

ably caused by the interband transition due to the pseudopotentials  $U_{111}$  and  $U_{200}$  with vertical transitions at 0.4 and 1.5 eV.<sup>21</sup> The influence of the band splitting and the relative difference between interband and intraband transitions become less when the energy transfer to an electron exceeds the band splittings. Therefore, it is expected that the plasmon dispersion will eventually, for increasing  $k$ , be well described within the jellium model—as is indeed the case. When  $k$  increases further and equals a Brillouin-zone-boundary vector, there is the theoretical possibility of the splitting of the plasmon dispersion curve and the formation of plasmon bands.<sup>22,23</sup> Our measurements<sup>24</sup> did not unravel this effect. For still greater  $k$ , the short-range correlation between the electrons influences the dispersion of the plasmon.<sup>6,25</sup>

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## Colored-Quark Version of Some Hadronic Puzzles

S. Nussinov

*Department of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel*

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A simple picture based on colored quarks and gluons yields  $\alpha_\rho(0) = \frac{1}{2}$ ,  $\alpha_\Delta(0) = 0$ , and  $\alpha_p(0) = 1$ , and explains various features of Pomeron exchange.

I adopt  $SU(3) \otimes SU(3)'$  where strong interactions are mediated by  $(\underline{1} \times \underline{8})$  colored gluons and hadrons are exact color singlet  $q; \bar{q}$  ( $\underline{3} \times \underline{3}; \underline{3}^* \times \underline{3}^*$ ) bound states.<sup>1</sup> A mechanism like the one of Casner, Kogut, and Susskind (CKS)<sup>2</sup> is assumed to generate spontaneous color neutralization which allows  $e^+e^- \rightarrow$  hadrons at large  $Q^2$  to be computed from the pointlike perturbation diagram. I suggest a generalization which may also provide a recipe

for hadronic two-body, large- $s$ , small- $t$  processes.

(i) The computation is first performed in the gluons quark basis. A transverse-momentum cutoff is introduced, hopefully making the CKS one-dimensional model more relevant. Ladder-type diagrams are naturally selected by the leading-logarithm approximation applicable to the cut-off theory.<sup>3</sup>

(ii) Bubbles are generated on each gluon line, and by a spontaneous distortion via attraction of oppositely colored lines I arrive at bound states composed only of  $\bar{q}q$  ( $qqq$ ) systems in all channels.

(iii) This spontaneous transition from quark to particle basis is assumed—just as in the one-dimensional quantum electrodynamic (QED) case<sup>2</sup>—not to introduce qualitative changes, at least as far as energy behavior up to a factor  $(\ln s)^{\text{const}}$ .

A critical assumption [for Sections (A) and (B) but *not* for (C)] is the correspondence between the perturbative gluon expansion (i) and the quark loop expansion and finally the expansion in particle number.<sup>4</sup> Since a diagram with  $N$  loops leads to states with at least  $N$  particles an inequality<sup>5</sup> (for the lowest  $N$ 's) is more naturally implied so that the results of (B) and (C) become upper bounds for intercepts. The fact that these are actually saturated prompts me to make the stronger assumption (iii).

(A) *Meson trajectory*.—Let us first consider the contribution to the  $\text{Im}f_{ab \rightarrow ab}(s, t=0)$  for meson-meson scattering (the non-Pomeron component) generated by intermediate states consisting of just two mesons, which in the particle basis is given (up to logarithms) by  $s^{2\alpha_R(0)-1}$ .

We next turn to the standard planar duality diagram (dressed up by binding gluons for the incoming particles) and look for the lowest-order gluon diagram such that (a) when bubbles are sprung up on the gluons and collapsed by the nonperturbative infinitely many-gluon attraction to oppositely oriented quark lines, it reduces to the diagram describing the process at hand; (b) it is leading in the logarithmic approximation. The one-gluon ladder, Fig. 1(a), is then naturally selected and the various stages for its distortion or dressing up are indicated in Figs. 1(b) and 1(c). Figure 1(a) computed as standard Feynman diagram with quark exchange yields  $s^{2J_q-1}$  (up to finite  $\ln s$  power) and comparing with the earlier expression we

find

$$\alpha_R(0) = J_q = \frac{1}{2}. \tag{1}$$

Since SU(3) breaking will be introduced later by explicit soft mass insertions, this is to be identified with the leading (i.e., nonstrange)  $\rho$  trajectory which for  $\alpha' \approx 1 \text{ GeV}^2$  yields  $m_\rho^2 = \frac{1}{2}$  a result which reflects just the spin  $\frac{1}{2}$  of the quarks and is independent of any parametrization. Multiparticle states will be analogously generated from higher ladder diagrams [Fig. 1(d)]. These make larger contributions at higher impact parameter values, sharpening the  $t$  distribution as  $s$  increases. The latter can be directly seen from the sum of ladders<sup>3</sup>

