Low-Frequency Noise in Tin Films at the Superconducting Transition*

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The $1/f$ noise spectra of current-biased tin films on glass substrates at the superconducting transition were in quantitative agreement with the semiempirical formula of Clarke and Voss. Tin films with an aluminum underlay to reduce the thermal boundary resistance between film and substrate had spectra that were flat at low frequencies. Spatial correlation of the noise was observed in both types of samples.

In a recent Letter,[1] Clarke and Voss reported measurements of $1/f$ voltage noise in current-biased metal films at room temperature. Their results imply that the noise arises from equilibrium temperature fluctuations that modulate the resistance of the film, and that the fluctuations obey the diffusion equation. According to the theory for an infinite thermally homogeneous medium, the average temperature of a bar-shaped subvolume of length l_1 and rectangular cross section $l_2 l_3$ ($l_1 \gg l_2 \gg l_3$) has a spectrum $S_T(f) \propto$ const for $f \ll f_1$; $\propto \ln(1/f)$ for $f_1 \ll f \ll f_2$; $\propto f^{-1/2}$ for $f_2 \ll f \ll f_3$; and $\propto f^{-3/2}$ for $f_3 \ll f$. We have defined $f_i = D/\pi l_i^2$, where D is the thermal diffusivity. $S_T(f)$ and the corresponding voltage spectrum $S_V(f)$ are normalized by setting

$$\int_0^\infty S_V(f)\,df = \bar{V}^2\beta^2 \int_0^\infty S_T(f)\,df = \bar{V}^2\beta^2 k_B T^2/C_V,$$

where $\beta = (1/R)\,dR/dT$ and C_V are the temperature coefficient of resistance and heat capacity of the subvolume, and $\bar{V}$ is the mean voltage across the bar. For a metal film on a glass slide, the measured spectrum was $1/f$. Clarke and Voss attributed the discrepancy between theory and experiment to the thermal inhomogeneity of the experimental system: Film and substrate have very different thermal diffusivities, and there is a substantial thermal boundary resistance between them. The data from the metal films were in excellent agreement with a semiempirical spectrum obtained by assuming that $S_T(f) \propto f^{-1}$ for $f_1 < f < f_2$:

$$S_V(f) = \bar{V}^2\beta^2 k_B T^2/[3 + \ln(f_2/f_1)]\,C_V f. \qquad (1)$$

In this Letter, we report the first systematic measurements of the low-frequency noise induced by intrinsic temperature fluctuations in tin films at the superconducting transition.[2] The dependences of $S_V(f)$ on β^2, $\bar{V}^2$, and sample volume ($\propto C_V$) are experimentally demonstrated. When the film is evaporated on a glass substrate, the observed $1/f$ noise is accurately predicted by Eq. (1) even though $\beta^2 T^2/C_V$ is nine orders of magnitude larger than in the room-temperature experiments. In particular, this result demonstrates that the theory correctly predicts the temperature dependence of the noise. When the tin deposition is preceded by a thin aluminum film, the thermal boundary resistance is greatly reduced, and the noise spectrum flattens off at frequencies below f_1, as predicted by a three-dimensional thermal-diffusion model. The spatial correlation of the noise is also substantially reduced, as expected when the film becomes strongly coupled to the substrate. These last two results are a graphic demonstration of the importance of the thermal boundary resistance in determining the noise spectrum.

Tin films typically 1000 Å thick were evaporated onto glass substrates and scribed to the desired dimensions; these samples will be referred to as type A. The substrate was clamped to a copper plate mounted in a vacuum can. The plate was linked to the can by a low thermal conductance to give a time constant of about 75 sec at 4 K. The long time constant filtered out bath temperature fluctuations at frequencies above 10^{-2} Hz. The can was immersed in liquid helium maintained at about 2 K by a manostat, and the copper plate was electrically heated to the transition temperature of the film. The noise spectrum was measured with a constant current I, typically 20 to 200 μA, flowing in the film.

