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Excitation of Lower-Hybrid Waves by a Slow-Wave Structure*

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We report the excitation of lower-hybrid waves by a multiple-ring slow-wave structure (four waves, $\lambda = 23$ cm) in a magnetized plasma. Wavelengths measured parallel and perpendicular to the magnetic field were in agreement with the theoretical dispersion relation. The waves propagated in a packet defined by the axial length of the slow-wave structure.

The feasibility of heating magnetically confined plasmas to thermonuclear temperatures by high-power radiofrequency sources with frequencies ω, near the lower-hybrid frequency ω_{LH}, has been investigated actively in recent years. In particular, it has been shown theoretically that an appropriately polarized electromagnetic wave launched near the surface of the plasma would propagate toward the lower-hybrid resonance layer as a quasielectrostatic cold-plasma wave.[1] This wave would convert near the layer into a short-wavelength ion plasma wave which would then propagate radially outward and would be readily absorbed, heating the plasma. Alternatively, the incoming wave could parametrically decay into short-wavelength hot-plasma waves and be absorbed.[2] To achieve these goals, two criteria must be met: (a) For wave penetration through the plasma surface without significant reflection (accessibility) we require a slow wave, $\omega/k_z < c$ (here $k_z = \vec{k}\cdot\vec{B}/B$ is the wave vector component parallel to the confining field $\vec{B} = B\hat{z}$ and c is the velocity of light).[3,4] (b) For heating large volumes of plasma, the incoming wave front in the z direction must be large.[2,5] These two criteria can be satisfied using either a finite-length slow-wave structure or a wave-guide array. In addition, because the cold-plasma-wave

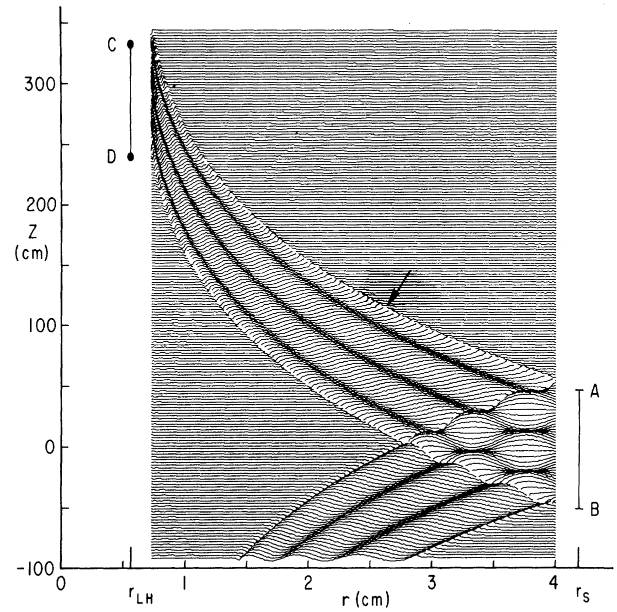

FIG. 1. Lower-hybrid waves generated by a slow-wave structure (located along *AB*) as predicted by theory (Ref. 5); $2\pi/k_z = 33.3$ cm, $n(r) = 10^{12}/[1 + (r/0.75)^2]$ cm^{-3}, $\omega/2\pi = 50$ MHz, $B = 2$ kG, He gas. Conical singularity (arrow) bounds the wave packet. The linearly converted hot-plasma wave (not shown) propagates radially outward from the line *CD*.

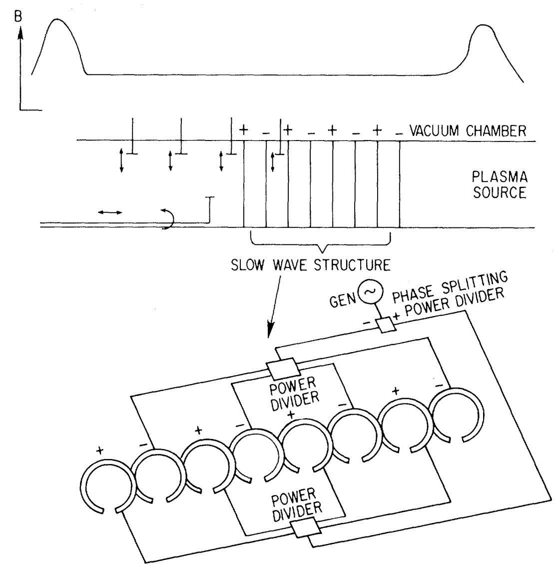

FIG. 2. Experimental setup. The eight rings driven by power dividers alternate in phase by 180°.