$$f^{\text{ladder}}(s, t) \approx s^{2J_q-1+g^2K(t)}$$

or

$$\alpha_R(t) = 2J_q - 1 + g^2K(t) = g^2K(t), \tag{1'}$$

where  $K(t)$  is to be computed from the  $\bar{q}q \rightarrow g \rightarrow \bar{q}q$  Born term. Consistency with  $\alpha_R(0) = \frac{1}{2}$  requires

$$g^2K(0) = \frac{1}{2}. \tag{2}$$

This also should be the value of  $\bar{g}^2$ , the effective coupling, for independent cluster emission in the part building Regge exchanges.<sup>6</sup>

Relation (2) fixing the dimensionless  $g^2$  may seem puzzling. However, it has been conjectured that some minimal critical  $g^2$  value is required to make the CKS model applicable in three dimensions. I conjecture that (2) coincides with the requirement that a zero-mass gluon will convert with probability 1 to some multibubble string, i.e.,

$$g^2K(0) + [g^2K(0)]^2 + \dots + [g^2K(0)]^n + \dots = 1, \tag{3}$$

an assumption needed to eliminate gluon lines from the physical intermediate states.

(B) *Baryon trajectories*.—Here I consider baryon-antibaryon annihilation via two mesons [Fig.

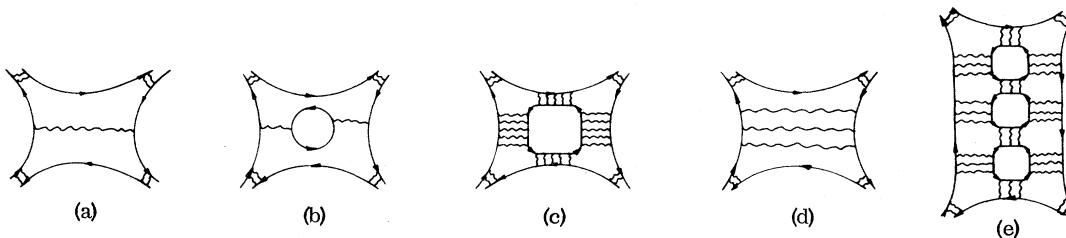


FIG. 1. (a) The  $mm$  duality diagram with one gluon in the  $s$  channel; (b) a bubble generated on the the gluon of diagram (a); (c) a duality-diagram description of the contribution of two-particle intermediate states to  $mm$  scattering; (d) the gluon ladder; (e) the evolution of the gluon ladder into the usual multiperipheral diagram.

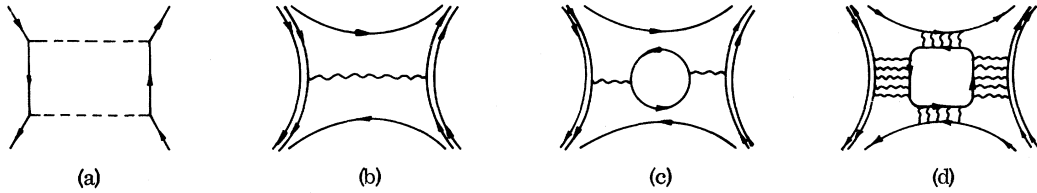


FIG. 2. (a) Two-physical-meson intermediate state in  $\bar{B}B$  annihilation; (b) the one-gluon diagram; (c) a bubble generated on the gluon of diagram (b); (d) the duality-diagram counterpart of (a).

2(a)] to be generated as above from the one-gluon diagram [Figs. 2(b) and 2(c)]. In analogy to the above we get  $s^{4J_q-3} = s^{2\alpha_B(0)-1}$  which fixes the leading baryon intercept at

$$\alpha_B(0) = 0, \quad m_\Delta^2 = 1.5, \quad (4)$$

with  $\alpha_B$  and  $\alpha_\Delta$  identified and  $\alpha' = 1$ .