The noise spectrum of a type-A sample is shown in Fig. 1, curve a. The dashed line is calculated[3] from Eq. (1). The background noise spectrum was typically one to five orders of magnitude below this spectrum, and was subtracted out. The agreement between the measured and calculated curves is good in this frequency range, although the measured slope is a little higher than f^{-1}. The value of the slope varied somewhat

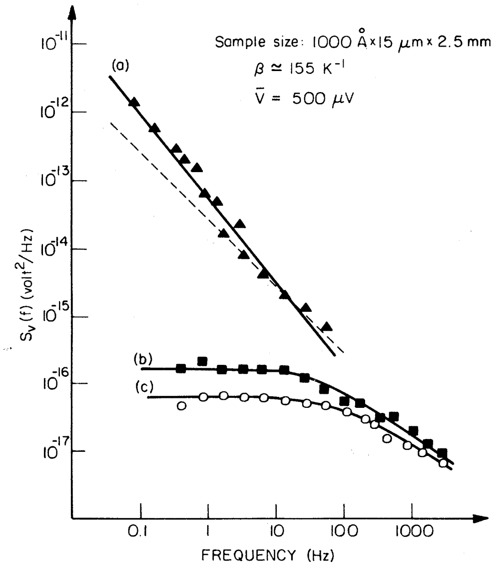

FIG. 1. $S_V(f)$ for tin films on (curve *a*) glass with no underlay (type *A*) and (curves *b* and *c*) sapphire and glass with aluminum underlays (type *B*). The dashed line is the prediction of Eq. (1) for type *A*.

from sample to sample, but was usually close to unity. In all cases, Eq. (1) predicted the magnitude of the spectrum to within a factor of 3.

We have investigated the dependence of $S_V(f)$ on the sample volume $\Omega = l_1 l_2 l_3$, dR/dT, and $\bar{V}$. In Fig. 2(a) we plot $S_V(1)$ against Ω^{-1} for five samples. In each case the temperature was set at the midpoint of the transition so that β was constant, and $\bar{V}$ was adjusted to the same value, 500 μV. In Fig. 2(b) we plot $S_V(10)$ against $(dR/dT)^2$ for one sample with the current $(\bar{V}/R)$ held constant. The various values of dR/dT were obtained by biasing the sample at different points on the transition curve.[4] In Fig. 2(c), we plot $S_V(10)$ against $\bar{V}^2$ for one sample, at the midpoint of the transition. In each of the three plots a line of slope unity has been fitted. The fits to the data are quite satisfactory[5] (the spectra of supposedly identical films often differ by as much as a factor of 3 in magnitude). The results confirm that $S_V \propto \bar{V}^2\beta^2/\Omega$. Equation (1) accurately predicts the amplitude for noise both in the earlier work[1] at room temperature and in the present work, although the total heat capacity changes by three orders of magnitude between 300 and 4 K. Consequently, there is strong evidence that $S_V \propto \bar{V}^2\beta^2/C_V$.

As a further test of the thermal diffusion mechanism, we measured the frequency-dependent spatial correlation of the noise. A tin film was scribed as shown in the inset of Fig. 3. The separation of the midpoints of the strips was 5 mm. The two noise voltages $V_1(t)$ and $V_2(t)$ generated by the current I were separately amplified, and the spectrum of their sum or difference measured. If $S_+(f)$ and $S_-(f)$ are the spectra of $V_1(t)+V_2(t)$ and $V_1(t)-V_2(t)$, the fractional correlation between the strips is $C(f)=[S_+(f)-S_-(f)]/[S_+(f)+S_-(f)]$. According to the diffusion model, the noise should be correlated at low frequencies, where $d \ll \lambda(f) = (D_e/\pi f)^{1/2}$, and uncorrelated at high frequencies, where $d \gg \lambda(f)$. Because of the inhomogeneous nature of the system, we have introduced an *effective* diffusivity, D_e. The measured behavior of $C(f)$, shown in Fig. 3, is in excellent agreement with this model. The observed changeover from correlated to uncorre-