dispersion relation depends only on the ratio k_z/k, i.e.,

$$\omega^2/\omega_{LH}^2 = 1 + (m_i/m_e)(k_z^2/k^2), \qquad (1)$$

in order to excite well-defined propagating waves, one must fix either k_z or the perpendicular wave-vector component k_x [where $\omega_{LH}^2 \equiv \omega_{pi}^2/(1+\omega_{pe}^2/\omega_{ce}^2)$ is the lower-hybrid frequency and ω_{pi} and ω_{pe} are the ion and electron plasma frequencies, respectively; ω_{ce} is the electron cyclotron frequency; m_i/m_e is the ion-to-electron mass ratio; and we assumed $\omega_{ci} \ll \omega_{pi}$, and $k_z \ll k_x$]. In particular, a sufficiently long slow-wave structure automatically imposes a well-defined k_z spectrum which ensures excitation of propagating waves in the x direction.[4,5]

There have been several attempts in the past few years to excite these slow cold-plasma waves, with the hope of eventually observing mode conversion.[6,7] However, it has been shown recently that the long capacitor plates used in these experiments should excite resonance cones rather than propagating slow waves.[5,8] This has been corroborated by recent experiments.[7] On the contrary, our theory predicts that a finite-length slow-wave structure will excite a set of well-defined propagating waves localized between the cones emanating from the ends of the structure.[5] An example of the results of this calculation is shown in Fig. 1.

In this Letter we wish to present what we believe is the first experimental observation of well-defined lower-hybrid waves excited by an external slow-wave structure.[9] By measuring both the axial and the radial wave numbers for a large range of frequencies and densities we have verified in detail the dispersion relation of these waves. In addition, we find that (a) the waves propagate in the x-z plane along the group-velocity trajectories as predicted by theory, and (b) the waves propagate in a packet defined by the axial extent of the slow-wave structure.

The experiments have been performed in both homogeneous and inhomogeneous plasmas, produced in the Princeton L4 device. The schematic of the experimental setup is shown in Fig. 2. Warm-electron plasmas were produced either by a hot-cathode discharge located near the end of the device, or by a coaxial, 450-MHz, 1-kW, cw rf source. Thus, plasma densities in the range $n = 10^9 - 10^{12}$ cm^{-3} could be obtained in helium gas at a filling pressure of $p \simeq 8\times10^{-4}$ Torr. The electron temperature was $T_e \lesssim 5$ eV, and the ion temperature $T_i \lesssim 0.1$ eV. Densities were measured by Langmuir probes and an 8-mm microwave interferometer. Temperatures were measured by a Langmuir probe. A number of radially and axially movable coaxial rf probes were

