Purely Algebraic Approach to the New Narrow Resonances

E. Takasugi and S. Oneda
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 23 December 1974)

The narrow width of $\psi(3105)$, as an uncharmed member of the 1^{--} meson multiplet of SU(4), is discussed in a purely algebraic manner without referring to quarks. A crucial test of this assignment of $\psi(3105)$ is proposed.

One of the possible places to put the new resonance[1] $\psi(3105)$ is as the uncharmed vector meson φ_c belonging to the fifteen-plet and singlet representation of SU(4) which contains ρ, K^*, φ, φ_c, ω, D^*, and F^*.[2] A popular anticipation is that φ_c is primarily a $c\bar{c}$ bound state and its decays into uncharmed particles are suppressed in the same way as the decays of $\varphi \sim s\bar{s}$ into nonstrange mesons are suppressed. One may then explain the narrow width of φ_c, provided that the most threatening decays such as $\varphi_c \to D\bar{D}$ and $F\bar{F}$ (D and F are the 0^- counterparts of D^* and F^*) are energetically forbidden.[2]

In this paper we demonstrate that one can entirely replace the *naive* quark-model argument by more powerful and precise sum rules which exhibit a remarkable interplay of masses, SU(4) mixing angles, and axial-vector matrix elements. Our only theoretical ingredients are (i) asymptotic SU(4), (ii) a part of chiral SU(4)⊗SU(4) algebra, and (iii) the usual simple mechanism of symmetry breaking.

To cope with broken SU(4), for our asymptotic SU(4) we assume that the annihilation operator $a_\alpha(\vec{k}, \lambda)$ of hadrons with physical SU(4) index α and helicity λ does transform *linearly* under SU(4) but only in the limit $\vec{k}\to\infty$.[3] Therefore, we define SU(4) mixing parameters in our asymptotic limit.[3] For example, the annihilation operators of φ, φ_c, and ω must be related (at $\vec{k}\to\infty$) *linearly* to the hypothetical SU(4)-representation a_8, a_{15}, and a_0 as follows[3]: $a_\varphi(\vec{k}) = \alpha a_8 + \alpha_c a_{15} + \alpha' a_0$, $a_\omega(\vec{k}) = \beta a_8 + \beta_c a_{15} + \beta' a_0$, and $a_{\varphi_c}(\vec{k}) = \gamma a_8 + \gamma_c a_{15} + \gamma' a_0$. In terms of Euler angles θ, φ, and ψ, the coefficients α etc. can be expressed as $\alpha = \cos\theta$, $\beta = \sin\theta\cos\psi$, $\gamma = \sin\theta\sin\psi$, $\alpha_c = -\sin\theta\sin\varphi$, $\beta_c = -\sin\psi\cos\varphi + \cos\theta\cos\psi\sin\varphi$, and $\gamma_c = \cos\psi\cos\varphi + \cos\theta\sin\psi\sin\varphi$.

With the imposition of the commutation relation (C.R.) $[V_i, V_j] = if_{ijk}V_k$ ($i,j,k = 1, 2, \ldots, 15$), asymptotic SU(4) implies that all the matrix elements of V_i, but considered *only* for the states of hadrons with infinite momentum, can take exact SU(4) values with the prescribed[3] modification by SU(4) mixing.

Next, thanks to the C.R. $[V_i, A_j] = if_{ijk}A_k$, and asymptotic SU(4), we find that "the axial-vector matrix elements, $\langle B_\alpha(\vec{k},\lambda)|A_i|B_\beta'(\vec{k},\lambda)\rangle$ with $\vec{k}\to\infty$, can also be parametrized by the usual prescription of exact SU(4) plus mixing."[3]

The usual assumption of SU(4) breaking (i.e., λ_0, λ_8, and λ_{15}) can be expressed by the *presence* of the exotic C.R.'s of the form $[\dot{V}_\alpha, V_\beta] = 0$ ($\dot{V}_\alpha = dV_\alpha/dt$), where α, β stands for such combinations as K^0, K^0; K^0, D^0; K^0, F^+; F^-, D^-; etc. Inserting these C.R.'s between the single vector[4] meson states (with $\vec{k}\to\infty$) and realizing the C.R.'s, the following *exact* mass constraints ($\rho^2 = m_\rho^2$, etc.) are obtained:

$$\alpha^2\varphi^2 + \beta^2\omega^2 + \gamma^2\varphi_c^2 = \tfrac{1}{3}(4K^{*2} - \rho^2), \tag{1}$$

$$\alpha X_\alpha \varphi^2 + \beta X_\beta \omega^2 + \gamma X_\gamma \varphi_c^2 = 2K^{*2} - \rho^2, \tag{2}$$

$$\alpha_c^2\varphi^2 + \beta_c^2\omega^2 + \gamma_c^2\varphi_c^2 = \tfrac{3}{2}(D^{*2} + \tfrac{1}{9}K^{*2} - \tfrac{4}{9}\rho^2), \tag{3}$$

$$K^{*2} - \rho^2 = F^{*2} - D^{*2}. \tag{4}$$

Here $X_\alpha \equiv \alpha - \sqrt{2}\,\alpha_c$, $X_\beta \equiv \beta - \sqrt{2}\,\beta_c$, and $X_\gamma \equiv \gamma - \sqrt{2}\,\gamma_c$. Noting $\alpha X_\alpha + \beta X_\beta + \gamma X_\gamma = 1$, we obtain from Eq. (2),

$$\alpha X_\alpha = \left(1 + \frac{\Delta^2}{\varphi_c^2 - \varphi^2}\right) - \left(\frac{\varphi_c^2 - \omega^2}{\varphi_c^2 - \varphi^2}\right)\beta X_\beta, \tag{5}$$

$$\gamma X_\gamma = -\left(\frac{\Delta^2}{\varphi_c^2 - \varphi^2}\right) + \left(\frac{\varphi^2 - \omega^2}{\varphi_c^2 - \varphi^2}\right)\beta X_\beta, \tag{6}$$

where

$$\Delta^2 \equiv \varphi^2 - 2K^{*2} + \rho^2.$$

We now assume that the chiral SU(4)⊗SU(4) breaking is also characterized by the presence of the *exotic* C.R.'s of the form $[\dot{V}_\alpha, A_\beta] = 0$, where α, β is the same as in the case of $[\dot{V}_\alpha, V_\beta] = 0$. This includes the popular *pure* $(\underline{4}, \underline{4}^*) \oplus (\underline{4}^*, \underline{4})$ breaking. We then study all the possible realizations of these C.R.'s (in our asymptotic limit) among 1^{--}. Only three independent sum rules emerge as constraints:

$$(\varphi^2-\rho^2)\alpha A+(\omega^2-\rho^2)\beta B+(\varphi_c{}^2-\rho^2)\gamma C=0, \qquad (7)$$

$$A=-\frac{X_\beta}{X_\alpha}\left(\frac{\varphi_c{}^2-\omega^2}{\varphi_c{}^2-\varphi^2}\right)B,\quad C=\frac{X_\beta}{X_\gamma}\left(\frac{\varphi^2-\omega^2}{\varphi_c{}^2-\varphi^2}\right)B. \qquad (8)$$

Here $A\equiv\langle\varphi|A_{\pi^-}|\rho^+(\vec{k})\rangle$, $B=\langle\omega|A_{\pi^-}|\rho^+(\vec{k})\rangle$, and $C=\langle\varphi_c|A_{\pi^-}|\rho^+(\vec{k})\rangle$ with $\vec{k}\to\infty$. By eliminating A, B, and C from Eqs. (7) and (8), we obtain one *important* mass constraint which, when combined with the mass relations, Eqs. (1) and (2), enables us to determine the mixing parameters *completely*.

We are now in a position to compare our sum rules with the naive quark model. Let us impose upon our above sum rules the "ideal" nonet mass constraints, $\rho^2=\omega^2$ and $\Delta^2=0$, i.e., $\varphi^2-K^{*2}=K^{*2}-\rho^2$. These constraints are actually *well* satisfied experimentally. With $\rho^2=\omega^2$, Eqs. (7) and (8) are satisfied by $X_\beta=0$, i.e., $\beta=\sqrt{2}\,\beta_c$, which also implies $A=0$ and $C=0$. With partial conservation of axial-vector current (PCAC), $A=C=0$ is equivalent to the *vanishing* of the $\varphi\to\rho\pi$ and $\varphi_c\to\rho\pi$ couplings. Next, with $\Delta^2=0$, Eqs. (5) and (6) imply that $\alpha X_\alpha=1$ and $\gamma X_\gamma=0$. We obtain $\gamma=0$, which implies that $\psi=0$, since $X_\gamma=0$ is definitely incompatible with the 1^{--} mass relation. Then Eq. (2) becomes an identity and the use of Eq. (1) finally leads to $\sin\theta=\sqrt{\frac{1}{3}}$ and $\sin\varphi=\frac{1}{2}$. We call these angles "ideal" and denote them by θ_i, φ_i, and ψ_i ($\sin\theta_i=\sqrt{\frac{1}{3}}$, $\sin\varphi_i=\frac{1}{2}$, and $\psi_i=0$).