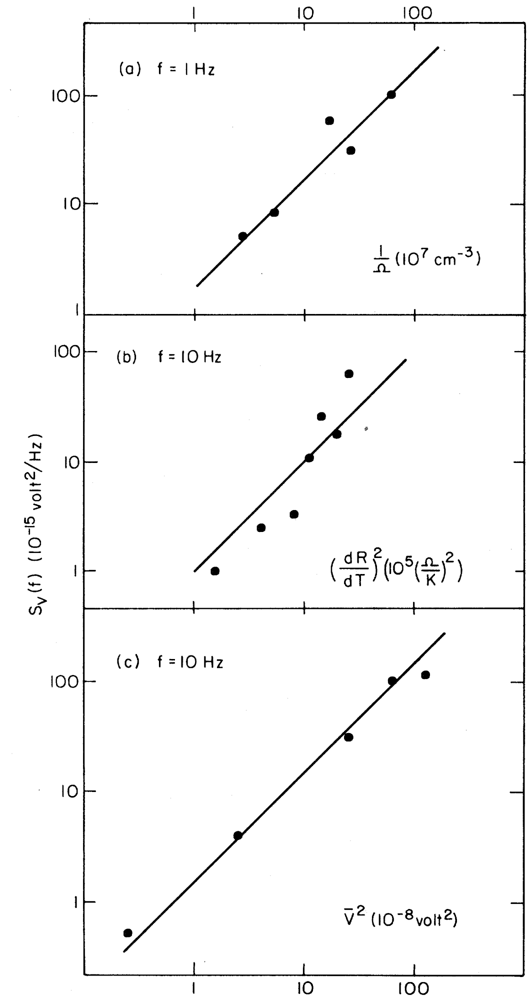

FIG. 2. $S_V(f)$ for type-*A* samples as a function of (a) Ω^{-1}, (b) $(dR/dT)^2$, and (c) $\bar{V}^2$.

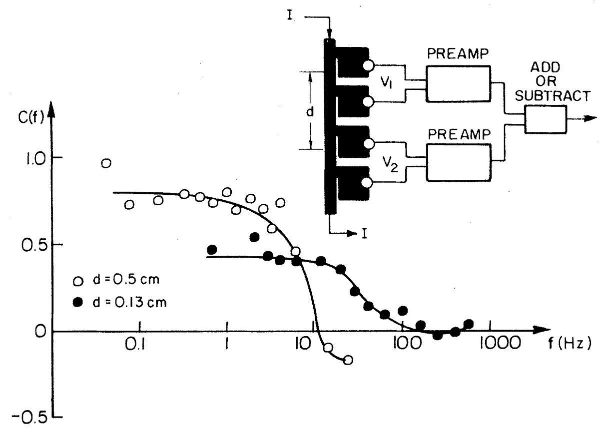

FIG. 3. Noise correlation for type-*A* (open circles) and type-*B* (black circles) samples. Inset is experimental configuration.

lated noise at about 8 Hz corresponds to $D_e \approx 6$ cm^2 sec^{-1}.

To study the effect of the thermal properties of the substrate on the spectrum, we made samples on single-crystal sapphire substrates. However, we found that the thermal boundary resistance between the tin films and the sapphire was extremely high: Self-heating due to the bias current was sufficient to raise the temperature of the film out of the transition range. We then made samples on both sapphire and glass with a 50-Å aluminum film deposited on the substrate prior to the tin evaporation; these will be referred to as type-*B* samples. The aluminum underlay did not significantly change the resistivity, transition temperature, or transition width of the tin film. The spectra of two type-*B* samples on sapphire and glass are shown in Fig. 1, curves *b* and *c*. A type-*B* spectrum is dramatically different from a type-*A* spectrum: Below about 30 Hz, it is flat, and very much smaller in magnitude. This result can be explained by assuming that the underlay greatly reduces the thermal boundary resistance so that a fluctuation in the film has a much greater probability of decaying across the boundary: Film and substrate become a closer approximation to a thermally homogeneous three-dimensional system. The diffusion model then predicts that the spectrum should be flat at frequencies below $f_1 \approx D_e/\pi l_1^2 \approx 30$ Hz (using the value of D_e from the correlation experiment). To our knowledge, this is the first "$1/f$" spectrum obtained for an electrical system that flattens off at low frequencies, as predicted by the diffusion theory. In a separate series of experiments in which we measured the heat required to raise the film temperature through a given interval, we found the thermal boundary resistance of type-*B* samples to be much less than that of type-*A* samples. Scanning electron micrographs showed that the tin nucleation was also very different in type-*A* and type-*B* films. The fact that type-*B* samples have essentially the same spectra when either sapphire ($D \sim 10^5$ cm^2 sec^{-1}) or glass ($D \sim 3$ cm^2 sec^{-1}) substrates are used implies that the boundary resistance, rather than the diffusivity of the substrate, plays the dominant role in determining the shape of the spectrum.

We also measured the correlation function $C(f)$ for type-*B* samples, using the same configuration as in the earlier correlation experiment. With $d = 5$ mm, we observed no correlation. When d was lowered to 1.3 mm, we obtained the correlation function shown in Fig. 3 (for a glass substrate). From the rolloff of $C(f)$ at 60 Hz, we deduce $D_e \sim 3$ cm^2 sec^{-1}. Notice that at low frequencies $C(f)$ is about 0.8 for type-*A* samples, but is only about 0.4 for type-*B* samples. In the case of poor thermal contact to the substrate, a fluctuation decays predominantly along the film; thus a high degree of correlation can be observed out to distances of 5 mm or more. On the other hand, with good thermal contact a fluctuation is more likely to decay into the substrate; thus the degree of correlation at low frequencies is lower, and can be observed only over relatively short distances.