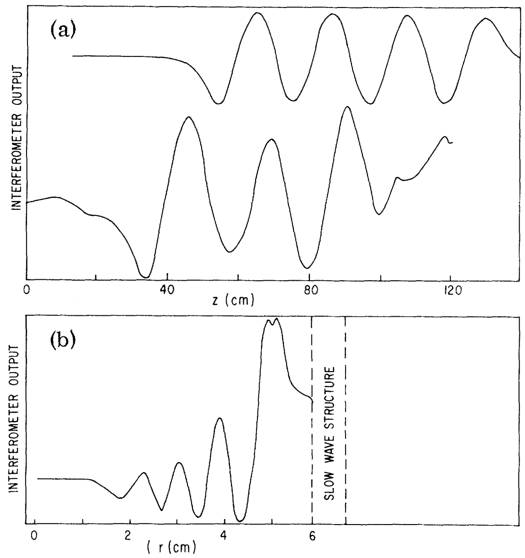

FIG. 3. (a) The top trace shows the vacuum field of the slow-wave structure (measured by an axial probe). $\omega/2\pi = 18$ MHz; $r = 3$ cm. The bottom trace shows a typical axial lower-hybrid wave field in the hot-cathode He plasma; $\omega/2\pi = 18$ MHz, $r = 3$ cm. (b) Typical radial lower-hybrid wave field in a hot-cathode He plasma; $\omega/2\pi = 20$ MHz; the axial position of the probe is 5 cm beyond the last ring.

used in conjunction with a radio interferometer to detect waves.

The 1-m-long slow-wave structure, shown in detail in Fig. 2, consisted of eight rings spaced 11.5 cm apart and connected to an rf source through transformers so as to have their phases alternate by 180°. This set up a four-wavelength-long axial wave train ($2\pi/k_z = 23$ cm) having $\omega/k_z \ll c$ for the range of frequencies used (8–50 MHz). The structure was placed near one end of the 160-cm-long uniform magnetic field region and waves were measured both inside and outside the ring structure.

In Fig. 3 we show typical interferometer traces from the axial and radial probes. As the frequency of the driving oscillator was increased, the radial wavelength (i.e., $2\pi/k_x$) increased whereas the parallel wavelength (i.e., $2\pi/k_z$) remained constant. In particular, the dominant Fourier component of k_z remained the same as in vacuum [c.f. Fig. 3(a)] verifying that the ring assembly acted as a true slow-wave structure. We also observed the following: (i) In the plasma the axial position of the wave packet shifted with radius [Fig. 3(a)]. (ii) Independent of frequency, the number of radial wavelengths was always the same as the number of axial wavelengths [Fig. 3(b)]. Both of these were in agreement with theoretical predictions (cf. Fig. 1).[5] In addition, using delay-line techniques we have verified that the waves were backward waves [i.e., $(\omega/k_x)\partial\omega/\partial k_x < 0$].

