the $\frac{3}{2}$-order family including the full membrane-polymer model with variable density, in order to compare to Π-A lipid monolayer experiments.[9]

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Specular Reflection of ^{4}He Atoms from the Surface of Liquid ^{4}He†

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Measurements with a pulsed ^{4}He atomic beam incident on the surface of liquid ^{4}He at $T=0.03$ K show that the probability of specular reflection depends only on the perpendicular momentum of the atom $\hbar k\cos\theta$ and that it varies smoothly from 4×10^{-2} to less than 10^{-4} for $k\cos\theta$ in the range from 0.05 to 0.6 Å^{-1}. The probability of diffuse inelastic scattering is less than 2×10^{-3}. Contrary to theoretical expectations, there is no significant change in the reflection probability at the roton threshold at $k=0.5$ Å^{-1}.

This Letter describes measurements of the elastic scattering probability of low energy (0.1 K $\lesssim \hbar^2k^2/2mk_B \lesssim$ 3 K) ^{4}He atoms incident on the free surface of superfluid ^{4}He. The probability of elastic scattering $R(k,\theta)$ was measured as a function of the angle of incidence θ and the momentum $\hbar k$ of the incident atoms. In addition, a search was made for atoms inelastically scattered from the surface which gives an upper limit for the probability of inelastic scattering, $D(k,\theta)$. The liquid-helium surface was varied in temperature between 0.03 and 0.12 K and no temperature dependence in R was observed. The experimental $R(k,\theta)$ is therefore characteristic of the ground state of the liquid and its surface. Our experiment is related to a number of theoretical discussions[1-6] of the process of condensation and evaporation at the surface of superfluid helium. It has been claimed[3,5] that $R(k,\theta)$ is related to the single-particle spectral density function in the liquid and therefore[3] perhaps to the elusive n_0, the fraction of atoms in the condensed state. This claim is based on the application of tunneling theory and supposes that the tunneling Hamiltonian which couples the vacuum states to the liquid is weak, implying that the probability of condensation $f(k,\theta)\equiv 1-R-D$ is small. Our results, on the contrary, show that, even for large θ and small k, $f(k,\theta)\simeq 1$ with R and D both small. We deduce that "weak" tunneling theory is probably invalid, and note that the effect of the finite thickness of the surface region and the effect of the surface excitations (ripplons) have not been included in any of the published theoretical calculations.

Apart from its possible connection with the condensate fraction, the reflection probability $R(k,\theta)$ is related to $\omega(k)$, the phonon spectrum of the liquid (shown in Fig. 1). As was pointed out by Anderson[2] and Widom and co-workers,[1] a condensing atom with momentum $\hbar k$ transfers an energy $\epsilon = L_0 + \hbar^2k^2/2m$ to the liquid, where $L_0/k_B = 7.16$ K is the latent heat at absolute zero. If the atom produces a single "roton," i.e., a phonon near the minimum in $\omega(k)$, a minimum kinetic energy $(\Delta - L_0)/k_B \simeq 1.5$ K is required. It has been predicted[1,2,5] that this will produce a discontinuity in the k dependence of $R(k,\theta)$ at $k=0.50$ Å^{-1}. The possible detection of this discontinuity was one of the reasons for undertaking

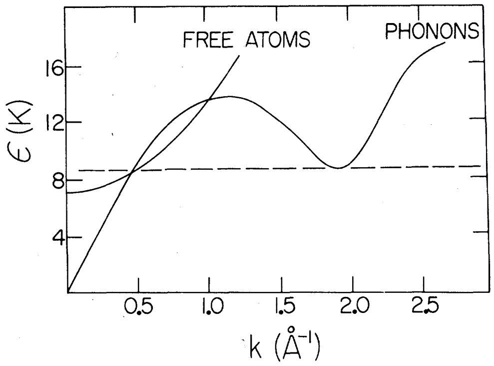

FIG. 1. The dispersion relation $\epsilon(k)$ for free atoms relative to their energy in the liquid. The phonon excitation spectrum is also shown. The dashed line is at the energy of the roton minimum.

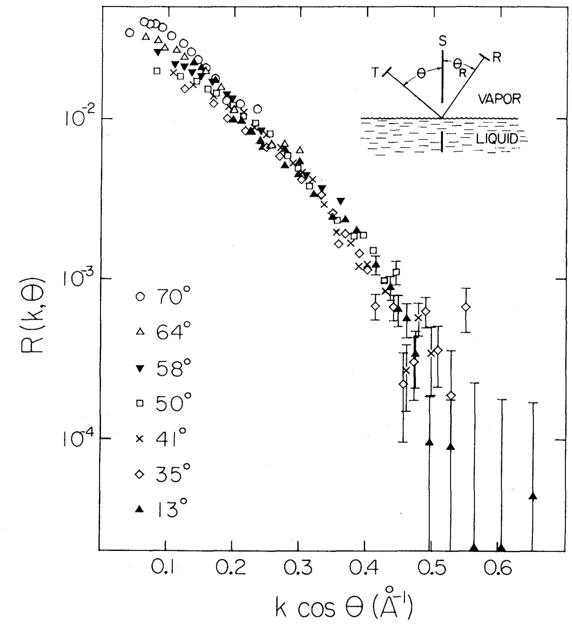

FIG. 2. The probability of specular reflection $R(k, \theta)$ as a function of the perpendicular momentum $\hbar k \cos\theta$. Overlapping points have been omitted. The inset shows the geometrical arrangement of the apparatus when measuring the reflected signal. To measure the direct signal the liquid level is lowered and the arms carrying the transmitter T and receiver R are horizontal.