Correspondence to the naive quark picture is as follows: Note that $\varphi_8=6^{-1/2}(u\bar{u}+d\bar{d}-2s\bar{s})$, $\varphi_{15}\simeq12^{-1/2}(u\bar{u}+d\bar{d}+s\bar{s}-3c\bar{c})$, and $\varphi_0=\frac{1}{2}(u\bar{u}+d\bar{d}+s\bar{s}+c\bar{c})$. The first rotation by the angle φ_i brings φ_{15} to the pure $c\bar{c}$ state, while the subsequent rotation by θ_i brings φ_8 and φ_0 to the pure $s\bar{s}$ and $\frac{1}{2}(u\bar{u}+d\bar{d})$ states, respectively. This configuration may be called "ideal," which, in our language, corresponds to the statement $\rho^2=\omega^2$, $\Delta^2=0$, $g_{\varphi\rho\pi}=0$, and $g_{\varphi_c\rho\pi}=0$.

In our purely algebraic approach the degree of *deviation* of any fifteen-plet and singlet mesons of arbitrary J^{PC} belonging to the same excitation s (ρ_s, φ_s, ω_s, φ_{cs}, ..., $F_s{}^*$) from the "ideal" structure can be measured by the deviations from our "ideal" constraints,[5] $\rho_s{}^2=\omega_s{}^2$ and $\varphi_s{}^2-K_s{}^2=K_s{}^2-\rho_s{}^2$. The strong point of our sum rules li in that we need not be confined to the simple "ideal" case. For example, it is of great interest to study whether our sum rules can accomodate the fifteen-plet and singlet 0^{-+} mesons whose structure is known to be far from "ideal."

For the fifteen-plet and singlet 1^{--}, the real mixing parameters (φ, θ, ψ) deviate only slightly from the ideal ones (φ_i, θ_i, ψ_i). Therefore, to a good approximation, we obtain from Eqs. (7) and (8)

$$X_\beta\simeq\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\omega^2-\rho^2}{\varphi^2-\rho^2}\right)\left(\frac{\varphi_c{}^2-\varphi^2}{\varphi_c{}^2-\omega^2}\right). \qquad (9)$$

Combining Eqs. (9) and (6) we obtain for γ

$$\gamma\simeq\left(\frac{2}{3}\right)^{1/2}\left\{\frac{\Delta^2}{\varphi_c{}^2-\varphi^2}-\frac{1}{2}\left(\frac{\omega^2-\rho^2}{\varphi^2-\rho^2}\right)\left(\frac{\varphi^2-\omega^2}{\varphi_c{}^2-\omega^2}\right)\right\}$$
$$\simeq1.0\times10^{-3}\text{ rad}. \qquad (10)$$

The value of γ is indeed very small, but its value is sensitive to the mass of the ρ and K^*. Even the electromagnetic correction to the masses is important. We have used $\rho=0.765$ and $K^*=0.894$ GeV. Since γ is, at any rate, small, the values of θ and φ can be evaluated reasonably well from Eqs. (1) and (2) using the masses of 1^{--}. We obtain $\theta\simeq40°$ and $\varphi\simeq27°$, compared with the "ideal" values of $\theta_i\simeq35°$ and $\varphi_i=30°$.

