the breaking $S_4{}^4 \times S_3{}^3$, we have an effective Hamiltonian $T_{[kl]}{}^{[ij]} S_j{}^l S_m{}^k$ which transforms as $\underline{15}$. Through this term the following decays are possible[15, 16]:

$$D^0 \to \pi^+\pi^-, 2\pi^0, K^+K^-, K^0\bar{K}^0, 2\eta, \pi^0\eta,$$
$$D^+ \to \pi^+\eta, K^+\bar{K}^0, \qquad (9)$$
$$F^+ \to \pi^+ K^0, K^+\pi^0, K^+\eta.$$

Even if we include all possible effects of the breaking, $(S_4{}^4)^m \times (S_3{}^3)^n$, the possible decays are included in Eqs. (8) and (9) under the assumption of the $\underline{45} \oplus \underline{45}^*$ suppression.

If the $\underline{45} + \underline{45}^*$ parts are not suppressed, contrary to the above assumption, then the D_1 meson decays to $\pi^{\pm}K^{\mp}$ and $K_2\pi^0$. Thus for charmed-particle searches it is crucial whether the $\underline{45}\oplus\underline{45}^*$ parts are suppressed or not.

If the $\underline{45}\oplus\underline{45}^*$ parts are suppressed, the decay modes including η may be substantial; from the above analysis we guess that the main modes of two-body decays may be $D^0\rightarrow\overline{K}^0\eta$, $D^+\rightarrow\pi^+\eta, K^+\overline{K}^0$ and $F^+\rightarrow\pi^+\eta$. The so-called energy crisis[17] may be due to the threshold effect of charmed mesons, decay products of which contain $\eta(550)$, since $\eta(550)$ decays to all neutral with the branching ratio 70%.

In this Letter I am not concerned with the origin of the $\underline{20}$ dominance for the current×current, but I comment that there are two possible explanations. One is that the $\underline{20}$ part is enhanced because of the short-distance behavior.[18] The other one is that the quarks obey Bose statistics.[19]

My final comment is as follows: The use of current algebra and the soft-pion technique for the S-wave amplitudes of the hyperon nonleptonic decays[20] leads to $\sqrt{3}\,\Sigma_0{}^+ + \Lambda_-{}^0 = \Xi_-{}^-$ and $\sqrt{6}\Lambda_-{}^0 + \sqrt{2}\,\Sigma_0{}^+ = \Sigma_+{}^+$. If we assume again the $\underline{20}$ dominance for the current×current, we obtain further $\Sigma_+{}^+ = 0$ (S wave), which reduces the above two relations to Eq. (2).[21]

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