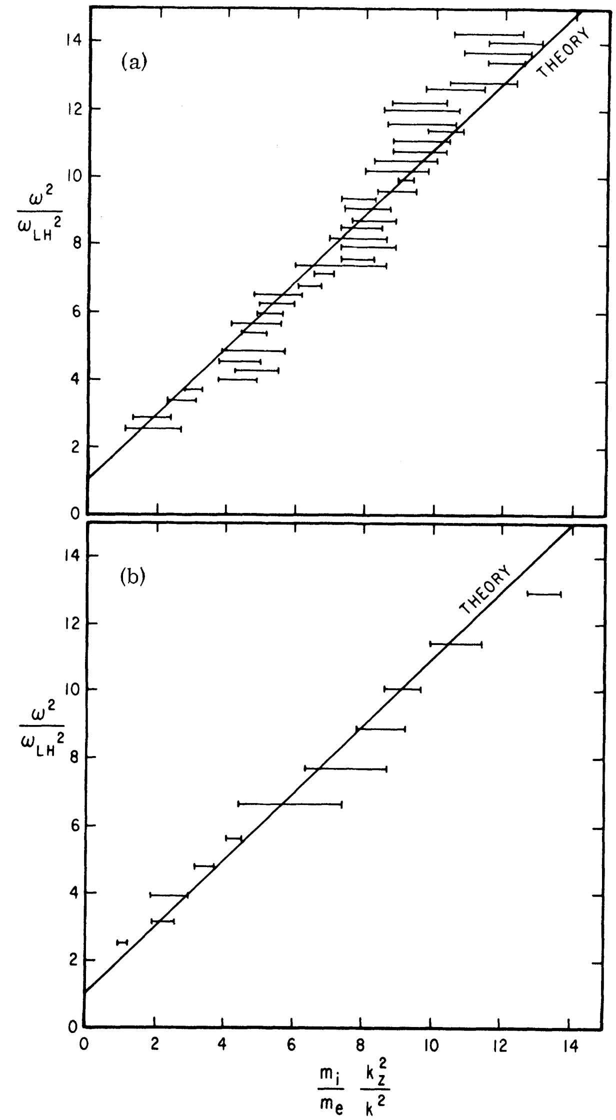

FIG. 4. (a) k_x versus ω; rf-produced He plasma. $2\pi/k_z = 23$ cm, $B = 2470$ G, and $n = 1.6\times10^{10}$ cm^{-3}. (b) Same as in (a) but for the hot-cathode–produced He plasma; $2\pi/k_z = 23$ cm, $B = 515$ G, and $n = 2.5\times10^{9}$ cm^{-3}.

In Fig. 4 we present the experimentally measured radial and axial wavelengths as a function of frequency. The solid lines represent the theoretical dispersion relation, Eq. (1). In Fig. 4(a) waves were generated in a rf-produced plasma in which the density was uniform to ±10% for a radius of 5.5 cm and then dropped linearly to zero in 1 cm. Wavelengths in the uniform-density region were measured for frequencies from 20 to 50 MHz. The error bars indicate the standard deviation from the mean wavelength. In Fig. 4(b) waves were generated in a hot-cathode discharge plasma with density uniform to ±12% for a radius of 4.5 cm and then dropping linearly to zero in 1.5 cm. Frequencies from 8 to 18 MHz were used. In Figs. 4(a) and 4(b) ω_{LH}^2 was adjusted for the best fit to the theoretical curve. This fitting, corresponding to adjusting the density, gives $n = 1.6\times10^{10}$ cm^{-3} (and $\omega_{LH}/2\pi = 13.3$ MHz) for the rf plasma and $n = 2.5\times10^9$ cm^{-3} (and $\omega_{LH}/2\pi = 5.0$ MHz) for the hot-cathode plasma, in good agreement with Langmuir-probe measurements. Measurements made in a nonuniform plasma showed that the radial wavelength was decreasing in regions of increasing plasma density, as predicted by Eq. (1). A comparison of the signals obtained from two radial probes showed that the wave packet moved radially inward with increasing axial distance from the slow-wave structure, in agreement with Fig. 1.

We found that as the lower-hybrid frequency was approached the wavelengths shortened considerably, and for $2\pi/k_x < 2.5$ mm the waves were damped out. Due to the damping associated with the high electron-neutral collision frequency ($\nu_{e0} \sim 2\times10^6$ sec^{-1}) and possibly to the background density fluctuations ($\tilde{n}/n_0 \lesssim 5\%$) we have not yet observed linear mode conversion. Our recent theoretical estimates showed that under the present experimental conditions electron-neutral collisions could provide strong damping for short-wavelength waves.[5]

In summary, we have experimentally demonstrated that cold lower-hybrid waves can be excited by an external slow-wave structure. The fundamental aspects of the theory of excitation (i.e., dispersion relation and wave-packet propagation) have been verified. We have also observed that in an inhomogeneous plasma k_x increases as the wave penetrates to regions of increasing plasma density, in accordance with Eq. (1). By reducing the collisional damping in future experiments we hope to obtain mode conversion at the lower-hybrid layer.

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Observation of Nonlinear Phase Shift of Electron Plasma Waves*

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The nonlinear phase shift which accompanies a large-amplitude electron plasma wave on a collisionless plasma column is demonstrated experimentally. The spatial evolution of the phase shift and the wave-number shift in a position-asymptotic limit are shown to be consistent with recent nonlinear theory.

It is now well established[1-5] that a large-amplitude electron wave (frequency ω_0, wave number k_0) propagating on a collisionless plasma may be accompanied by nonlinear oscillations of the wave amplitude. In general, however, the nonlinearities give rise to changes in the phase of the waves as well as in its amplitude. In the small-amplitude limit the wave phase θ at position x and time t is expressed in the form $\theta = k_0 x - \omega_0 t$. The nonlinear phase shift $\delta\theta$ in a wave of