bers of these bands are also plotted. Comparison of experiment with theory suggests that the 2.87-MeV state is likely the 3^- member of the 1^- band and that the new member of the 2.97-MeV doublet is likely the 4^- member of the 2^- band. The comparison also suggests that either the 3.59- or 3.68-MeV state is the 4^+ member of the 1^+ band, with perhaps a slight preference for the 3.59-MeV level.

Clearly, one or both members of the 4.20-MeV doublet have high spin. In any case, one member must have $J^\pi \geq 4^-$ or $\geq 5^+$. Thus a state here is a candidate for identification as the 4^- member of the 1^- band or the 5^- member of the 2^- band, or the 5^+ member of the 1^+ band or the 6^+ or 7^+ member of the 2^+ band. If one member is 4^-, the other is probably 5^-, 6^+, or 7^+, while if one is 5^-, the other is probably either 4^-, 4^+, 5^+, or 6^+. It is thus very likely that one of the members of this doublet is a 6^+ state.

The 4.51-MeV state appears to be a good candidate for the 4^- member of the 1^- band, or the 6^+ member of the g.s. band. One of the members of the 4.6-MeV doublet may be the 5^- member of the 2^- band, or the 5^+ member of the 1^+ band, or the 7^+ member of the g.s. band. If the 7^+ state is not contained in the 4.20-MeV doublet, then one of the 4.6-MeV states is the only other good candidate below 5 MeV. However, if the two 4.6-MeV states have comparable spins, then neither need be larger than 3. The 4.73- and 4.76-MeV states are candidates for either the 4^- member of the 1^- band, or the 5^+ member of the 1^+ band, or the 6^+ member of the g.s. band. If one of the 4.9-MeV states has low spin, the other might be the 5^+ member of the 1^+ band. Clearly, the γ decays of these levels must be studied in order to pin down their spins. But the present reaction provides a powerful tool for determining which states may have high spin.

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[1]E. C. Halbert, J. B. McGrory, B. H. Wildenthal, and S. P. Pandya, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1971), Vol. 4, p. 315.

[2]B. H. Wildenthal, private communication.

[3]J. B. McGrory and B. H. Wildenthal, Phys. Rev. C 7, 974 (1973).

[4]F. Ajzenberg-Selove, Nucl. Phys. A190, 1 (1972).

[5]R. R. Betts, H. T. Fortune, and R. Middleton, Phys. Rev. C 8, 660 (1973).

[6]R. R. Carlson and R. L. McGrath, Phys. Rev. Lett. 15, 173 (1965).

[7]J. L. Wiza, H. G. Bingham, and H. T. Fortune, Phys. Rev. C 7, 2175 (1973).

[8]F. Ajzenberg-Selove, H. G. Bingham, and J. D. Garrett, Nucl. Phys. A202, 152 (1973); J. D. Garrett, F. Ajzenberg-Selove, and H. G. Bingham, Phys. Rev. C 10, 1730 (1974).

[9]H. T. Fortune and H. G. Bingham, Phys. Rev. C 10, 2174 (1974).

[10]H. T. Fortune and R. R. Betts, Phys. Rev. C 10, 1292 (1974).

[11]D. J. Crozier and H. T. Fortune, Phys. Rev. C 10, 1697 (1974).

Superdense Matter: Neutrons or Asymptotically Free Quarks?

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We note the following: The quark model implies that superdense matter (found in neutron-star cores, exploding black holes, and the early big-bang universe) consists of quarks rather than of hadrons. Bjorken scaling implies that the quarks interact weakly. An asymptotically free gauge theory allows realistic calculations taking full account of strong interactions.

There are several astrophysical and cosmological situations where one needs the equation of state for matter of densities greater than 10^{15} g cm^{-3}: in particular, the center of a neutron star,[1,2] the early phases of the big-bang universe,[3] and black-hole explosions.[4] However, such densities might at first sight appear to be outside the range of normal physics, so that nothing can be said. In this note, we explain how recent developments in high-energy physics suggest precisely the opposite. Specifically, in the popular asymptotically free gauge theories of strong interactions, one can calculate thermodynamic quantities properly.

We first give arguments leading to this idea. It is commonly believed that hadrons consist of quarks[5-7] despite the apparent nonexistence of free quarks.[8] There are two main reasons for this belief. First, a quark model explains[5,6] many properties of the hadron spectrum, and of strong-interaction decays. Secondly we have Bjorken scaling[7] in the deep inelastic scattering of leptons by nucleons. Basically, this indicates that hadrons consist of pointlike objects (partons) which interact weakly with each other when close together. Analysis of the data indicates that partons are the fractionally charged spin-$\frac{1}{2}$ Gell-Mann–Zweig quarks. Since free quarks are not observed,[8] it is assumed that they are permanently bound in hadrons[9] by a mechanism as yet unknown, but much speculated on.