We now estimate the φ_c decays. The dominant hadronic decays will presumably proceed through the two-body decays involving resonances. The use of our result about parametrization of matrix elements[3] and PCAC with $f_\pi\simeq f_K\simeq f_\eta$ prescribes an unambiguous angular-momentum barrier factor.[4] Constraints from the exotic C.R., $[\dot{V}_{K^0},A_{\pi^-}]=0$, help to reduce the unknown amplitude. $\varphi_c\to B\pi$ is related to $\omega\to B\pi$. We obtain

$$\frac{\Gamma(\varphi_c\to B\pi)}{\Gamma(B\to\omega\pi)}\simeq\frac{3}{2}\left(\frac{\omega^2-\rho^2}{\varphi^2-\rho^2}\right)^2\left(\frac{\varphi^2-\omega^2}{\varphi_c{}^2-\omega^2}\right)^2\left(\frac{p_{\varphi_c}}{p_B}\right)^3. \qquad (11)$$

Here we have used $K^{*2}-\rho^2=K_B{}^{*2}-B^2$ obtained from $[\dot{V}_{K^0},A_{\pi^-}]=0$ and we have neglected K_A-K_B mixing.[6] Only pion PCAC is involved. We predict that $\Gamma(\varphi_c\to B\pi)\simeq13$ keV. [However, this estimate is very sensitive to the ρ mass, since in the "ideal" limit $\Gamma(\varphi_c)=0$.] We thus expect that $\varphi_c\to B\pi\to\omega\pi\pi$ is certainly an observable mode of φ_c decay, though it is forbidden in the "ideal" limit.

$\varphi_c\to KK$, $K^{**}(1420)K$, K_AK, and K_BK are related to $K^*\to K\pi$, $K^{**}\to K^*\pi$, $K_A\to K^*\pi$, and $K_B\to K^*\pi$, respectively,[6] through the small angle γ. For example, we obtain a relation, $\langle K^+|A_{K^+}|\varphi_c(\vec{k})\rangle=-(\frac{2}{3})^{1/2}\gamma\langle K^+|A_{\pi^+}|K^{*0}\rangle$ for $\vec{k}\to\infty$. $\varphi_c\to\rho\pi$, $\varphi\eta$, $\omega\eta$, and K^*K are related to $\varphi\to\rho\pi$, neglecting small η-η'-η_c mixing. Since $\Gamma(\varphi\to\rho\pi)$ is not well known, we choose to predict $\Gamma(\varphi\to\rho\pi)$ from $\Gamma(\varphi_c\to\rho\pi)$. We subtract from $\Gamma(\varphi_c\to\text{all})$ all the partial rates of main modes of φ_c decays other

TABLE I. The predicted partial rates of main hadronic decays of φ_c. These rates are quite sensitive to the choice of the masses of ρ, K^*, ω, and φ. $\Gamma(\varphi_c \to K_A(1240)K)$ must await the determination of $\Gamma(A_1 \to \rho\pi)$. $\Gamma(\varphi_c \to B\pi)$ and $\Gamma(\varphi_c \to K_B K)$ are sensitive also to the mass of K_B. The mass of K_B is determined from our relation $K^{*2} - \rho^2 = K_B^2 - B^2$, neglecting K_A-K_B mixing.

Decay mode of φ_c	Predicted partial width
$B\pi$	13 keV
$K_B K$	35 keV
KK	10 keV
$K^{**}K$	2.8 keV
$K_A(1240)K$	$3.5\times10^{-4}\Gamma(K_A \to K^*\pi)$
$\varphi\eta$	$0.01\Gamma(\varphi_c \to \rho\pi)$
$\omega\eta$	$0.1\Gamma(\varphi_c \to \rho\pi)$
K^*K	$0.4\Gamma(\varphi_c \to \rho\pi)$
$\rho\pi$	$0.7\Gamma(\varphi \to \rho\pi)$

than the $\varphi_c \to \rho\pi$, $\varphi\eta$, $\omega\eta$, and K^*K. Using asymptotic SU(4) and the C. R. $[V_i, V_\mu{}^j(x)] = i f_{ijk} V_\mu{}^k(x)$, we predict[7] that $\Gamma(\varphi_c \to \bar{e}e + \bar{\mu}\mu) \simeq 3$ keV. The rate of the most important photon decay $\varphi_c \to \eta_c\gamma$ has been estimated[8] to be $\simeq 9$ keV.

All the partial rates of hadronic φ_c decays discussed above become *zero* in our ideal limit ($\rho^2 = \omega^2$ and $\Delta^2 = 0$). Since the real situation is close to the ideal limit, these particular rates are sensitive to the actual input values of Δ^2 and $\rho^2 - \omega^2$. Our crude estimate shown in Table I should not be taken too seriously. A slight change in the relevant masses of ρ, K^*, ω, and φ can, for example, reduce the numbers listed by a factor 10. Therefore, even the neglected SU(2)-breaking effect may play a role for these rates.

