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High-Frequency Driven Ion-Cyclotron Instability

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The mechanism of the high-frequency driven ion-cyclotron instability is investigated and shown, for a Q-machine plasma column with symmetric external excitation, to correspond to a three-wave resonant process in which an electron-plasma wave decays into an electrostatic ion-cyclotron wave and an oppositely propagating electron-plasma wave, all waves being in their lowest-order mode.

One method of ion heating in magnetized plasmas involves driving an instability with a high-frequency pump field at a frequency ω_0 high compared with the ion-plasma and ion-cyclotron frequencies, ω_{pi} and ω_{ci}, and low compared with the electron-plasma frequency, ω_{pe}. The product frequencies close to threshold are ω_1 and $\omega_0-\omega_1$, where ω_1 is observed[1] to be slightly above ω_{ci}. If the interaction were a three-wave resonant process, matching of frequencies and of wave numbers in the longitudinal and transverse directions would be expected. Alternatively it is possible that the low- and high-frequency decay products observed are parametrically coupled[2] by an effectively zero-wave-number pump field. The dispersion diagram for a finite plasma with these two types of pump field can be represented as in Fig. 1. This Letter reports measurements which elucidate the mechanism of this instability as observed in a Q-machine plasma.

Typical experimental conditions in our thermally ionized sodium plasma[3] are $n_e=3\times10^7$ cm^{-3} and $B_z=2$ kG, giving $\omega_{pe}/2\pi=49$ MHz, $\omega_{pi}/2\pi=240$ kHz, and $\omega_{ci}/2\pi=134$ kHz. A large-ampli-

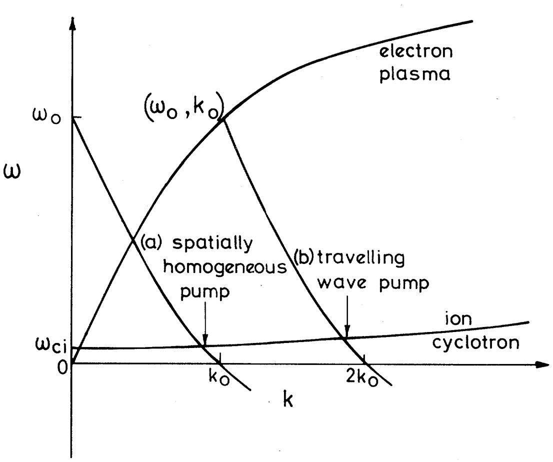

FIG. 1. Dispersion diagram (not to scale) for electrostatic ion-cyclotron waves $\epsilon(\omega,k)=0$ showing the points of minimum threshold for nonlinear excitation (a) by an electrostatic ($k_0=0$) pump field producing in addition an electron-plasma wave $\epsilon(\omega-\omega_0,k)=0$; (b) by an electron-plasma wave $\epsilon(\omega_0,k_0)=0$ producing in addition an electron-plasma wave $\epsilon(\omega-\omega_0,k-k_0)=0$. The arguments of the dielectric constant used above have the effect of showing the oppositely propagating ($k<0$) electron-plasma waves inverted and drawn in this quadrant.

tude electric field in the frequency range $\omega_0 \sim (0.1-0.3)\omega_{pe}$ is generated by applying an rf voltage to a 1-cm-long, 5-cm-diam metal ring electrode surrounding and outside the 80-cm-long, 3-cm-diam plasma column maintained on the axis of a 15-cm-diam stainless-steel vacuum vessel; in general this generates a very-long-wavelength vacuum field, as well as exciting a long-wavelength electron-plasma wave on the column.

The nonlinearly produced frequencies could be identified as corresponding to particular waves if wavelengths could be measured by interferometry. In practice any spatial dependence of the pump field imposes itself on the amplitudes of the product waves and variations of phase, particularly at ω_1, proved difficult to determine in the absence of a well-defined reference signal. We have attempted to identify the low-frequency product, ω_1, by stimulating the instability below threshold, using separate excitation. This requires independent linear excitation of waves which can propagate in the plasma at ω_1 with appropriate wave numbers, viz. ion-acoustic waves and electrostatic ion-cyclotron waves.