A neutron has a radius[10] of about 0.5–1 fm, and so has a density of about 8×10^{14} g cm^{-3}, whereas the central density of a neutron star[1,2] can be as much as 10^{16}–10^{17} g cm^{-3}. In this case, one must expect the hadrons to overlap, and their individuality to be confused. Therefore, we suggest that matter at such high densities is a quark soup. In such a system, long-range interactions are screened because of many-body effects,[11] and hence no problems will arise for any peculiar infrared behavior of quark binding forces. At short range, Bjorken scaling implies that the interaction is weak enough to use perturbation theory.[12] Bjorken scaling sets in[7] at $\lesssim 1$ GeV, so that short range starts at not very much less than a nuclear radius. Thus, plausibly, calculations should work at least above 5×10^{16} g cm^{-3}, and our picture is valid above nucleon density.

An obvious candidate for a detailed model is an asymptotically free quantum field theory.[12,13] In such a theory, the coupling constant is essentially small at short distances, and large at long distances. This is caused by renormalization effects, and is the opposite of what happens in quantum electrodynamics.

An asymptotically free theory of strong interactions is realized[12-14] by quarks bound by Yang-Mills fields (gluons).[15,16] The theory is symmetric under an SU(3) group which commutes with the ordinary SU(3) strong-interaction symmetry group. This new SU(3) group is called color, and was introduced so that baryons can be formed from quarks in an s-wave state. Quarks come in color triplets; the gluons are a single color octet, and are massless. The massless gluons and the asymptotic freedom presumably give bad long-distance behavior. The hypothesis[12-14] is that this confines the quarks, and that the only realizable free states must be color singlets.

The fields are as follows:

(a) Quark fields,

$$q_{ai}=\begin{pmatrix} u_r & d_r & s_r & \dots \\ u_w & d_w & s_w & \dots \\ u_b & d_b & s_b & \dots \end{pmatrix}. \tag{1}$$

Each component is a Dirac spinor. The color index is a, and i is the ordinary hadron symmetry index; the dots indicate possible charmed quarks needed to give satisfactory weak interactions.[13,17]

(b) An octet of color gauge fields $A_{\alpha\mu}$ ($\alpha=1,\dots,8$ and μ is a Lorentz index).

The Lagrangian is

$$\mathcal{L}=i\bar{q}_{ai}\gamma^{\mu}D_{ab\mu}q_{bi}-\bar{q}_{ai}M_{ij}q_{aj}-\tfrac{1}{4}F_{\alpha\mu\nu}F_{\alpha}^{\mu\nu}+\text{gauge terms}, \tag{2}$$

where the gauge terms are *à la* Fadeev-Popov.[16] The mass matrix M_{ij} is color symmetric. As usual, we have minimal coupling:

$$D_{ab\mu}=\delta_{ab}\partial_{\mu}-\tfrac{1}{2}ig\lambda_{ab}{}^{\alpha}A_{\alpha\mu}, \tag{3}$$

$$F_{\alpha\mu\nu}=\partial_{\mu}A_{\alpha\nu}-\partial_{\nu}A_{\alpha\mu}+gf_{\alpha\beta\gamma}A_{\beta\mu}A_{\gamma\nu}, \tag{4}$$

where g is the coupling constant and $\lambda_{ab\alpha}$ and $f_{\alpha\beta\gamma}$ are the generators and structure constants of SU(3). The β function of the renormalization-group equation is[12]

$$\beta=-g^{3}(33-2k)/48\pi^{2}, \tag{5}$$

where k is the number of quark triplets. Thus we have asymptotic freedom if $k<16$. Physically acceptable theories need only have $k=3$ or 4.