We note, however, that the ratio, $\Gamma(\varphi_c \to \rho\pi)/\Gamma(\varphi \to \rho\pi) \simeq 0.8(\varphi^2 - \omega^2)(\varphi_c^2 - \omega^2)^{-2}(p_{\varphi_c})^3(p_\varphi)^{-3} \simeq 0.7$, is *not* small, although $\Gamma(\varphi_c \to \rho\pi)$ and $\Gamma(\varphi \to \rho\pi)$ are zero in the ideal limit. If we choose,[8] rather arbitrarily, $\Gamma(\varphi_c \to \text{all}) \simeq 100$ keV, we obtain $\Gamma(\varphi_c \to \rho\pi) < 30$ keV which predicts $\Gamma(\varphi \to \rho\pi) < 40$ keV. This implies that $\varphi \to \rho\pi$ can *at most* be only 10% of the $\varphi \to 3\pi$ decay listed in the compilation,[9] barring a possible large effect of SU(2) breaking.

Our crude estimate of the ratio $\langle\varphi_c|A_{\pi^-}|\pi^+\pi^0\rangle/\langle\omega|A_{\pi^-}|\pi^+\pi^0\rangle$ is of the order $\gamma/\alpha \simeq 10^{-3}$, which will yield a negligibly small rate for the direct $\varphi_c \to 3\pi$ decay.

If $\varphi \to \rho\pi$ is the main cause of $\varphi \to 3\pi$ decay, we think that the assignment of $\psi(3105)$ to φ_c is in serious trouble, *unless* mixing between our 1^{--} and the neglected higher-lying 1^{--} meson plays a role. On the other hand, if the $\varphi \to 3\pi$ is not dominated by $\varphi \to \rho\pi$, the assignment $\psi(3105) = \varphi_c$ is indeed very compelling! We urge the experimentalists to settle this question.

Finally, our mixing angles and mass relations, Eqs. (3) and (4), predict the masses of the charmed 1^{--} meson masses at $D^* = 2.32$ and $F^* = 2.36$ GeV. For baryons the SU(4) mass relations obtained from $[\dot{V}_\alpha, V_\beta] = 0$ should hold in a *mass-squared* form.[3,10] In our scheme there is, *at present,* no serious argument against (but also no compelling argument for) assigning the other new resonance $\psi(3695)$ to the φ_c counterpart of $\rho'(1600)$.

We thank Professor G. A. Snow who sparked this investigation and also our colleagues at the University of Maryland for informative discussions.

[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); C. Bacci *et al.*, Phys. Rev. Lett. 33, 1408 (1974).

[2]We have used the notation introduced in a recent paper by M. K. Gaillard, B. W. Lee, and J. L. Rosner, Fermilab Report No. 74/86-THY (Rev. Mod. Phys., to be published). Earlier but probably not complete references for SU(4) will be found there. We follow the assignment of quarks from S. L. Glashow, J. Iliopoulos, and I. Maiani, Phys. Rev. D 2, 1285 (1970).

[3]This is an extension to SU(4) of previous work by S. Matsuda, S. Oneda, H. Umezawa, and co-workers. For a review, see S. Oneda and S. Matsuda, in *Fundamental Interactions in Physics, 1973 Coral Gables Conference,* edited by B. Kurşunoğlu and A. Perlmutter (Plenum, New York, 1973), p. 175. See also S. Oneda, H. Umezawa, and S. Matsuda, Phys. Rev. Lett. 25, 71 (1970).

[4]E. Takasugi and S. Oneda, Phys. Rev. Lett. 31, 1274 (1973).

[5]If we find one more constraint, $\rho_s^2 = \omega_s^2$ and $\varphi_s^2 - K_s^2 = K_s^2 - \rho_s^2$ may not be independent. Work along this line is found in Ref. 3.

[6]$A_1 \to \rho\pi$ and $K_A \to K^*\pi$ are not well known. Some information may be found in T. Laankan and S. Oneda, Phys. Rev. D 9, 3098 (1974).

[7]E. Takasugi and S. Oneda, Phys. Rev. Lett. 34, 988 (1975).

[8]$\Gamma(\varphi_c \to \text{all}) = 125 \pm 50$ keV was given by L. Clavelli and T. C. Yang, University of Maryland Technical Report No. 75-046, 1974 (unpublished).

[9]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).

[10]S. Oneda and E. Takasugi, Phys. Rev. D 9, 3113 (1974).