Equation (4) is reproduced if we consider instead backward scattering and use (1). Next let us consider the ladder-multimeson diagrams which fix  $\alpha_{\text{annihilation}}(t)$ , the effective trajectory built by the  $\bar{B}B$ -mesons intermediate states<sup>7</sup>:

$$\alpha_{\text{annihilation}}(t) = 4J_q - 3 + \frac{22}{14}g^2K(t), \quad (5)$$

and from a similar treatment for backward  $mB$  exchange

$$\begin{aligned} \alpha_B(t) &= 3J_q - 2 + g^2K(t) \\ &= -\frac{1}{2} + g^2K(t) = \alpha_R(t) - \frac{1}{2} \end{aligned} \quad (6)$$

I took coefficients of  $K(t)$  as suggested by the color model,<sup>9</sup> in all three cases (1'), (5), and (6) (rather than having them in a 1:4:2 ratio as an Abelian gluon model would imply) thus predicting that the baryon and meson trajectories be indeed parallel.<sup>10</sup>

Equations (6) and (2) predict asymptotic annihilation cross behaving as  $s^{-13}/14 \sim s^{-1}$ .

(C) *Pomeron exchange.*—The Pomeron is identified with two- or more-gluon exchanges [Fig. 3(a)] (one is forbidden by color neutrality of hadrons—a nice feature absent in Abelian models). Neglecting temporarily the gluon ladders [Fig. 3(f)], I find the following:

(a) The Pomeron is a fixed singularity at  $\alpha = 1$ . Its intercept reflects the spin-1 gluon rather than the saturation of unitarity (Froissart) bounds. Indeed, the fact that  $\sigma_{el}/\sigma_{tot} \approx 0.2$  and not  $\frac{1}{2}$  may indicate that we are not yet in the energy regime where saturation is manifest.

(b) A symmetrical coupling of the two gluons to a color singlet is required, yielding even  $C$  exchange. For three or more gluons more than one

coupling to singlet is available, some of which have odd charge conjugation and could couple like an  $\omega$  to a single quark line. In coupling these objects to  $\bar{q}q$  composites, opposite contributions result when we permute all  $g-q$  and  $g-\bar{q}$  vertices leading us to expect their decoupling here and also to the color neutral baryons.

(c) Consideration of color neutrality for physical states suggests that the intermediate states dual to Fig. 3(a) do not decompose into distinct forward and backward moving systems, i.e., that they correspond to the “pionization” rather than the “diffractive” component of multiparticle production. The latter is, as usual, to be identified with the iterated Pomeron exchange. The ratio between the two diagrams is naively expected to be  $\approx [g^2K(0)]^2$  consistent with a diffractive part

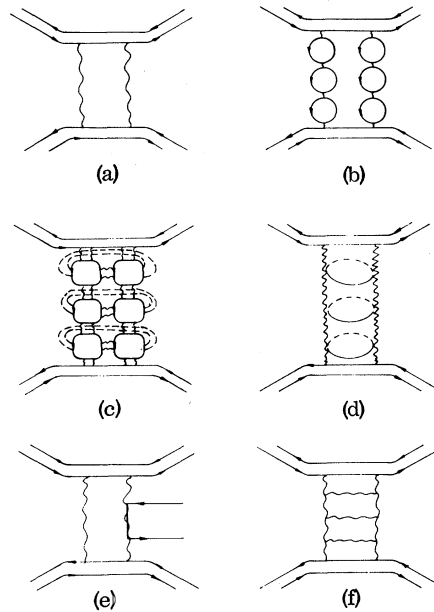


FIG. 3. (a) The Born approximation for the Pomeron; (b)–(d) various stages of generating physical intermediate states corresponding to (a); (e) the Pomeron-Pomeron-particle vertex; (f) the multigluon ladder.

smaller by a factor of 4 as compared with the SRC component.

Let us form  $n$  bubbles symmetrically on each of the exchanged gluon lines [Fig. 3(b)]. Letting such a system evolve under mutual attraction of oppositely oriented lines we will get as the most stable configuration the cylindrical arrangement of Fig. 3(c) which after  $90^\circ$  rotation (with respect to the cylindrical axis) reproduces just the configuration [Fig. 3(d)] suggested for the Pomeron by Lee and Veneziano.<sup>6</sup> It is nice that the quark-basis computation from 3(a) yields a fixed cut dual to all these states.

(d) If we adopt an extremely naive molecular analog picture of hadrons interacting via the Van der Waals forces generated by Fig. 3(a) we can make semiclassical calculations for the total meson-meson cross sections. These are proportional to  $g^4 R^2$ , where  $R^2$  is the mean space separation of the  $q$  and  $\bar{q}$  in the meson, which in turn is proportional to  $\alpha'$ . The qualitative feature of  $\sigma \approx \alpha'$  is nice because it suggests that the small proton-Pomeron cross sections (or small triple-Pomeron coupling) be correlated with  $\alpha_p'(0) \approx 0.2$ .

