step is basic to the definition of ionicity in terms of the dielectric constant and the entire structure rests on it. In contrast, the dielectric constant derived in the bond-orbital model[3] shows an additional dependence upon polarity due to the spatial separation of the bonding and antibonding states. This yields a dependence of $\epsilon_1 - 1$ upon the band gap of the form $(\hbar\omega_g)^{-3}$ in an isoelectronic series, or a proportionality of the dimensionless oscillator strength to $\hbar\omega_g^{-1}$. Note that bond length does not vary appreciably in such a series.) It is desirable to state the problem clearly and to seek a test of these significantly different results. We should not selectively bury the results we do not like by introducing new parameters to absorb the discrepancy.

The oscillator strength for a particular transition can be written in terms of the matrix elements of the coordinate between those states[4] or, with use of a familar identity,[5] in terms of the matrix elements of the gradient. The latter form is

$$f_{ij} = 2\hbar^2 |\langle i|\nabla|j\rangle|^2/m\hbar\omega_{ij}.$$

The assumption upon which Phillips has based his theory, then, is that the square of the matrix element, $\langle i|\nabla|j\rangle^2$, is proportional to the band gap, $\hbar\omega_g = \hbar\omega_{ij}$ for the two-level model. The bond-orbital model predicts it to be independent of the gap. The square of the matrix element has long been known experimentally to be independent of polarity, or ionicity, in an isoelectronic series,[6] in support of the bond-orbital model but contrary to Phillips's assumption. The recent calculations by Chelikowsky and Cohen, shown in Phillips's Table I,[2] independently support this conclusion as have earlier calculations. This simply means that some of the oscillator strength is transferred to higher energies. There seems not to be theoretical nor experimental support for the assumption upon which the ionicity theory is based. It may have seemed plausible at the outset and there may not have been motivation to test it subsequently. It appears now that the assumption was incorrect and I do not find the current effort to rescue it convincing.

[1] J. C. Phillips, Rev. Mod. Phys. 42, 317 (1970).

[2] J. C. Phillips, preceding Comment (Phys. Rev. Lett. 34, 1196(C) (1975)].

[3] W. A. Harrison, Phys. Rev. B 8, 4487 (1973); W. A. Harrison and S. Ciraci, Phys. Rev. B 10, 1516 (1974). Note that in the latter the defining standard for the gap was taken to be the absorption peak E_2, not the dielectric constant.

[4] See, for example, J. M. Ziman, *Principles of the Theory of Solids* (Cambridge Univ. Press, Cambridge, England 1964), p. 225.

[5] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), 3rd ed., p. 404.

[6] P. Lawaetz, Phys. Rev. B 4, 3460 (1971); see in particular the concluding paragraph. Noted also by J. C. Phillips, *Bonds and Bands in Semiconductors* (Academic, New York, 1973), p. 121.

Is a "New Scaling Hypothesis in High Energy Collisions" Needed?

B. E. Y. Svensson and L. Sollin
Department of Theoretical Physics, University of Lund, Lund, Sweden
(Received 26 December 1974)

The experimental results used by Dao *et al.* to support a "scaling-in-the-mean" hypothesis for semi-inclusive processes can also be reproduced by a simple *Ansatz* for the invariant cross section obeying Feynman scaling, Koba-Nielsen-Olesen multiplicity scaling, and factorization in longitudinal and transverse momenta.

In a recent Letter, Dao *et al.*[1] observed a striking regularity in semi-inclusive production of π^- in pp collisions. They studied the production cross section as a function of the prong number n, the pion longitudinal momentum p_L, and its transverse momentum p_T. With $\langle p_L\rangle_n$ denoting the mean value of the longitudinal momentum for a given prong number, they find that the differential cross section[2] $(\langle p_L\rangle_n/n\sigma_n)\, d\sigma_n/dp_L$ is a function only of the ratio $p_L/\langle p_L\rangle_n$ and is independent of both n and the total energy s; this may be called "scaling-in-the-mean" for the longitudinal cross section. An analogous observation was made for the transverse-momentum distribution. Dao *et al.* use this observation to propose scaling-in-the-mean as a general property for semi-