While the use of field theory in nonrelativistic many-body problems is well known,[11] only recently has the relativistic case been examined.[18,19] We follow here the methods of Bowers, Campbell, and Zimmerman.[18]

Thermodynamic quantities like the pressure P, the energy density $\rho = E/V$, the number density N/V, and the baryon number density $B/V = \frac{1}{3}N/V$ are obtained from the propagator. We work initially at zero temperature, which is a sensible high-density limit, and our units have $\hbar = c = 1$. The Fermi momentum of quarks of type i is $p_{\mathrm{F}i}$ ($i = u, d, s$). To derive the leading behavior of P, ρ and B/V for large p_{F} we write a renormalization-group equation. Using standard methods,[20] we have

$$\left[\sum_i p_{\mathrm{F}i}\frac{\partial}{\partial p_{\mathrm{F}i}} - \beta(g_R)\frac{\partial}{\partial g_R} + M_R\gamma_M(g_R)\frac{\partial}{\partial M_R} - D_X\right]X(p_{\mathrm{F}i}, g_R, M_R, \mu) = 0, \tag{6}$$

where X stands for P, ρ, N/V, or B/V and D_X is the mass dimension of X. We use a multiplicative mass renormalization, and γ_M is a matrix on the components of the quark-mass matrix. The solution is[20]

$$X(\kappa p_{\mathrm{F}i}, g_R, M_R, \mu) = \kappa^{D_X}X(p_{\mathrm{F}i}, g(\kappa), M(\kappa), \mu), \tag{7}$$

where κ is a number which scales up the Fermi momenta. Our theory is asymptotically free, so $g(\kappa) \to 0$ as $\kappa \to \infty$. Hence the form of $X(p_{\mathrm{F}i})$ for large p_{F} should be the same as for a free-quark gas. In renormalization-group arguments of this sort, difficulties may arise from infrared divergences, since $M(\kappa) \to 0$ as $\kappa \to \infty$.[21] However, interaction with the quark gas gives the gluons an effective mass; this effect is the same as Coulomb screening.[11] There are two relevant masses: the inverse screening length λ, and the plasma frequency ω_p. Our calculation of λ and ω_p used the diagram of Fig. 1. When $M \ll p_{\mathrm{F}}$ we find $\lambda^2 = \frac{1}{2}\sum_i g^2 p_{\mathrm{F}i}^2/\pi^2$ and $\omega_p^2 = \sum_i \frac{5}{24} g^2 p_{\mathrm{F}i}^2/\pi^2$. Thus, the Fermi momentum acts as an infrared cutoff: Infrared divergences worsen by a logarithm in each order[22] (giving an extra factor $\ln g$), but have an extra coupling factor g^2.

As a first approximation, we examine the equations of state for a free-quark gas. Now according to light-cone considerations,[23] quark masses are small, perhaps even $M_u = M_d \approx 5$ MeV, $M_s \approx 100$ MeV. Charmed quarks are presumably much more massive, and we ignore them. For large Fermi momenta[24] we have

$$B/V = \tfrac{1}{3}N/V = \tfrac{1}{18}d\sum_i p_{\mathrm{F}i}^3/\pi^2, \tag{8}$$

$$P = \tfrac{1}{24}d\sum_i p_{\mathrm{F}i}^4/\pi^2, \tag{9}$$

$$\rho = E/V = \tfrac{1}{8}d\sum_i p_{\mathrm{F}i}^4/\pi^2, \tag{10}$$

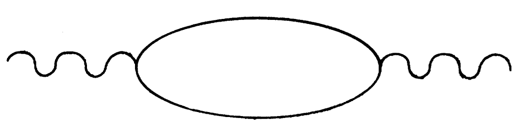

FIG. 1. Diagram giving mass to the gluons.

where d is the degeneracy factor, which in this case is 6, from the spin and color degrees of freedom.

The relative Fermi momenta for the u, d, and s quarks, electrons, and muons are determined[2] by requiring zero total electric charge and that the system be in equilibrium for the decays $d \rightleftharpoons u + l + \bar{\nu}$, $s \rightleftharpoons u + l + \bar{\nu}$, where l is e^- or μ^-, and $\bar{\nu}$ is the appropriate antineutrino. The neutrinos interact weakly, so that in a neutron star they escape,[2] and as usual $p_{\mathrm{F}\bar{\nu}} = 0$. As stated earlier, the lowest possible density at which our picture works is nucleonic; then Eqs. (8) and (10) indicate that the Fermi momenta are above the s threshold, if $M_s \lesssim 300$ MeV. When $p_{\mathrm{F}} \gg M_s$ we find that

$$N_u = N_d = N_s, \quad N_e = N_\mu = 0. \tag{11}$$

We now qualitatively compare the free-quark equations of state (8)–(10) with those for free nucleons. The difference arises because the degeneracy factor is 6 instead of 2, and because $B = \frac{1}{3}N$ rather than $B = N$: The pressure and energy density at a given baryon number density are both multiplied by $3^{2/3}$ at high density. Also, strange-particle production always happens.