(e) Experimentally, the Pomeron-Pomeron-particle couplings are small. This may now be related [Fig. 3(e)] to the suppression of  $\bar{q}q$ -gluon annihilations even at masses  $\approx 0.8$ -1 GeV which is necessary for ideal nonets and the Zweig rule.<sup>8</sup>

(f) Including multigluon ladders, we find

$$\alpha_p(t) = 2J_q - 1 + g^2 H(t), \quad (7)$$

where  $H(t)$  is now the  $t$  projection of the  $g+g \rightarrow g-g+g$  link. To explain why the intersecting storage ring cross section rise is relatively small we have to fix  $\alpha_p(0) - 1 \sim g^2 H(0)$  to be small [as compared to  $g^2 K(0) \approx \frac{1}{2}$ ]. This can be achieved by an appropriate choice of  $m_g^2/m_q^2$ . It will simultaneously reduce the slope [ $\alpha_p'(0) \approx H'(0)$ ] in accordance with experiment.

(g) It has been speculated that the newly discovered narrow particles correspond to gluonic states. If this conjecture is indeed true, then analogy to the Pomeron explains their large spacing ( $\sim 1/\alpha'$ ) and small hadron cross sections.

To sum up I have suggested above a particular method to implement a colored gluon-quark model which explains some features of small- $t$ , high- $s$ , two-body reactions provided we adopt a CKS-like mechanism accounting for the transition to physical states. Should there emerge eventually a concrete theoretical foundation for the model unifying the various aspects of duality and color

confinement mechanisms in three dimensions, we would find a solution to the preceding (perhaps more compact) colored quark representation of certain hadronic puzzles.

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*Note added.*—After completing this work I learned that F. Low has constructed a detailed model for the Pomeron viewing it as a multigluon exchange.<sup>11</sup>

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<sup>2</sup>A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. **31**, 792 (1973). The model is based on Schwinger's one-dimensional QED.

<sup>3</sup>R. J. Eden, P. V. Landshoff, D. Olive, and J. C. Polkinghorne, *The Analytic S Matrix* (Cambridge Univ. Press, Cambridge, England, 1966), p. 131.  $K(t)$  is the  $\bar{q}q$  bubble in the contracted (two-dimensional) ladder. Intercept predictions remain intact also if we do not use  $t$  cutoffs as may be appropriate for quarks not originating from the incident hadrons.

<sup>4</sup>A "particle" here means a  $\bar{q}q$  system with a finite invariant mass which we represent pictorially by two nearby horizontal  $q$  lines. Similarly two nearby vertical  $\bar{q}q$  lines represent a Regge trajectory which can propagate finite  $t$  only.

<sup>5</sup>We consider positive specific contributions to  $\text{Im}f_{ab \rightarrow ab}(s, t=0)$ .

<sup>6</sup>H. Lee, Phys. Rev. Lett. **30**, 719 (1973); G. Veneziano, Phys. Lett. **43B**, 413 (1973).

<sup>7</sup> $\alpha_{\text{annihilation}}(t)$  corresponds to the exotic  $\bar{B}B$  states predicted by J. L. Rosner, Phys. Rev. Lett. **21**, 950 (1968).

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<sup>9</sup>For a baryon exchange color neutrality forces the  $qq$  on one side of the ladder to couple to  $\underline{3}^*$  so that we have the same gluon coupling as in a  $\bar{q}q$  ladder. This is not so for annihilation where the  $qq$  ( $\bar{q}q$ ) on opposite sides could also couple to  $\underline{6}$  ( $\underline{6}^*$ ), respectively. The coefficients are proportional to the values of the second-order Casimir operators for the representation. The  $\frac{23}{14}$  in Eq. (5) is a weighted average of  $C_2(3)$  and  $C_2(6)$ .

<sup>10</sup>So far, we have not discussed the nonleading  $\alpha_\pi$  and  $\alpha_N$ . The pion exchange does not correspond to identifiable positive contributions to  $\text{Im}f_{ab \rightarrow ab}(s, t=0)$ . The quark model suggests antiparallel spin addition, so that the analog of Eq. (1') is  $\alpha_\pi(t) = -1 + g'^2 K(t)$ . The  $V \rightarrow P$  Fierz coefficient is twice as large as the corresponding one for  $V \rightarrow V$  so that using Eq. (2) we find  $g'^2 = 2g^2$ ,  $\alpha_\pi(0) = 0$ . The relation  $\alpha_\Delta(0) - \alpha_N(0) \approx \frac{1}{2} \approx \alpha_\rho(0) - \alpha_\pi(0)$  presumably reflects a similar effect of spin reversal in the [ $(qq \rightarrow \underline{3}^*) + q(=\underline{3})$ ] ladder for baryons.

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