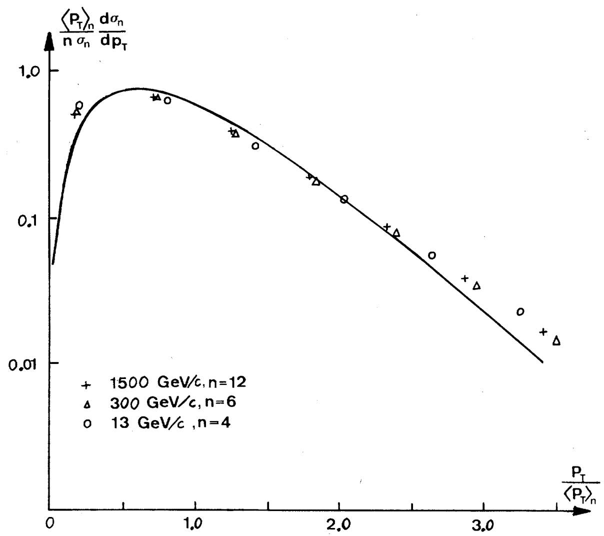

FIG. 1. The curve represents the fit to experimental data made by Dao *et al.* (see Ref. 1). The points are the results of calculations with our *Ansatz*. The points at 300 GeV/c and $n = 22$ fall practically on top of the $n = 6$ points and are not shown. The discrepancies between experiment and our results are within the experimental errors.

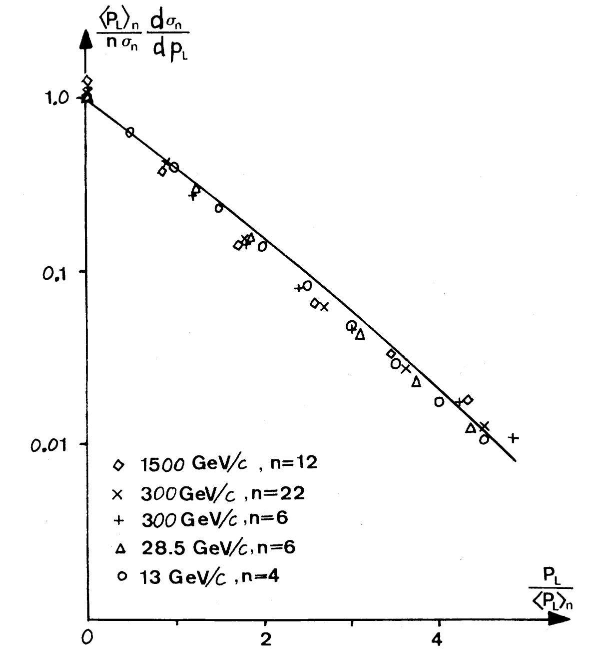

FIG. 2. As in Fig. 1, the curve is a fit to the data of Dao *et al.* and the points are our results.

inclusive processes.

Are there any relations between "scaling-in-the-mean" results and other regularities in semi-inclusive reactions? We think in particular of the fact that, to an accuracy of about 30% or better, the invariant semi-inclusive cross sections seem to obey both Koba-Nielsen-Olesen (KNO) and Feynman scaling,[3,4] i.e., depend on the multiplicity only through the variable $n/\langle n\rangle$ and on the longitudinal momentum only through the variable $x = 2p_L/\sqrt{s}$. Furthermore, within say an accuracy of about 20%, it seems as if the invariant differential cross section $E\,d^3\sigma_n/d^3p$ factorizes[5] into a function of x and another function of p_T.

To see the connection between these results and the observation by Dao *et al.*, we made the simple *Ansatz*

$$\frac{1}{\sigma_n} E \frac{d^3\sigma_n}{d^3p} = A \exp\left(-\frac{x}{a}\right) \exp\left(-\frac{p_T}{b}\right), \qquad (1)$$

where A, a, and b depend only on the KNO variable $n/\langle n\rangle$; this form is then consistent with KNO's proposal for semi-inclusive cross sections.[4] The parameters a and b are to be adjusted to fit the data. We find an acceptable fit with $a = 0.025(\langle n\rangle/n + 1)$ and $b = 0.17$ GeV; the value of A is determined by the normalization condition[4]

$$n\sigma_n = \int (d^3\sigma_n/d^3p)\, d^3p\ . \qquad (2)$$

From this *Ansatz* we then calculate the special cross section used by Dao *et al.* It should be noted that they use *noninvariant* cross sections. Furthermore, since the experimental values[1] of $\langle p_L\rangle_n$, 0.9–0.4 GeV/c, are comparable to the value ≈ 0.33 GeV/c for $\langle p_T\rangle_n$, we have to use the full energy factor $E = (p_L^2 + p_T^2 + m_\pi^2)^{1/2}$ in converting our invariant cross section to a noninvariant one.

Figures 1 and 2 give the results. For clarity in presentation we use the curves that Dao *et al.* find to fit their results, while the output of our calculations is given as points in the diagrams. These points are well within the spread of the experimental points.

As is seen, the experimental results used by Dao *et al.* to support their scaling-in-the-mean hypothesis are well reproduced by the simple *Ansatz* (1). We have not made any attempt to obtain a best fit to the data with the expression (1), since a preferable procedure would be to plot the (partially integrated) invariant cross section instead. Our sole purpose is to draw attention to the fact that the experimental regularity behind the scaling-in-the-mean hypothesis can be reproduced by use of other current ideas on inclusive reactions.