Note that according to Eqs. (8)–(10), a baryon number density of 10^{40} cm^{-3} corresponds to a Fermi momentum for each quark species of ~ 1 GeV, a pressure of 2×10^{37} dyn cm^{-2}, and a density of 6×10^{16} g cm^{-3}. In our model $P = \frac{1}{3}\rho$ as $\rho \to \infty$. This is a consequence of the freedom of quarks at high density. We note that what we have said here is also relevant at nonzero temperature. The high-density relation $P = \frac{1}{3}\rho$ remains valid at all temperatures.[24] Assuming that quark masses are small, we see that production of quark-antiquark pairs will matter at lower temperatures than in conventional models. Also we might expect superfluidity and superconductivity, since the interquark forces are attractive in some channels.

Our basic picture then is that matter at densities higher than nuclear consists of a quark soup. The quarks become free at sufficiently high density. A specific realization is an asymptotically free field theory. For such a theory of strong interactions, high-density matter is the second situation where one expects to be able to make reliable calculations—the first is Bjorken scaling. Calculations become better as the density increases. Our main argument is, however, independent of details like choosing the quark masses, or the exact number and type of quarks. Since infrared problems cause trouble in Yang-Mills theories,[25] it is noteworthy that the gluons acquire a mass from many-body effects.

Other models used for high-density matter assume the observed hadrons to be basic entities. This applies to field theory models[18] and the Hagedorn model.[1,26] From our point of view, these ignore the fundamental physics, and tend to involve much over-counting of particle states.

The calculations given in this paper are clearly neither complete nor rigorous. We hope to consider in a future paper both fundamental field-theoretic problems and the astrophysical and cosmological implications of our ideas.

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[1]B. Carter and H. Quintana, Astrophys. Lett. 14, 105 (1973); V. Canuto, "Equations of State at Ultrahigh Densities, Pt. II" (to be published).

[2]Ya. B. Zel'dovich and I. D. Novikov, *Relativistic Astrophysics, Vol. 1: Stars and Relativity* (Univ. of Chicago Press, Chicago, Ill., 1971).

[3]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[4]S. Hawking, "Particle Creation by Black Holes" (to be published).

[5]H. J. Lipkin, Phys. Rep. 8C, 173 (1973).

[6]J. L. Rosner, Phys. Rep. 11C, 191 (1974).

[7]F. J. Gilman, in *Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, 1974,* edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975).

[8]E.g., Yu. M. Antipov *et al.*, Phys. Lett. 30B, 576 (1969); G. Gallinaro and D. Morpurgo, Phys. Lett. 23, 609 (1966).

[9]E.g., K. Johnson, Phys. Rev. D 6, 1101 (1972); A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974); H. B. Nielson and P. Oleson, Nucl. Phys. B61, 45 (1973).

[10]H. I. Miettinen, CERN Report No. TH. 1906 (to be published).

[11]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), and references therein.

[12]D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973), and 9, 980 (1974); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[13]S. Weinberg, Rev. Mod. Phys. 46, 255 (1974), and references therein.

[14]S. Weinberg, Phys. Rev. D 8, 4482 (1973).

[15]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[16]E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).

[17]A. De Rújula, H. Georgi, S. L. Glashow, and H. R. Quinn, Rev. Mod. Phys. 46, 391 (1974).

[18]R. L. Bowers and R. L. Zimmerman, Phys. Rev. D 7, 296 (1973); R. L. Bowers, J. A. Campbell, and R. L. Zimmerman, Phys. Rev. D 7, 2278, 2289 (1973), and 8, 1077 (1973).

[19]C. W. Bernard, Phys. Rev. D 9, 3312 (1974); L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974); S. Weinberg, Phys. Rev. D 9, 3357 (1974); R. E. Norton and J. M. Cornwall, "On the Formalism of Relativistic Many-Body Theory" (to be published).

[20]S. Weinberg, Phys. Rev. D 8, 3497 (1973); G. 't Hooft, Nucl. Phys. B61, 455 (1973); J. C. Collins and A. J. Macfarlane, Phys. Rev. D 10, 1201 (1974).

[21]See T. Appelquist and E. Poggio, Phys. Rev. D 10, 3280 (1974).

[22]T. Kinoshita, J. Math. Phys. (N.Y.) 3, 650 (1962).

[23]R. L. Jaffe and C. H. Llewellyn Smith, Phys. Rev. D 7, 2506 (1973); H. Leutwyler, Phys. Lett. 48B, 431 (1974).

[24]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, London, England, 1959).

[25]S. Weinberg, Phys. Rev. 140, B516 (1965).

[26]R. Hagedorn, Astron. Astrophys. 5, 184 (1970).