In conclusion, the exciton dynamics of the singlet-ground-state system Pr can be understood almost completely in terms of the RPA theory combined with the damping effects due to scattering on single-ion fluctuations, as calculated to first order in the high-density expansion. This work provides the first evidence of this effect in a real magnetic system. This fundamental understanding of the excitations in Pr may be valuable for the analysis not only of other singlet-ground-state systems of magnetic and nonmagnetic nature, but also of more complicated systems, since the formalism can easily be extended to arbitrary level schemes.

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# New Theory of Coercive Force of Ferromagnetic Materials*

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We have enlarged the theory of the coercive force to include the wave nature of the ferromagnetic domain walls. Specifically, we obtain an explicit equation for the magnetic field required to move a 180° ferromagnetic domain wall across a planar defect such as a grain boundary. Using this expression, we have been able to classify successfully both low- and high-coercive-force materials.

Experimentally, the coercive force in a ferromagnetic material is defined as that external magnetic field required to demagnetize the specimen. The coercive force is a sensitive property of ferromagnetic materials and provides an important criterion in the selection of these materials for practical use. It ranges from 0.002 Oe in supermalloy transformers and 0.5 Oe in iron-silicon power transformers to 10 000 Oe in new high-stability magnets such as $Co_5Sm$. Because of the nonlinear nature of the relevant mathematical equations, theory has not kept pace with experimental results. Traditionally, the coercive force is determined by simply equating the change in the magnetostatic field energy to the maximum height of the energy barrier.[1] Thus, the effects of the width of the domian wall relative to the width of the energy barrier are neglected. Recently, it has proved difficult to apply this elementary approach to such high-coercive-force materials as $Co_5Sm$. Because of this, the highly successful continuum approximation of micromagnetics has been questioned when applied to narrow ferromagnetic domain walls.[2]

We have enlarged the theory of the coercive force of ferromagnetic materials within the framework of micromagnetics to include the wave nature of the ferromagnetic domain walls and have formulated a new theoretical approach to determine this force. We apply our work to bulk materials where the equations first presented by Landau and Lifshitz[3] are applicable. Our theory explains how different materials (possessing the same type of defects, such as grain boundaries) can have widely different coercive forces. It also shows that such planar defects as grain boundaries give the correct order of magnitude for the very-high-coercive-force materials such as $Co_5Sm$ as well as for lower-coercive-force materials such as iron and cobalt.

We consider specifically the problem of a 180° domain wall in an infinite medium divided into

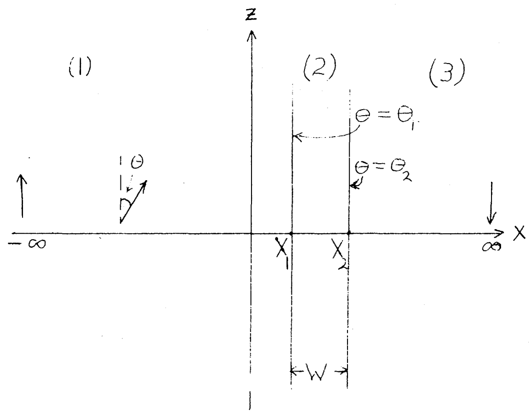


FIG. 1. Geometry of medium showing three regions.

three regions as illustrated in Fig. 1. Regions 1 and 3 are identical and represent the homogeneous material while region 2 represents the energy barrier characterized by an abrupt change in the properties of the magnetic materials and caused by some defect such as a grain boundary. We shall show that, if the defect region differs from regions 1 and 3 by a fixed percent change in the magnetic energy constants independent of the materials considered, our resultant expression for the coercive force varies with different materials over a range of $10^7$ in agreement with experiment.

We do not take into account the reduction of wall energy by nonmagnetic inclusions such as appear in the Kersten inclusion theory[4] nor those changes in wall energy due to sinusoidal magnetostress variations resulting from inhomogeneous internal strains as developed by Kondorsky[5] and Kersten.[6] Neither of these can account for the wide range of coercive forces found in modern magnetic materials. We recognize that, for the case where region 2 represents a grain boundary, magnetic poles will form at the boundary and thus demagnetizing fields will be present. Néel[7] and Goodenough[8] have considered the effect of these fields on the coercive force. We expect these results to be significant for materials with small crystalline anisotropy where they may constitute a major portion of the coercive force. However, their results cannot account for the four-orders-of-magnitude change in the coercive force in going from iron to $Co_5Sm$. Finally, we do not consider the effects of domain nucleation on the coercive force.