We should also remark that rather than using the total charged-particle multiplicity n as we have done, it might be more consistent to use the number n_- of negative pions. However, since n_- is roughly proportional[6] to n we do not expect any large corrections to our results from using this variable. Nor would such modifications of the KNO scaling variable as those proposed[7] to get a more accurate fit to the multiplicity distribution affect the essence of our results.

One of us (B.E.Y.S.) wants to thank Professor K. Tanaka for initiating discussion on the possible relevance of the scaling-in-the-mean law.

[1]F. T. Dao, R. Hanft, J. Lach, E. Malamud, F. Nezrick, V. Davidson, A. Firestone, D. Lam, F. Nagy, C. Peck, A. Sheng, R. Poster, P. Schlein, W. Slater, and A. Dzierba, Phys. Rev. Lett. 33, 389 (1974). See also D. B. Lichtenberg, Phys. Rev. Lett. 33, 520 (1974).

[2]Dao *et al.* claim to use $(\langle p_L\rangle_n/\sigma_n)d\sigma_n/dp_L$, but a consistent normalization [see Eq. (2)] requires the extra factor $1/n$.

[3]Z. M. Ma, R. Engelmann, R. Singer, L. Voyvodic, and J. Whitmore, Phys. Rev. Lett. 31, 1320 (1973); Y. Cho, R. Engelmann, T. Fields, L. Hyman, R. Singer, L. Voyvodic, and J. Whitmore, Phys. Rev. Lett. 31, 413 (1973).

[4]Z. Koba, H. B. Nielsen, and P. Olesen, Phys. Lett. 38B, 25 (1972); Z. Koba, in CERN Report No. CERN 73-12, Proceedings of the CERN-JINR School of Physics, Aarhus, Denmark, 1973 (unpublished).

[5]H. Bøggild *et al.*, Nucl. Phys. B27, 285 (1971).

[6]See, e.g., R. Slansky, Phys. Rep. 11C, 99 (1974).

[7]A. J. Buras, J. Dias de Deus, and R. Møller, Phys. Lett. 47B, 251 (1973).

Comment on a Narrow Meson Resonance at 1932 MeV*

D. Weingarten and S. Okubo
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 18 February 1975)

A number of characteristics of a narrow resonance observed in $\bar{p}p$ scattering can be explained by assuming that it is a nucleon-antinucleon system, rather than quark-antiquark. This object, together with similar states in other baryon-antibaryon channels, could be instrumental in the saturation of duality constraints. Relations between our proposal and models for ψ and ψ' are discussed.

In addition to the now well-known ψ and ψ', at least one other very narrow meson resonance with mass greater than 1100 MeV seems reasonably well confirmed.[1] This object, which we will refer to as θ, appears as a bump in the total cross sections for $\bar{p}p$ and $\bar{p}d$ scattering and has a mass and width of

$$M_\theta = 1932 \pm 2 \text{ MeV}, \quad \Gamma = 9^{+4}_{-3} \text{ MeV}.$$

The mass of θ is sufficiently far above the masses of the lowest octet of 0^- mesons that we would expect the phase space available for multimeson decays to be extremely large, and therefore the value of Γ to be correspondingly large. For comparison, the widths[2] of $g(1680)$ and $\omega(1675)$ are 180 ± 30 MeV and 140 ± 20 MeV, respectively. Thus although θ seems perhaps less unusual than ψ and ψ', its narrow width is still surprising and needs explanation, particularly in relation to the various models offered for ψ and ψ'. In the present note, we will give semiquantitative arguments which suggest that the narrow width of θ can primarily be explained by the assumption that θ is a resonant $\bar{N}N$ (antinucleon-nucleon) system in much the same sense that deuterium is an NN bound state.[3] Thus part of our result is that θ appears to be exotic. We will also suggest that if there are a number of other $\bar{B}B$ (antibaryon-baryon) resonances and bound states, then these objects may be responsible for the saturation of duality in the $\underline{27}$, $\underline{10}$, and $\underline{10}^*$ $\bar{B}B$ channels. Finally we will show that an alternative interpretation of θ as a $\bar{q}_5q_5$ system (where q_5 is a new fifth quark), similar to the many recent interpretations[4] of ψ and ψ' as $\bar{q}_4q_4$, appears somewhat unlikely, while on the other hand, to the extent that a reasonable guess concerning magnetic form factors can be believed, it would seem that ψ and ψ' probably are not $\bar{B}B$ systems as some authors[5] have proposed.

The parameters of θ given by Carroll *et al.*[1] were obtained by fitting the observed bump in $\bar{p}p$ cross sections with a Breit-Wigner form. The maximum enhancement at the peak of the resonance is $\sigma_{max} = 18^{+6}_{-3}$ mb. Then if $\sqrt{s}$ is the c.m.-system energy of a $\bar{p}p$ scattering experiment,