the present measurements.[7]

The experiment is based on a technique discovered by Meyer *et al.*[8] A description of the apparatus and of some preliminary experiments has been published by Eckardt *et al.*[9] A pulsed beam of atoms is produced by applying a 20–40-μsec burst of heat Q to the "transmitter," a square of resistance board 8 mm $\times$ 8 mm, covered with the saturated helium film. Typically Q is 0.1 to 0.4 erg so that only a fraction of an atomic layer of helium is evaporated. The transmitter T is mounted on a movable arm 40 mm long so as to vary the angle of incidence of the beam of atoms with respect to the surface of the liquid (see inset to Fig. 2). Those atoms which are reflected through the "window" in the screen S can be detected by the receiver R which is a similar resistor board used as a low-temperature bolometer. The bolometer and its associated electronics have a time constant which varies with the receiver temperature and which is ~30 μsec at the normal operating temperature of 175 mK. This is sufficiently fast for the velocity distribution of the atoms to be determined from the received power as a function of the time. The reflection coefficient is obtained by taking the ratio of the reflected signal to the direct signal obtained when the receiver and transmitter are in direct line of sight through the window. The observed velocity distribution in the direct beam depends on the heat input Q, but it is nearly Maxwellian and for $Q = 0.4$ erg has a temperature of about 0.6 K.

Tests were made to verify that the ratio $R(k, \theta)$ for the elastically scattered signal was independent of the input energy Q and the operating temperature of the receiver which was varied from 120 to 200 mK with the liquid bath at ~30 mK.

The fact that the atoms are specularly (and therefore elastically) scattered from the liquid surface was carefully verified by varying both the angle of incidence θ and of reflection θ_R. The boundaries of the reflected beam agreed with the assumption of specular reflection to within about 1°, the angular accuracy of the apparatus. In the shadow where $\theta_R \neq \theta$ and no specular reflection is expected the signal dropped to within the noise, so that we could place an upper limit on the fraction of atoms which were inelastically scattered. Assuming, conservatively, that the inelastic scattering is completely diffuse, this fraction is less than 2×10^{-3} of the incident atoms. The sensitivity of this measurement is less than that for the elastic fraction because of the greater solid angle accessible to diffusely scattered atoms.

The liquid sample of about 300 cm^3 had been refined to remove all but 0.4×10^{-9} of ^{3}He. We estimate, using the data and theory of Eckardt *et al*,[10] that the uppermost atomic layer in the liquid contained less than 0.5×10^{-3} of ^{3}He. In our earlier experiments[9] the sample, of normal

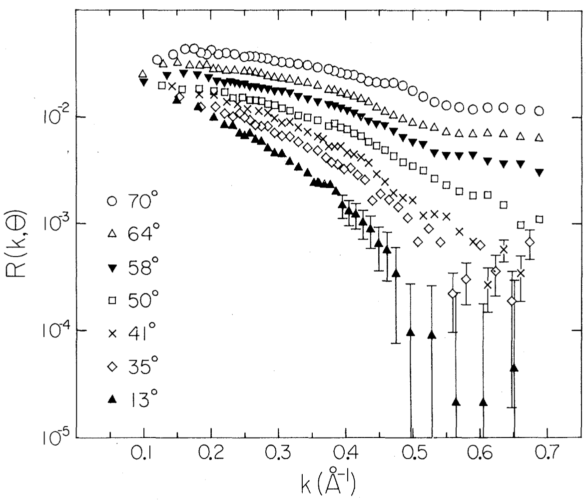

FIG. 3. The probability of specular reflection $R(k, \theta)$ as a function of the wave number k and angle θ of the incident atom. The error bars represent the statistical error and are shown when greater than twice the size of the point. Systematic errors are probably about the same size.

purity ($\sim 1\times10^{-7}$ of ^{3}He), was covered at low temperature with ~ 0.1 monolayer of ^{3}He, and substantial inelastic scattering was observed. Further experiments are planned to study the effect of adsorbed ^{3}He on both scattering probabilities.