It is not necessary to solve the general dynamical equations of motion for the magnetic moment in the presence of an applied field. Instead, we examine the static equilibrium solutions to determine whether, for specific properties of the barrier and of the material, the domain wall is pinned. For a given barrier there exists a continuous set of solutions of pinned domain walls corresponding to a range of external magnetic fields. We postulate that the maximum external field for which there exists a static-domain-wall solution corresponds to the coercive force for that particular obstacle.[9] Conversely, for a given external magnetic field, one can find the minimum width of region 2 (holding the other characteristics of the medium constant) for which there exist static-domain-wall solutions and obtain that barrier for which the given applied field is the coercive force.

We first obtain an implicit equation for the magnetic field required to move a ferromagnetic domain wall across the barrier. We solve this under the restraint that the magnetostatic energy due to the external field is much less than the anisotropy energy of region 2. This restraint is not a limiting factor for most practical materials.

Assume that the magnetic moments lie in the plane of the coordinates perpendicular to the $x$ axis and are functions of the $x$ axis only (i.e., neglect any variation in anisotropy direction between regions). Denote the angle between the magnetic moment and the fixed axis $z$ by $\theta$. Then, using the notation of Landau and Lifshitz,[3] we write (a) the energy density due to inhomogeneity in the distribution of the directions of the magnetic moments (i.e., the exchange energy) as $\frac{1}{2}\alpha\theta'^2$, where $\theta' = d\theta/dx$; (b) the magnetic anisotropy energy density due to the presence of an axis of easy magnetization as $\frac{1}{2}\beta\sin^2\theta$; and (c) the magnetostatic energy density as $-HS\cos\theta$, where $S$ is the magnitude of the magnetic moment density and $H$ is the applied field strength. Note that the contribution to the field strength due to the interaction between the magnetic moments is zero when all the magnetic moments are distributed perpendicular to the $x$ axis and when $\theta$ depends only on $x$.

As discussed above, we look for static equilibrium solutions, i.e., the energy of the crystal,

$$\int(\tfrac{1}{2}\alpha\theta'^2 + \tfrac{1}{2}\beta\sin^2\theta - HS\cos\theta)\,dv,$$

must have a minimum. Integrating the Euler equation for this integral yields the three non-

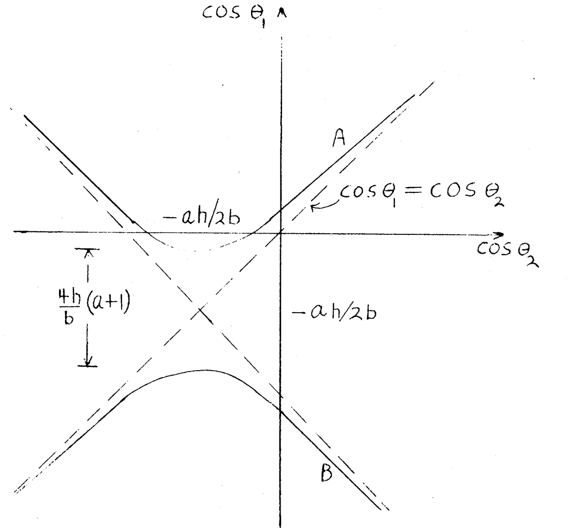


FIG. 2. Sketch of relationship between the angles $\theta_1$ and $\theta_2$ for arbitrary positive values of $a$, $b$, and $h$. Note: Curve $A$ need not cross the abscissa.

