Unphysical Solutions of Yang's Gravitational-Field Equations

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A static, spherically symmetric solution of Yang's vacuum gravitational field equations is given which predicts incorrect values for experimental observations. It is argued that Yang's field equations must be supplemented by further restrictions on the class of allowable space-times.

In a recent paper Yang[1] proposed vacuum gravitational field equations of the form

$$R_{ij;k} - R_{ik;j} = 0, \qquad (1)$$

where R_{ij} is the Ricci tensor and the semicolon denotes covariant differentiation. In a recent note[2] I discussed certain aspects of the static, spherically symmetric solutions of these equations and, in addition, mentioned that seemingly unphysical solutions of (1) had been found.[3] However, these unphysical solutions appear in coordinate systems which do not lend themselves readily to simple physical interpretation. Arguments to accept such solutions as physically meaningful are therefore not particularly strong. In contrast to these solutions, I shall now present a new solution of (1) whose physical interpretation is obvious and whose physical predictions are sufficiently incorrect to necessitate restricting allowable solutions of (1) to certain classes of space-times.

Consider the metric

$$ds^2 = -(1+M/r)^{-2}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + (1+M/r)^{-2}dt^2. \qquad (2)$$

This metric satisfies (1) exactly.[4] It possesses a singularity at $r=0$ and has the proper Newtonian correspondence limit so that we must interpret M as the gravitational mass. In addition the metric is flat at infinity but curved for finite values of r. Thus all reasonable criteria for a physically interesting metric are satisfied. However, the equations of motion for (2) predict no light deflection about a massive body and an anomalous perihelion shift of $\frac{1}{16}$ the Einsteinian value but in the retrograde direction. These predictions are in harsh contrast to observed values and imply that (2) must not be allowed as a physically acceptable solution of (1).

To exclude (2) from the theory implies a restriction upon the permissible class of space-times. It can be shown that (2) is conformally flat and may be transformed into the metric

$$ds^2 = -(1-M/r)^2 \times [dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) - dt^2]. \qquad (3)$$

An examination of the unphysical metrics found by Thompson and Kilmister seems to confirm that conformally flat solutions of (1) have undesirable physical properties. We believe that such solutions cannot be allowed and must be eliminated from the theory by a constraint. However, the elimination of conformally flat solutions does not rid the theory of all unphysical metrics. For example,

$$ds^2 = -(1-2M/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + dt^2 \qquad (4)$$

is another unphysical curved solution I have found. This metric not only violates the astronomical tests but does not possess the proper Newtonian correspondence limit and violates the principle of equivalence.[5] The metric (4) is not conformally flat but satisfies $R=0$. We therefore see that the unphysical solutions are not isolated to the conformally flat set of space-times.

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[4]The fact that (2) satisfies (1) has recently been given by A. H. Thompson, Phys. Rev. Lett. 34, 507 (1975).

[5]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 184.