Recent experiments[4] have demonstrated simultaneous excitation of these modes using a coil to modulate the confining magnetic field. By electrostatically screening the coil, coupling to the ion-acoustic mode can be suppressed so that the ion-cyclotron wave alone is excited. Ion-acoustic waves can be excited independently using a small probe immersed in the plasma. Interferograms of these ion modes at one frequency are shown in the inset to Fig. 2, which shows typical dispersion results obtained for the cyclotron wave. The theoretical curve given for comparison is derived from the expression below for the conditions in our experiment; in particular ion drift is included. The ion-drift velocity v_d has been measured using the waves which were observed to propagate below the ion-cyclotron frequency. As shown in Fig. 2 this mode approaches asymptotically the Doppler-shifted ion-cyclotron frequency $\omega = \omega_{ci} + kv_d$, from which v_d can be determined.

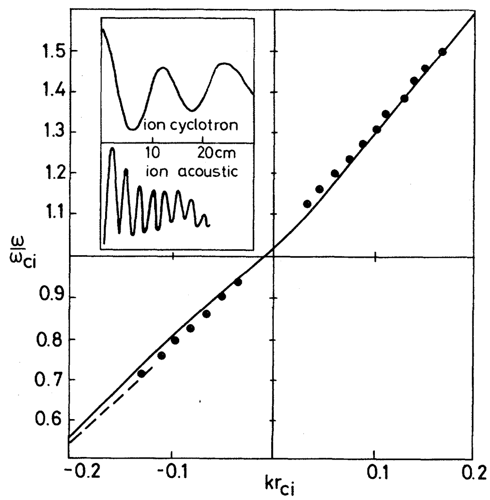

FIG. 2. Experimental dispersion of linear ion-cyclotron waves compared with theory. The inset shows typical interferograms for screened-coil-excited ion-cyclotron waves and wire-probe-excited ion-acoustic waves at the same frequency $\omega/2\pi = 170.0$ kHz, $\omega/\omega_{ci} = 1.28$ kHz. The asymptote $\omega = \omega_{ci} + kv_d$ is shown by the dashed line; $v_d = 3v_i$.

The approximate theoretical dispersion relation used is

$$\epsilon(\omega,k) = 1 + \frac{1}{(k_\parallel^2 + k_\perp^2)\lambda_{De}^2}\left\{1 - \frac{1}{2}\left[\frac{k_\parallel^2 v_i^2}{(\omega - k_\parallel v_d)^2} + \frac{k_\perp^2 v_i^2}{(\omega - k_\parallel v_d)^2 - \omega_{ci}^2}\right]\exp\left[-\frac{k_\perp^2 v_i^2}{2\omega_{ci}^2}\right]\right.$$
$$\left. - \frac{k_\perp^2 v_i^2}{8\omega_{ci}^2}\exp\left[-\frac{k_\perp^2 v_i^2}{2\omega_{ci}^2}\right]\left[\frac{T_{i\perp}}{T_{i\parallel}} + \frac{\omega_{ci}}{\omega - \omega_{ci} - k_\parallel v_d}\right] Z'\left[\frac{\omega - \omega_{ci} - k_\parallel v_d}{k_\parallel v_i (T_{i\parallel}/T_{i\perp})^{1/2}}\right]\right\} = 0, \qquad (1)$$

where $k_\parallel$ is the wave number parallel to the magnetic field, $k_\perp$ is an effective perpendicular wave number taken as 2.4/(radius), λ_{De} is the electron Debye length, v_d is the ion drift velocity, $T_{i\parallel}$ and $T_{i\perp}$ the ion temperatures parallel and perpendicular to the field, $v_i = (2k_B T_{i\perp}/m_i)^{1/2}$ is the ion thermal velocity, and Z' is the derivative of the plasma dispersion function.[5] This equation has been derived from the general dispersion relation for electrostatic waves in warm plasma[6] retaining only terms significant near $\omega = \omega_{ci}$. The anisotropic ion temperatures, used to approximate the truncated parallel ion distribution in a single-ended Q-machine, are $2T_{i\parallel} = T_{i\perp} = T_e = 2500$°K.