linear equations,

$$-\alpha_i\theta'^2+\beta_i\sin^2\theta-2HS_i\cos\theta=C_i, \qquad (1)$$

where the subscript $i$ denotes the region of the material and where the quantities $\alpha$, $\beta$, and $S$ have the same values in regions 1 and 3. Imposing the boundary conditions $\theta(-\infty)=\theta'(-\infty)=\theta'(+\infty)=0$ and $\theta(+\infty)=\pi$ yields the values $-HS_1$ for the constant $C_1$ and $+HS_1$ for $C_3$. Further, imposing continuity conditions for $\theta$ and $\alpha_i\theta'$ at the two in interfaces $x_1(\theta_1)$ and $x_2(\theta_2)$ yields the equations

$$[(ha/2b)+\cos\theta_1]^2-[(ha/2b)+\cos\theta_2]^2 = 2h(a+1)/b, \qquad (2)$$

and

$$W=(\alpha_2/\beta_2)^{1/2}\int_{\theta_1}^{\theta_2}d\theta\,[\sin^2\theta-h\cos\theta + b\sin^2\theta_1-ah\cos\theta_1+h(a+1)]^{-1/2}, \qquad (3)$$

where $W=x_2-x_1$, $a=(\alpha_1S_1/\alpha_2S_2)-1$, $b=(\alpha_1\beta_1/\alpha_2\beta_2)-1$, and $h=2S_2H/\beta_2$. Equation (2) is sketched in Fig. 2. We note that the lower branch, labeled $B$, of this curve corresponds to values of $\theta_1>\theta_2$ and is therefore not considered here. Equations (2) and (3) determine $h$ implicitly in terms of $W$ and $\theta_1$. Maximizing $h$ with respect to $\theta_1$, we can in principle obtain the coercive $h$ for a given $W$.

As an illustration, we consider first the trivial case where the applied field $H$ is zero. Then, from Eq. (2), $\cos\theta_1=\cos\theta_2$, i.e., when $\theta_2$ ranges from 0 to $\pi/2$ rad, $\theta_1=\theta_2$ and $W=0$, while from $\pi/2$ to $\pi$ rad, $\theta_1=\pi-\theta_2$ and $W>0$. Thus, there exist solutions $W(\theta_1,\theta_2)$ for $H=0$ with the minimum value being $W=0$. This shows that any barrier, no matter how small, is sufficient to pin the wall in the absence of a magnetic field.

We next consider the more general case corresponding to $h$ small but finite. As is shown in Table I, most physical cases of interest fall within this range. Using this approximation, we rewrite Eq. (3) in the form

$$W\simeq\left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\int_{\cos^2\theta_1}^{\cos^2\theta_2}\frac{d(-\cos^2\theta)}{\sin 2\theta(\sin^2\theta+b\sin^2\theta_1)^{1/2}}, \qquad (4)$$

where we have dropped those terms in the integrand proportional to $h$. To evaluate the integral, we note from Eq. (2) that

$$\cos^2\theta_1\simeq\cos^2\theta_2+2h(a+1)/b, \qquad (5)$$

and we approximate the integrand by substituting $\theta_1$ for $\theta$. We obtain the result

$$H\simeq\frac{1}{2}\frac{\beta_1}{S_1}\frac{W}{\delta_1}\left(\frac{\alpha_1}{\alpha_2}-\frac{\beta_2}{\beta_1}\right)\sin^2\theta\cos\theta, \qquad (6)$$

where $\delta_1=(\alpha_1/\beta_1)^{1/2}$ is the usual ferromagnetic-domain-wall width parameter for the material. In accordance with our prescription for the coercive force, we maximize the quantity $\sin^2\theta\cos\theta$ and get $\tan\theta_c=2^{1/2}$, and the coercive force

$$H_c=3^{-3/2}\frac{\beta_1}{S_1}\frac{W}{\delta_1}\left(\frac{\alpha_1}{\alpha_2}-\frac{\beta_2}{\beta_1}\right). \qquad (7)$$

The factor $\beta_1/S_1\delta_1$ in Eq. (7) represents the resistance of the material to the coercive force and $W[(\alpha_1/\alpha_2)-(\beta_2/\beta_1)]$ is a measure of the pinning strength of the barrier. We note the factor, $W/\delta_1$, indicating the importance to the coercive force of the ratio of the defect width to the wall width. For high-anisotropy materials such as $Co_5Sm$, this ratio increases to order 1 even for such narrow barriers as grain boundaries.