These results, together with the room-temperature measurements,[1] constitute very strong evidence in support of an equilibrium thermal-fluctuation model for $1/f$ noise. However, it is clear that a more detailed understanding of the decay of fluctuations in a thermally inhomogeneous system is still required in order that one may explicitly *calculate* the $1/f$ spectrum observed when there is poor thermal contact between the film and its substrate. The result of such a calculation should closely resemble the semiempirical Eq. (1), except perhaps for the geometrical factor $[3 + \ln(f_2/f_1)]$.

We are indebted to Richard F. Voss for the use of his computer programs and for many helpful conversations.

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[1] J. Clarke and R. F. Voss, Phys. Rev. Lett. 33, 24

(1974).

[2] Other authors have reported $1/f$ noise at the superconducting transition; for example, R. M. Katz and K. Rose, Proc. IEEE 61, 55 (1973); R. P. Huebener and D. E. Gallus, Phys. Rev. B 7, 4089 (1973).

[3] In calculating the total heat capacity, we have assumed that the electronic contribution is the average of the values in the normal and superconducting states just above and below the transition temperature.

[4] We have not corrected for the fact that the electronic heat capacity differs for each value of dR/dT: The correction is small compared with the measurement errors.

[5] $S_V(f) \propto \overline{V}^2$ for almost all linear systems. F. N. Hooge [Phys. Lett. 29A, 139 (1969)] has shown that $S_V(f) \propto 1/\Omega$ for metals and semiconductors.

Short-Time-Scale Measurement of the Low-Temperature Specific Heat of Polymethyl Methacrylate and Fused Silica*

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Low-temperature specific-heat measurements of amorphous silica and polymethyl methacrylate have been performed on a 10^{-4}-sec time scale using a novel heat-pulse technique. The data disagree with a recent theoretical prediction of Phillips and of Anderson *et al.* This prediction is based on the assumption that tunneling states are the origin of both the linear temperature dependence of the specific heat and the T^2 temperature dependence of the thermal conductivity characteristic of all glasses at low temperatures.

The temperature dependence of the specific heat of a wide variety of amorphous dielectric materials is of the form[1,2]

$$C = c_1 T + c_3 T^3 \ (0.1\ \mathrm{K} < T < 2\ \mathrm{K}). \tag{1}$$

This behavior cannot be understood with the Debye model. Even c_3 exceeds the Debye prediction (based on measurements of the sound velocity) by between 30% and 200% depending on the material.[2,3] The thermal conductivity of amorphous dielectrics in the range below 1 K varies as $T^{1.9\pm0.1}$ and is, at a given temperature, of the same order of magnitude for most glasses independent of chemical composition or preparation.[1-3] The physical mechanism responsible for the temperature dependence of the thermal properties (and in particular the linear term in the specific heat) is still poorly understood. It has been suggested that both the linear specific-heat anomaly and the thermal conductivity may be a result of two-level atomic or molecular tunneling states which are presumed to be intrinsic to the amorphous state.[4,5] These localized states enhance the specific heat above the Debye specific heat and resonantly scatter phonons in such a way as to account for the observed thermal conductivity. One of the consequences of the tunneling model is that the specific heat should be a function of the time duration of the measurement. This effect arises because there is a time τ required for the tunneling states to come into thermal equilibrium with the phonons. When the measurement time t is reduced to a value comparable with τ a reduction in the specific heat should occur since only those states which have had an opportunity to interact with the phonons will influence the measurement. It has been shown[6] that for a constant density of states $\overline{P}$ (which is required to explain the linear specific-heat anomaly) the tunneling model predicts a specific heat which increases logarithmically with t. In the dominant-phonon approximation ($\hbar\omega = 3k_B T$) the coefficient c_1 of Eq. (1) has the form

$$c_1 \propto \ln(4t/\tau_m) \quad (t \gg \tau_m), \tag{2}$$

where τ_m^{-1} is the maximum rate at which tunneling states of a given energy splitting can relax. The quantity τ_m is related to the phonon lifetime τ_{ph} which can be derived from the thermal conductivity κ. Using published data[1,2,7] for $\overline{P}$ and κ of fused silica and polymethyl methacrylate (PMMA) and calculating τ_{ph} in the dominant-phonon approximation we find that τ_m varies roughly as T^{-3} and at 0.1 K has values of 3.2 and 0.8 μsec for fused silica and PMMA, respectively. This value of τ_m for fused silica agrees well with that