To demonstrate stimulation, measurements were first made on the decay-producing ion-acoustic waves[7]—a three-wave resonant process. Figure 3(a) shows the measured amplitude of the

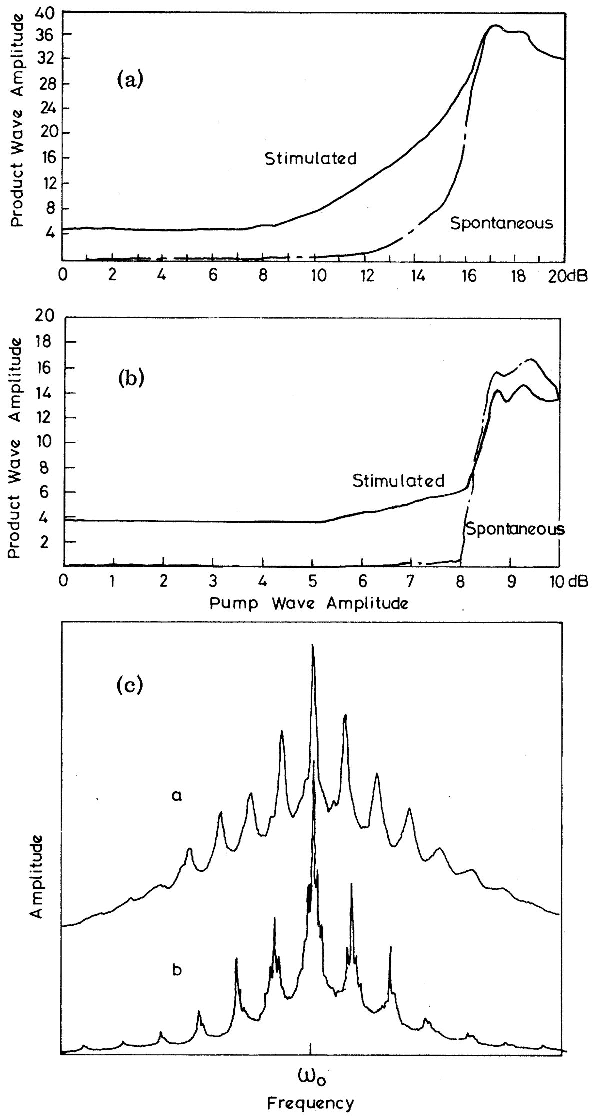

FIG. 3 (a) Amplitude of the ion-acoustic instability, at frequencies ω_1, as a function of relative pump-wave amplitude with and without stimulation; $\omega_0/2\pi = 24$ MHz. (b) Similar data for the ion-cyclotron instability. (c) Spectra of the fluctuations in the plasma with the pumps well above threshold, curve a, for the ion-cyclotron instability $\omega_0/2\pi = 11$ MHz, scan = 2 MHz; curve b, for the ion-acoustic decay instability $\omega_0/2\pi = 24$ MHz, scan = 1 MHz.

ion-wave decay product of this instability as a function of the initial electron-plasma-wave amplitude, both with and without separate linear excitation of an ion-acoustic wave at the appropriate frequency set by the matching conditions. Excitation at frequencies off resonance showed no interaction through stimulation.

Attempts to stimulate the ion-cyclotron instability (derived at pump frequencies appreciably below those used for the ion-acoustic decay experiment) by separate excitation of ion-acoustic waves using the wire probe did not produce interaction, showing that the stimulation is selective as to wave type. However, Fig. 3(b) shows the effect of simultaneously exciting an ion-cyclotron wave in the presence of the pump field, where stimulation of this instability is clearly evident. The threshold for spontaneous decay, estimated by comparison with that for ion-wave decay,[7] corresponded typically to $e\varphi/k_BT_e \sim 0.3$, where φ is the amplitude of the electrostatic driving wave.

The high-frequency spectrum centered on the pump frequency, observed well above threshold, is shown in Fig. 3(c). It closely resembles that reported previously[1] and, apart from the frequency scale, is essentially the same as displayed by the ion-acoustic wave decay, also shown in Fig. 3(c), curve b.