In Table I, we tabulate our results for the whole range of coercive forces for ferromagnetic materials—both soft and hard. A detailed calculation of the exchange and anisotropy within a grain boundary is not feasible. In accordance with our earlier discussion, however, we have assumed the particular set of values $\alpha_1/\alpha_2=1.2$ and $\beta_2/\beta_1=0.7$ which seem reasonable and which, together with the value of 12 Å for $W$, normalize our parameter $W[(\alpha_1/\alpha_2)-(\beta_2/\beta_1)]$ to yield the correct value for the coercive force of iron–4%-

TABLE I. Values of the coercive force due to grain boundaries. We assume the values, $W=1.2\times10^{-7}$ cm, $\alpha_1/\alpha_2=1.2$, $\beta_2/\beta_1=0.7$, and $S_1/S_2=1$.

| Material | $S_1$ Magnetization | $2\beta_1$ Anisotropy (ergs/cm$^3$x10$^{-5}$) | $2\alpha_1$ Exchange (ergs/cmx10$^6$) | $\delta_1$ -Domain Wall Width (cmx10$^6$) | h Expansion Parameter | $H_c$-Coercive Force (Oersteds) | Observed Coercive Force |
|---|---|---|---|---|---|---|---|
| Supermalloy | 630. | 0.015 | 1.5 | 32. | 0.001 | [a]0.0004 | 0.002 |
| Permalloy | 860. | 0.02 | 2.0 | 32. | 0.001 | [a]0.0006 | 0.05 |
| Nickel | 485. | 0.7 | 0.5 | 2.7 | 0.012 | 0.3 | 0.7 |
| [b]Iron-Si3% | 1590. | 3.7 | 2.2 | 2.4 | 0.0045 | 0.2 | 0.1 |
| Iron-Si4% | 1570. | 3.2 | 2.1 | 2.6 | 0.013 | 0.5 | 0.5 |
| Iron | 1707. | 4.8 | 2.4 | 2.2 | 0.014 | 0.7 | 1. |
| Cobalt | 1400. | 45. | 4.7 | 1.0 | 0.034 | 20. | 10. |
| [c]Alnico | 915. | 260. | 2.0 | [c]0.28 | 0.12 | 600. | 600. |
| $Co_5Sm$ | 800. | 1500. | 2.0 | 0.12 | [d]0.27 | [d]9000. | 10000. |

[a]For these materials, the coercive force may be dominated by magnetostatic effects. In particular, for permalloy, the rapid quenching of the disordered state should produce high stress fields and a Kondorsky-Néel—type contribution to the coercive force.

[b]This material is grain oriented. Therefore, we use $W=4\times10^{-8}$ cm.

[c]Modern theory suggests a spin-rotation mechanism rather than domain-wall motion for Alnico. We have used an effective anisotropy taken from theoretical estimates of the intrinsic coercive force.

[d]We have also obtained lower and upper bounds (see text).

silicon. Keeping this characterization of the defect constant, we see from the last two columns of Table I that our theoretical results agree remarkably well with the experimental values. Our largest value of $h$ occurs for $Co_5Sm$ [i.e., use of Eq. (7) yields $h=0.274$ and $H_c=9000$ Oe for this material]. Using Eqs. (2) and (3), we have also found an upper and lower bound for $Co_5Sm$. The upper bound is obtained by setting $\theta_1=0$ and finding that $h$ for which $\theta_2$ ceases to have real values. This occurs when $h=0.325$ corresponding to $H_c=10\,700$ Oe. The lower bound is found by replacing $\theta$ by $\theta_1$ and dropping all terms containing $h$ in the integrand of Eq. (3). For $\theta_1=35°$ and $\theta_2=86°$ we obtain a solution to Eqs. (2) and (3) with $h=0.211$. The coercive force, $H_c$, is therefore at least 6900 Oe. Thus, for $Co_5Sm$ where we might question our approximation that $h$ is small, Eq. (7) yields an $H_c$ within the lower and upper bounds obtained for this material.

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