The results for $R(k,\theta)$ are shown in Fig. 3 as a function of incident wave number k and angle θ. We now point out the following features of these data:

(1) The probability of reflection for a very slow atom ($k\to0$) seems to approach a limit of about 0.05. At first sight this is surprising since, in the one-dimensional quantum-mechanical problem of a particle approaching an attractive, smooth potential step, the reflection coefficient $\to 1$ as $k\to 0$. This follows if $k\lambda \ll 1$ where λ is a length characterizing the width of the step. Then, by a change of scale, the Schrödinger equation can be transformed to that of an infinite, sharp potential step which has $R=1$. The de Broglie wavelength of the slowest atoms measured ($2\pi/k\sim 60$ Å) is certainly large compared to characteristic distances in the liquid (~ 1 Å), but there is a larger characteristic length associated with the Van der Waals potential outside the liquid surface. In the presence of the Van der Waals potential the Schrödinger equation may be written as $d^2\psi/dz^2+(k^2+\lambda/z^3)\psi=0$, where z is the distance from the liquid surface and $\lambda = 20$ Å. We surmise that at still lower k, such that $k\lambda \ll 1$, the reflection coefficient may again rise and approach unity.

(2) There is no sharp decrease or indeed any significant structure in R at the roton "threshold" at $k=0.5$ Å^{-1}. This is a most interesting feature of the results since the theoretical prediction rests mainly on the assumption that single-phonon processes are important. We deduce that multiple excitation or ripplon process are dominant.

(3) The reflection coefficient is always small compared to unity. This fact coupled with the very low upper limit for D ($<2\times10^{-3}$) means that "weak" tunneling theory is invalid. It also means that the "accommodation coefficient" $\bar{f}$, i.e., the probability of condensation f averaged over a Maxwellian distribution of k and θ, is nearly unity. This is in agreement with some,[11] but not all,[12] earlier measurements of the accomodation coefficient. Averaging $f=1-R-D$ over a Maxwellian distribution at $T=0.6$ K we find $\bar{f}=0.989 \pm 0.002$. We also deduce, from detailed balance, that the distribution of atoms evaporated from the liquid in equilibrium with the vapor will be extremely close to Maxwellian (within $\sim 1\%$). Actual measurements,[7,9] particularly the angular distribution reported in Ref. 9, show deviations larger than 1%, which are due to lack of internal equilibrium in the liquid.

(4) The reflection coefficient $R(k,\theta)$ is, within quite small deviations, a function of the perpendicular momentum $\hbar k\cos\theta$ alone. This is shown in Fig. 2. For the elastic scattering of neutrons at the surface,[13] $R(2k\cos\theta)$ is related to the Fourier transform of $n(z)$, the density distribution at the surface, but among other things this depends on the weakness of the neutron-He scattering. There is no corresponding simple theory for atomic scattering, but we hope that the presentation of these new data will encourage the development of such a theory.

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Plasma Heating by a Rotating Relativistic Electron Beam

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A rotating relativistic beam of energy 500 keV and current 20–40 kA is injected into a preformed plasma of density n_p between 10^{12} and 10^{14} cm^{-3}. It is observed that optimum coupling of beam energy to the plasma occurs at $n_p >> n_b$, where n_b is the beam density. The peak plasma temperature is about 1 keV. The heated plasma undergoes radial oscillations characteristic of the magnetosonic mode. The results are interpreted with a new theoretical model.

Recent experiments[1-6] on plasma heating by straight relativistic electron beams have generally shown[1-4] that the optimum energy transfer efficiency from the beam to the plasma occurs at $n_p \simeq n_b$. At plasma densities much greater than the beam density ($n_p/n_b \geqslant 100$), the energy coupling efficiency drops to not more than a few percent[1-3] per meter of plasma length. These results are consistent with predictions based on the nonlinear theory of the two-stream instability.[7] In view of these facts, it is of importance to find different schemes which will be characterized by a relative high coupling efficiency for $n_p \gg n_b$. Such schemes would be in particular useful for heating dense plasmas confined in cusped[8] magnetic fields or in short magnetic mirrors. In this Letter, we report preliminary experimental results supported by theory which show that when a rotating relativistic electron beam is injected into a magnetically confined plasma the optimum energy transfer efficiency occurs near $n_p \simeq 10^{14}$ cm^{-3}, i.e., two orders of magnitude greater than the beam density.

The 500-keV electron beam is emitted from a 3.8-cm-o.d., 0.6-cm-thick annular carbon cathode. After the 50-nsec-duration electron pulse passes through the 1.2×10^{-3}-cm-thick titanium anode foil, it enters a 7-cm-long drift region which contains air at ~400-mTorr pressure. The drift region is terminated at the symmetry plane of the cusp by a second 1.2×10^{-3}-cm-thick titanium foil. A soft-iron plate serves to reduce the transition width of the cusp. At the cusp, the beam gains a rotational velocity V_θ which is comparable to its axial velocity V_z. Upon passing through the cusp the beam enters the plasma, which is confined by a uniform magnetic field. The 60-cm-long plasma is produced by a critically damped Z discharge in a 10-mTorr-pressure He gas and has a peak density of 10^{14} cm^{-3} and an initial temperature of about 1 eV.

The energy content of the plasma after injec-