To determine the nature of the instability, a comparison of the wave number, k_1, of the low-frequency wave ω_1 with the wave number, k_0, of an electron-plasma wave at the pump frequency ω_0 suffices, since from Fig. 1, $k_1 \lesssim k_0$ for the parametric case, and $k_1 \lesssim 2k_0$, for the three-wave resonant case. For typical experimental conditions, $B_z = 2$ kG, $\omega_{pe}/2\pi = 50$ MHz, $\omega_0/2\pi = 11$ MHz, we find $\omega_1 = 159$ kHz and measurements of the wave number of an ion-cyclotron wave, linearly excited at this frequency, give $k_1 = 0.350$ cm^{-1}. Direct measurement of the electron-plasma wave gives $k_0 = 0.177$ cm^{-1}. Both measurements agree well with the theoretical dispersion of ion-cyclotron and electron-plasma waves, respectively, propagating in a finite plasma column in the lowest-order mode, and this is confirmed by direct measurement of the radial phase of these waves. Using narrow-band rf amplifiers it proved possible to filter signals at the frequency $\omega_0 - \omega_1$ from the large pump-wave signal and, using two detectors, one movable, to obtain the wavelength of the high-frequency product wave. Comparison with a directly excited wave at the same frequency showed it to lie on the dispersion curve for the lowest-order electron-plasma wave. Clearly the process is a resonant decay involving matching of the longitudinal wave numbers, and in this case all waves involved are in the lowest-order, radial, azimuthally symmetric mode, in contrast to previous suggestions.[1]

At constant pump frequency the dependence of

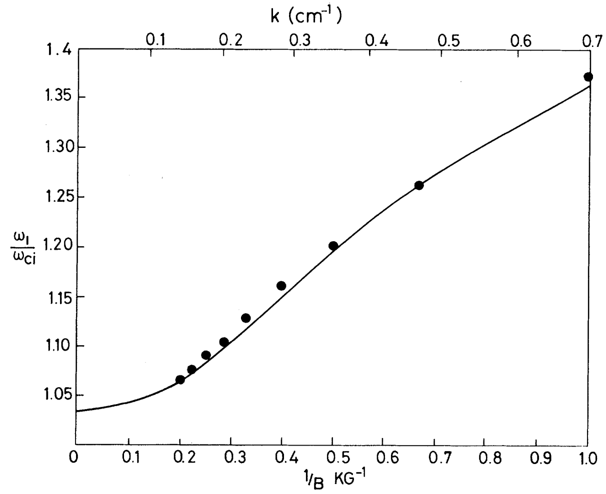

FIG. 4. The normalized instability frequency ω_1/ω_{ci} as a function of the magnetic field B (full circles), compared with the measured linear dispersion of ion-cyclotron waves at 2 kG (solid line).

the instability on the ion-cyclotron frequency has been considered by varying B_z. Figure 4 shows ω_1, scaled to the ion-cyclotron frequency, plotted against B_z^{-1} and compared with the linear dispersion of ion-cyclotron waves, measured at 2 kG, by taking the k scale so that the two coincide for $B_z = 2$ kG. It is concluded that k_1 is determined by k_0 and that ω_1/ω_{ci} varies with $k_1 v_i/\omega_{ci}$ according to the linear dispersion.

Observations of the instability were also carried out at higher densities ($\sim 5\times10^9$ cm^{-3}) in a similar single-ended potassium Q machine. The parameter $p \equiv \omega_{pe}^2/\omega_{ce}\omega_0$ determines[1] whether the driving field is predominantly in the direction of the pump field E_z, i.e., axial ($p<1$), or in the direction $\vec{E}_r \times \vec{B}_z$, i.e., azimuthal ($p>1$). This parameter was varied over the range 0.2 to 30 and the ratio ω_1/ω_{ci} was comparable with that observed for smaller values of p and was not dependent on it. Thus the product waves did not show the variation in frequency which would be predicted by (1) if azimuthal modes were involved at higher values of p.

We conclude that the instability is a three-wave resonant decay which can be excited over a range of conditions in the lowest-order modes, i.e., with all waves azimuthally symmetric.

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